S-shaped probability weighting and hyperbolic preference reversal - an intimate relationship

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Abstract

This paper [1] argues that intertemporal decision experiments have to take into account the unavoidable uncertainty surrounding the realization of gains and losses in the future. It is shown that an intertemporal state-dependent expected utility model (generating S-shaped probability weighting by considering anticipated emotional reactions to the resolution of uncertainty in addition to expected utility of wealth) is capable to explain various stylized facts. (1) S-shaped probability weighting induces imputed hyperbolic discount rates. (2) The 'sign-effect' (losses are discounted less than gains) follows from the same set of assumptions. (3) The 'magnitude effect' (small outcomes are discounted more than large ones) emerges for losses, but not for gains. Within the domain of comparatively small ('lab-like') gains a 'magnitude effect' can be rationalized by incorporating small anticipated transaction costs. (4) As a corollary, a poor subject will show higher (lower) imputed discount rates for gains (losses) than the rich.

Key words: Hyperbolic time preferences, Decision Making under Uncertainty, Probability Weighting
JEL classification: D11, D81, D91

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Introduction

This paper tries to demonstrate that 'hyperbolic preferences', the 'sign-effect', the 'magnitude effect' (*Frederick, Loewenstein and O'Donoghue, 2002*) and 'S-shaped probability weighting' can be seen as intimately related phenomena, provided uncertainty is accepted as an essential aspect of intertemporal decision making.

While the idea that uncertainty of delayed outcomes should be used as a starting point to explain the present bias in intertemporal decision making is not a new one (c.f. *Green and Myerson, 1996; Souzou, 1998*, *Dasgupta and Maskin, 2005*), the following paper goes beyond these approaches by deducing hyperbolic preferences, preference reversal and some other stylized facts within a common framework of intertemporal state dependent expected utility theory inducing S-shaped probability weighting. The 'sign-effect' (losses are discounted less than gains) follows quite naturally from the same set of assumptions. The 'magnitude effect' (small outcomes are discounted more than large ones) emerges for losses, but not for gains. By incorporating small transaction costs (similar to the 'waiting cost' story presented by *Dasgupta and Maskin, 2002)* something like a 'magnitude effect' appears within the domain of comparatively small ('lab-like') gains, however. The model predicts that poor subjects will have higher discount rates with respect to gains and lower discount rates with respect to losses than rich ones. A key message is that probability weighting and hyperbolic discounting will be positively correlated traits. As a common root for the phenomena observed relative disappointment aversion under conditions of uncertainty is identified.

1 S-shaped probability weighting

S-shaped probability weighting has evolved as a 'stylized fact' from many carefully designed decision experiments under uncertainty (*Gonzalez and Wu, 1999*; *Bleichrodt and Pinto, 2000*; *Harbaugh. et al., 2002*; *Fehr-Duda et al., 2006*). Different parametric versions of this weighting function had been suggested in the literature (*Quiggin, 1982; Gul, 1991*; *Tversky and Kahnemann, 1992*; *Lattimore et al. 1992, Prelec, 1998, Walther,. 2003).* 'Probability weighting' is capable of explaining several anomalies of decision making under uncertainty, like the famous Allais-paradox (*Allais, 1953*) or the 'common-ratio' effect (*Cubitt et al., 1998*). However, this does not resolve the question of why rational subjects depart from standard expected utility of wealth maximization, in particular why they substitute 'wrong' probabilities for the correct

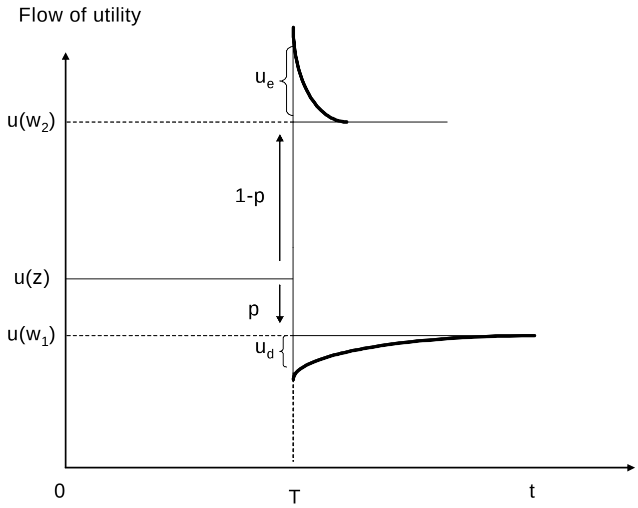

Fig. 1. The binary prospect

ones [2].

The NREU-model (*Walther, 2003*) tried to address this conundrum by deducing (instead of postulating) S-shaped probability weighting from an intertemporal, state-dependent expected utility framework. The basic idea is simple. At the moment of decision the subject anticipates that resolution of uncertainty induces not only a *permanent* flow of utility from wealth effects ($u'(w) > 0, u''(w) < 0$), but also from *temporary and fading* emotional shocks, referred to as 'elation' and 'disappointment'. Fig. 1 illustrates the decision tree under such circumstances for a binary lottery ticket $L = (p, w_1, w_2)$, where p is the probability of the event $w_1 < w_2$. The flows of utility from elation, u_e, and disappointment, u_d, induced by resolution uncertainty at T are anticipated to be temporary and fading [3].

The model generates the following probability weighting function,

[2] See also *Starmer (2000)* for an overview of different non-expected utility theories emerging from 'stylized facts'.

[3] Subjects anticipate that temporary emotional reactions will take place if something 'extraordinary' happens 'unexpectedly', against all odds. ("Such an incredible luck!", "Such a pity!"). This idea is formalized in a simple way in the Appendix.

$$q(p) = p\frac{1 + (1 - p)\mu}{1 + (1 - p)p(\gamma + \mu)} \tag{1}$$

$$\gamma = \frac{\delta\alpha}{\delta + \theta} \tag{2}$$

$$\mu = \frac{\delta\beta}{\delta + \rho} \tag{3}$$

The distortion weights $\gamma \geq 0$ and $\mu \geq 0$ (referred to as the 'elation' and 'disappointment' parameter respectively) are related by definition to the *exponential* rate of time preference, $\delta \geq 0$, the impact parameters $\alpha \geq 0$ and $\beta \geq 0$ (scaling the (dis-)utility effects of anticipated emotional shocks at the moment of resolution), and the exponential rates of 'depreciation' for the utility flows from 'elation', $\theta \geq 0$, and 'disappointment', $\rho \geq 0$. The last two parameters serve to specify the relative 'emotional stability' of the subject, which might be different with respect to elation and disappointment.

The intuitive message behind this weighting mechanism is simple: The 'cool investor with the long-term view' behaves like a pure expected utility of wealth maximizer ($q(p) = p$), while disappointment and/or elation sensitive subjects systematically depart from this reference model.

Fig. 2 illustrates[4] the S-shaped distortion of probability weights for the case of $\mu > \gamma$, which is caused by anticipated longer-lasting disappointment ($\rho < \theta$). As can be seen, a higher rate of time preference, δ, increases the weight of disappointment (μ), *and* elation (γ). Short-run, fading emotions get more important relative to long-run wealth effects for this subject. Note that the point of intersection with the 45°-line, where emotional distortions are just neutralized, is also shifting downwards. Within the 'disappointment interval' ($0 < p < \mu/(\mu + \gamma)$), the weighted probability of loss, $q(p)$, is higher than the true probability. Within the 'elation interval' ($\mu/(\mu + \gamma) < p < 1$) the weighted probability of gain is higher than the true probability.

One might ask, whether this type of probability weighting could be supported by some evolutionary arguments, similar in spirit to the route pursued by *Robson and Samuelson (2007)* in their attempt to explain the evolution of intertemporal preferences. A simple argument - relevant for a 'hunter' economy perhaps - might be the following one: As high gains and high losses have to be regularly shared between related individuals (e.g. parents and offspring), while small gains and small losses can (and will) be borne individually, it seems to be evolutionary advantageous, if individual decision making under uncertainty takes into account those externalities. Probability weighting acts precisely in that direction, as anticipated emotions penalize (subsidize) for given expected values of wealth negatively (positively) skewed outcomes relative to the pure

[4] Maple 11.0 was used to produce the numerical simulations and plots presented below.

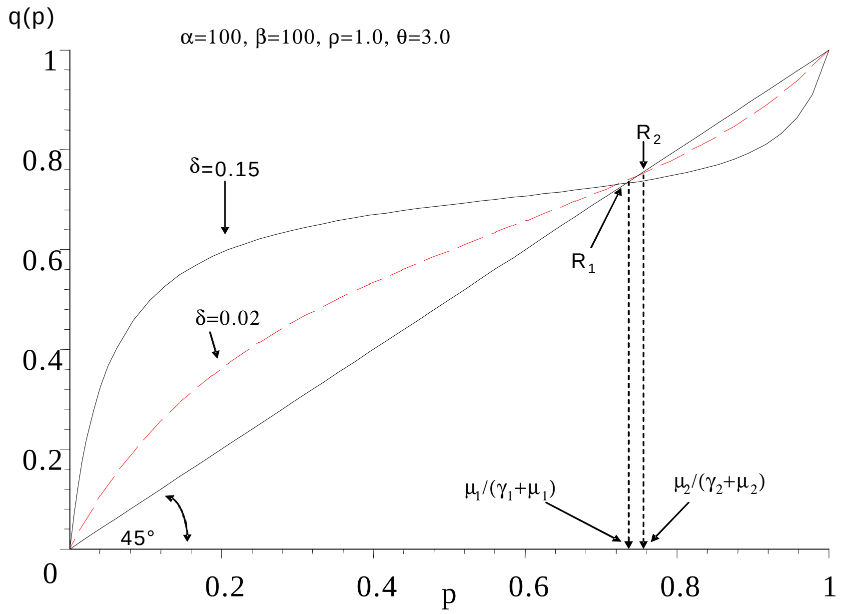

Fig. 2. Probability weighting

expected utility of wealth maximizing behavior. One might therefore argue that probability weights add a subtle internalization of externalities to individual risk aversion [5].

2 Hyperbolic preferences and S-shaped probability weighting

Similar to inverse S-shaped probability weighting, the hypothesis of 'hyperbolic time preferences' was stimulated by the development of experimental economics (*Ainsly, 1992*; *Harvey, 1986*; *Laibson, 1997*; *Loewenstein and Thaler, 1989*; *Frederick, Loewenstein and Prelec, 2002*; *Anegeletos et al. 2001*; *Harris and Laibson, 2001*). The stylized facts emerging from intertemporal decision experiments are (1) a time-dependent discount rate, where the immediate future is discounted more heavily than the more distant future [6], (2) the possible appearance of a 'preference reversal' phenomenon, (3) a sign-effect (losses are discounted at lower rates than gains), (4) a 'magnitude effect' (large outcomes are discounted less than smaller ones) and (5) poor subjects are more impatient with regard to gains (*Green et al., 1996 b*).

[5] This hypothesis might be investigated by biologists and economists alike looking at different species (sexes) caring in different ways for their offspring and sharing their gains and losses in various ways.

[6] *Read (2003)* presents various possible formalizations of hyperbolic discounting. Those models assume the subjective discount function to be a generalized hyperbola $\Phi(t) = (1 + \alpha t)^{-\frac{\beta}{\alpha}}$ with $\alpha, \beta > 0$.

So far, the various attempts to model such a behavior by introducing time dependent uncertainty with regard to realization (*Souzou, 1998, Bommier (2006)*) or realization time (*Dasgupta and Maskin, 2005)* seem to have failed to explain these various observations within a unique framework.

Souzou, (1998) and (in a similar spirit) *Robson and Samuelson (2007)* show that a present bias can be evolutionary advantageous if future consumption may disappear before it can be realized. They do not offer an explanation for the 'preference reversal', the 'sign effect' or the 'magnitude effect', however. *Dasgupta and Maskin (2005)* try to explain preference reversal within a model, where the realization time of (certain) payoffs is uncertain. Under specific assumptions with respect to the probability densities of realization over time they are able to explain (dynamically consistent) preference reversal.

Rubinstein's (2001) approach is very different from others (and our's). He is pointing to some perception bias (caused by the subjects attempt to reduce complexity via identifying 'similarity'). While I do not doubt that such a bias might be present in many different settings, it remains to be seen whether this concept is capable of explaining the wide variety of stylized facts presented above.

In the following it will be shown that the appearance of hyperbolic preferences in intertemporal decision experiments (in spite of 'true' exponential discounting) can be easily explained by the interaction of two factors: Time dependent uncertainty with regard to the fulfillment of intertemporal contracts as a necessary, and S-shaped probability weighting induced by relative disappointment aversion as a sufficient condition.

Why is residual uncertainty of contracts accompanying delayed payments unavoidable, even if careful experimental design tries to eliminate it? Suspected ex post opportunism of the experimenter might be one, but hopefully not the most important reason. However, even if the check is trustworthy, it might get lost during the resolution period or the subject might anticipate the risk of being unable to cash it - perhaps due to uncertainty of lifetime as such[7].

Let us now construct a simple intertemporal decision experiment designed for a subject characterized by the particular type of 'emotional' probability

[7] *Bommier (2006)* proved that the assumption of hyperbolic absolute risk aversion with regard to length of life is sufficient for explaining hyperbolic discounting. The presence of pure time preference is not a necessary assumption. It is doubtful, however, that such a view is capable of explaining available experimental evidence for young probands (students) and comparatively short resolution lags relative to residual lifetime. Furthermore, one strange implication of his model is that - for exogenously given mortality patterns - rich subjects will be more impatient than poor subjects with regard to gains.

weighting presented above (and shortly summarized in the Appendix).

The proband, endowed with initial wealth z, has a well-behaved utility of wealth function, $u'(w) > 0, u''(w) < 0$. The experimenter asks for the preference ordering of $(z + g_{T+\tau}, T + \tau)$ vis-a-vis $(z + g_T, T)$, given $g_{T+\tau} > g_T$. Let us assume that at each point of time the expected probability of contract survival, $\lambda > 0$, is constant and the survival events are independently distributed over time. Therefore, the ex ante probability of a contract break down, p, will be an increasing function of the length of the contract period $T + \tau$.

$$p = 1 - \lambda^{T+\tau} \tag{4}$$

The discounted flow of utility from the certainty equivalent wealth, s, of a prospect resolving at $T + \tau$ will be

$$U(s_{T+\tau}) = \int_0^{T+\tau} e^{-t\delta} u(z)\, dt + \int_{T+\tau}^{\infty} e^{-t\delta} \left(q(p)u(z) + (1 - q(p))u(z + g_{T+\tau})\right) dt \tag{5}$$

Let us assume that an offer g_T is fixed by the experimenter. Then we can use equations (6) to determine the value $\widetilde{g}_{T+\tau}$, which has to be offered to make the subject just indifferent between getting g_T at resolution lag T and $\widetilde{g}_{T+\tau}$ at time $T + \tau$. Corresponding to the observed values of $\widetilde{g}_{T+\tau}$ and g_T an imputed rate of time preference, ξ, can be defined via equation (7).

$$U(s_{T+\tau}) = U(s_T) \tag{6}$$

$$g_T = e^{-\xi\tau} \widetilde{g}_{T+\tau} \tag{7}$$

For simplicity's sake, let $u(w) = \ln w$. The parameters of the probability weighting function are the same as in fig. 2 ($\delta = 0.15, \alpha = 100, \beta = 100, \rho = 1.0, \theta = 3.0$). Setting arbitrarily $\tau = 1$, $g_T = \$\ 3000$, $\lambda = 0.95$, we can plot an implicit relationship between the 'imputed rate of time preference', ξ, and the resolution lag T. Fig. 3 illustrates this relationship for three different levels of wealth. For the rich subject ($z = 10^5$), the imputed rate starts at approximately 60 %, falls dramatically within a few periods and approaches the lower limit given by the 'true' exponential discount rate. This is obviously very similar to the pattern revealed in experimental settings.

Interestingly, due to our assumption of declining marginal utility of wealth (very) poor subjects suffer from (much) higher imputed discount rates relative to the same absolute gain than the rich, in spite of being equipped with the same fundamental preferences. This result is in full accordance with empirical findings by *Green et al., 1996.*

The essential role of uncertainty is illustrated in fig. 4. The hyperbolic function becomes flatter as the confidence in contract fulfillment increases.

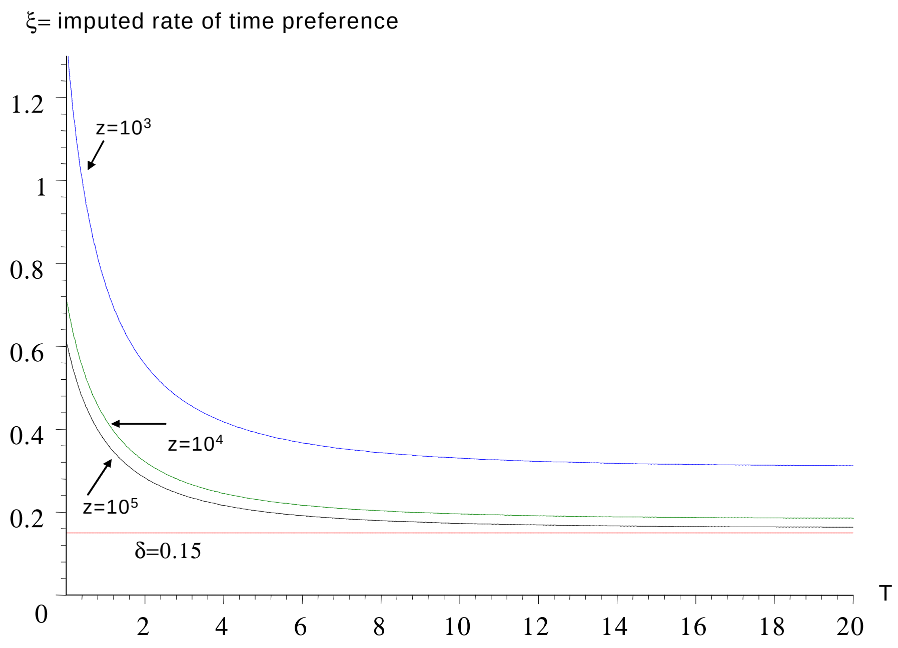

Fig. 3. The hyperbolic rate of time preference

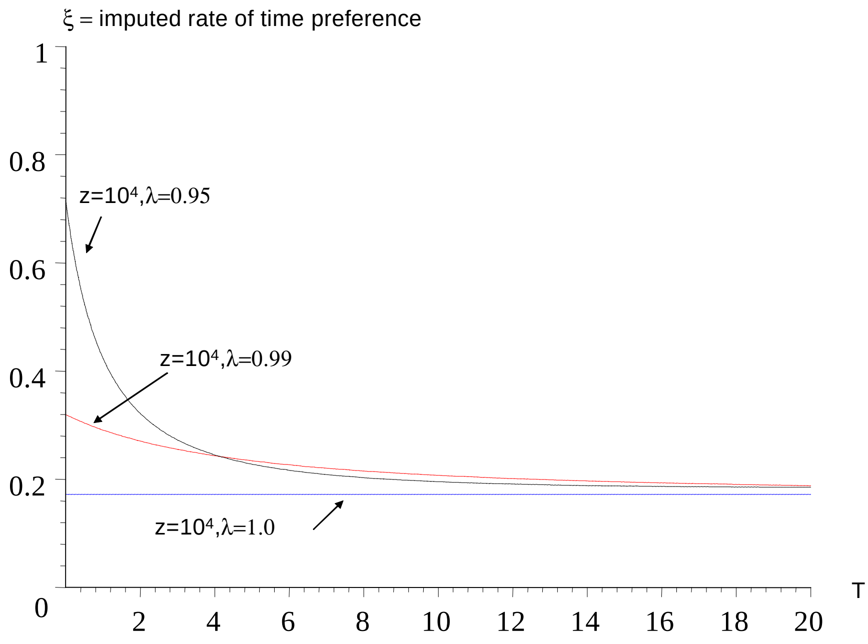

Fig. 4. Uncertainty and hyperbolic preferences

The essential role of disappointment aversion is illustrated in fig. 5. This figure compares the reactions of two subjects, differing only by their ability to master the temporary experience of disappointment.

The presented model makes a clear-cut, empirically testable prediction: The

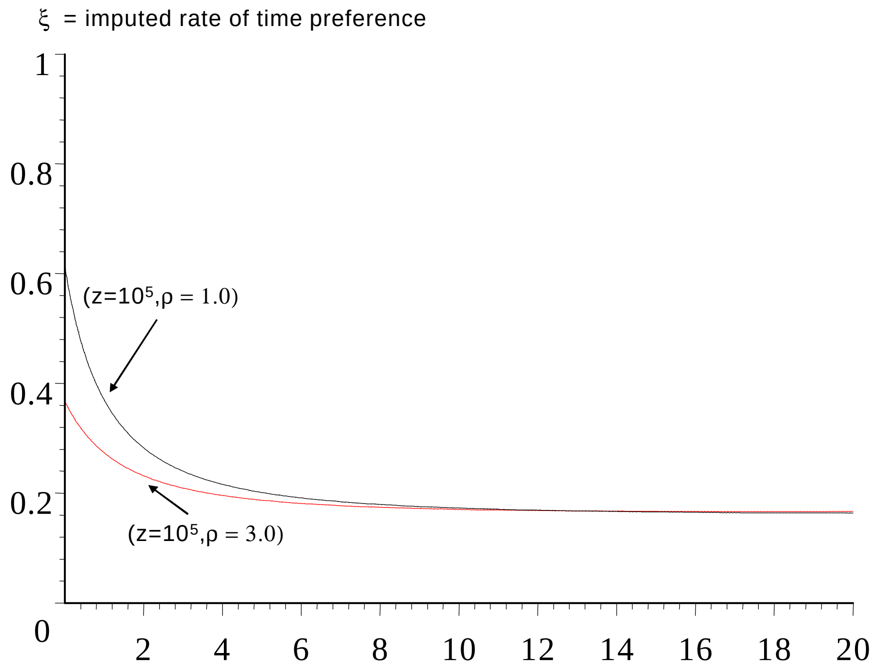

Fig. 5. Relative disappointment aversion differs

same type of subjects departing from pure expected utility of wealth maximizing behavior by showing S-shaped probability weighting will also reveal time dependent discount rates and hyperbolic discounting.

3 The "Sign Effect" (gains are discounted more heavily than losses)

Let us investigate, how the subject acts when a trade off is offered of a loss now against a loss in the future. For instance, in *Thaler's (1981)* pioneering study he asked subjects to imagine they had received a traffic ticket that could be paid now or later. Thaler and many other others (e.g. *Loewenstein, 1987*) have found that the discount rates for losses are systematically lower than for gains. As will be shown, those observations fit perfectly well into the presented model.

Let us assume again, that a small chance exists that the intertemporal contract (i.e. bearing the loss in the future) fails to get realized. Referring to Thaler's example, the subject might hope to evade the fine by bureaucratic negligence, by moving to another state and so on. Therefore, we will assume that the probability of contract fulfillment at resolution time, $T + \tau$, is equal to

$$p = \lambda^{T+\tau} \tag{8}$$

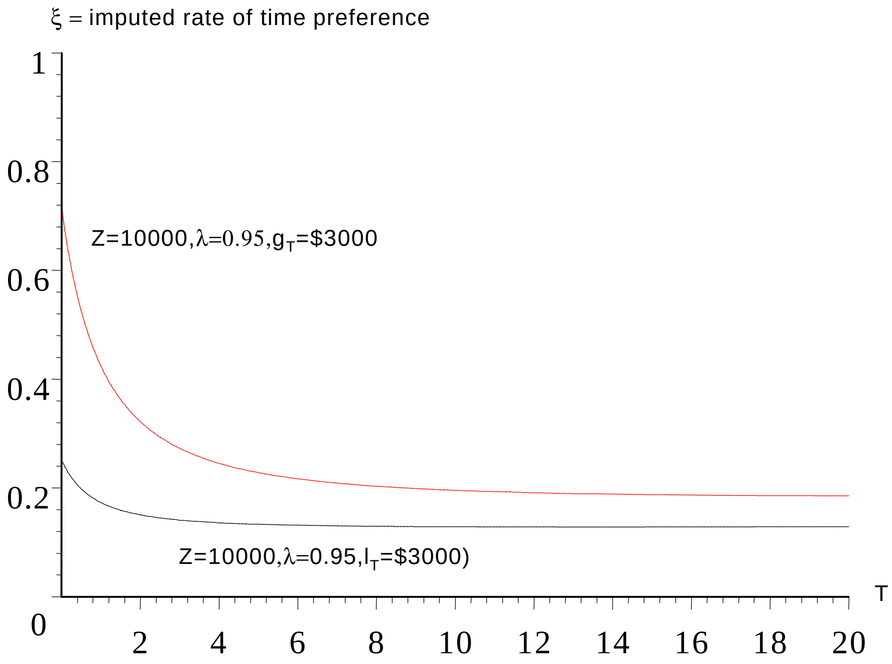

Fig. 6. The 'sign-effect'

To be indifferent between a future loss, $l_{T+\tau}$ and an earlier loss l_T, the following condition must be fulfilled.

$$\int_0^{T+\tau} e^{-t\delta} u(z)\, dt + \int_{T+\tau}^{\infty} e^{-t\delta} \left(q(\lambda^{T+\tau}) u(z - \tilde{l}_{T+\tau}) + (1 - q(\lambda^{T+\tau})) u(z) \right) dt = \left(\int_0^{T} e^{-t\delta} u(z)\, dt + \int_{T}^{\infty} e^{-t\delta} \left(q(\lambda^{T}) u(z - l_T) + (1 - q(\lambda^{T})) u(z) \right) dt \right) \quad (9)$$

By eliminating $\tilde{l}_{T+\tau}$ from equation (9) and using the definitional equation for the imputed discount rate

$$l_T = e^{-\xi\tau} \tilde{l}_{T+\tau} \quad (10)$$

an implicit relationship between ξ and T can be found. Let us take a look at this relationship for the same parameter values as above to compare the imputed discount rates for losses ($l_T = \$3000$) and for gains ($g_T = \$3000$).

The result is shown in fig. 6 and seems to be in accordance with empirical observations: imputed rates of time preference are lower for losses than for gains. Intuitively, relative disappointment aversion increases imputed discount rates for gains, while it is virtually absent for losses, as the future gains/losses will happen with high probability anyway. Note that experiments based on losses seem to give somewhat more reliable figures for the 'true', exponential rate of time preference.

Let us compare the subject portrayed in fig. 6 with another subject, ten times as rich. If both subjects get offered the same absolute amount of gain or loss, the rich subject shows lower imputed rates of time preference than the

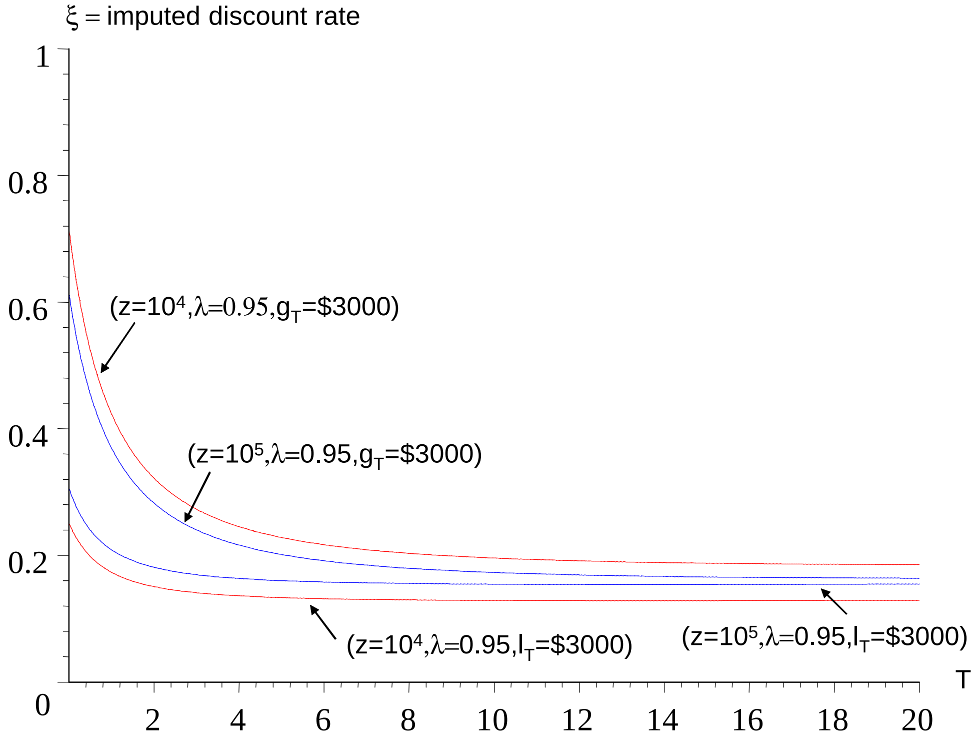

Fig. 7. Imputed discount rates for gains and losses

poor with regard to gains, but higher imputed rates with regard to losses. Note however, that in spite of the lower imputed rate of time preference, the maximum additional loss the poor subject is willing to accept *relative* to it's wealth $(\widetilde{l}_{T+\tau} - l_T)/z$) for getting a delay is much higher nevertheless[8]. As can be shown, for our particular case of logarithmic utility of wealth, the poor and the rich will show the same imputed discount rates if the *ratio of gains and/or losses relative to wealth* is equal for both.

4 The "Magnitude Effect" (small outcomes are discounted more than large ones)

The clear-cut prediction that 'poor subjects' will reveal higher rates of discounting relative to gains is equivalent to the prediction that 'small' gains (relative to present wealth) should be discounted *less* than 'large' ones. However, many studies have found, quite to the contrary, that 'large' gains are discounted at a lower rate (e.g. *Thaler, 1981*; *Kirby, Petry and Bickel, 1999*; *Loewenstein, 1987*). Is it possible to reconcile these observations with the

[8] Let us set $T = 0$, $\tau = 1$. The poor (rich) subject prefers to delay the loss as long as the additional loss induced does not exceed 5.7 % (0.7 %) of its wealth.

implications of our model?

The answer is a tentative 'yes', if anticipated transaction costs of accepting delayed instead of immediate payments are taken into account (e.g. efforts of 'mental bookkeeping' and organizing the payment procedures). While admittedly somewhat ad hoc, a very similar argument was presented by *Dasgupta and Maskin (2002)*.

Let us simulate Thaler's (1981) pioneering study with the help of our model and the transaction cost argument. In Thaler's study respondents were, on average, indifferent between \$15 immediately and \$60 in a year, \$250 immediately and \$350 in a year, and \$3000 immediately and \$4000 in a year, implying discount rates of 139 percent, 34 percent, and 29 percent respectively.

The behavioral parameters of the model are postulated to be the same as above. The subject considered is assumed to be rich ($z = 10^5$) relative to the promised payments[9]. It might be plausible that the anticipated costs of 'mental bookkeeping' are negatively related to human capital endowment (and therefore also to wealth), while the opportunity costs of the time used to cash the check might be positively related. For the sake of simplicity, let us assume that the transaction costs, k, associated with a delayed contract are fixed.

In terms of our model, Thaler's experiment implies ($T = 0, \tau = 1$). Assuming the same parameter values as above (and setting $k = 20$), we have to solve equations (11) and (12) for the variables g_1 and ξ.

$$\int_0^1 e^{-t\delta}u\left(z-k\right)dt + \int_1^\infty e^{-t\delta}\left(q(1-\lambda)u(z-k) + (1-q(1-\lambda))u(z-k+g_1)\right)dt = \int_0^\infty e^{-t\delta}u(z+g_0)dt \quad (11)$$

$$g_0 = e^{-\xi}g_1 \quad (12)$$

The subject will be indifferent between \$15 immediately and \$46 in a year, \$250 immediately and \$355 in a year, and \$3000 immediately and \$3998 in a year, implying observed discount rates of 112 percent, 35 percent and 29 percent - very similar to the pattern observed in Thaler's setting. However, at higher amounts of money offered ($g_0 = 20000$) the imputed discount rate rises again (up to 31 percent). The reason is simple: For high amounts of money at risk disappointment aversion gets more weight than the small transaction costs involved. Anticipated transaction costs of implementing the intertemporal contract might dramatically increase observed imputed discount rates for small (lab-like) gains.

Let us again compare a poor ($z = 10^3$) and a rich ($z = 10^5$) subject. Prefer-

[9] Relative to the potential outcomes in real experiments (were actual payments are offered in the lab) all probands are probably very rich.

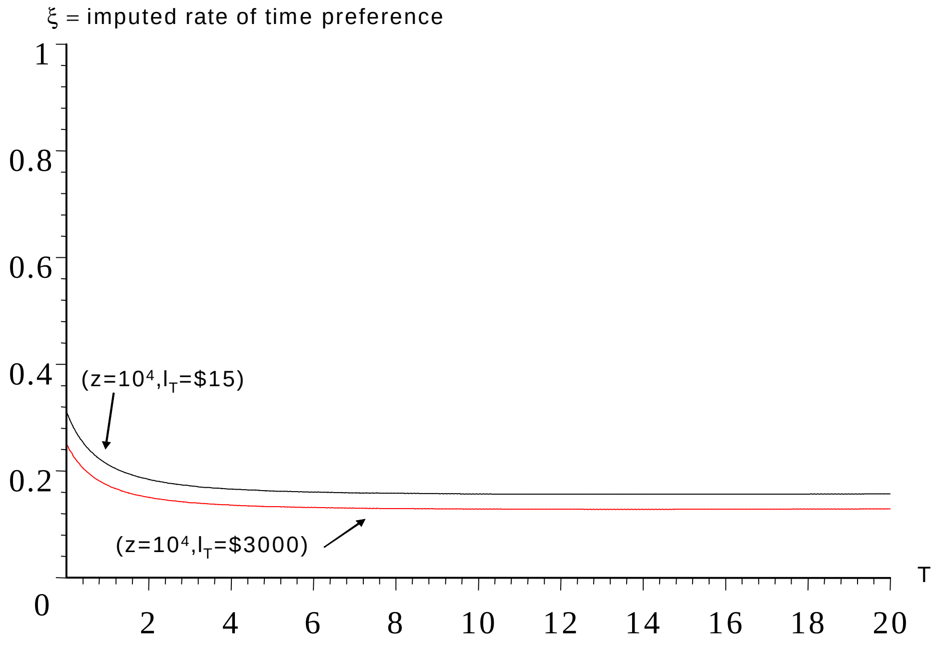

Fig. 8. The magnitude effect with respect to losses

ences and transaction costs are assumed to be equal, as are the alternatives offered. Imputed discount rates of the poor subject (128 percent, 39 percent and 56 percent) are higher than the rich one's, illustrating again the U-shaped nature of the discounting pattern relative to the size of the gains if fixed transaction costs are present. Generally speaking, the model suggests that the 'magnitude effect' appearing in the lab (relative to gains) should not be generalized prematurely to large gains, as payments offered in the lab are presumably small relative to the subject's total wealth.

With regard to losses the 'magnitude effect' can be predicted without any further reference to transaction costs: Large losses are discounted at lower imputed rates than smaller losses. Fig. 8 illustrates this result.

5 Preference reversal for gains and losses

Preference reversal appears within our framework quite naturally. Fig. 9 shows for a rich and a poor person both equipped with the same preference parameters (as above) the net utility gain for waiting at time T one further period to get g_{T+1} instead of g_T. (The gain $g_{T+1}$was calculated to induce indifference between g_{T+1} and g_T for the rich person at $T = 1$). The model predicts that for equal absolute gains the poor subject's present bias will be more pronounced, as can be seen. Let us take a look at the case of delaying the loss under similar circumstances. Fig. 10 shows that the rich person is somewhat more willing to

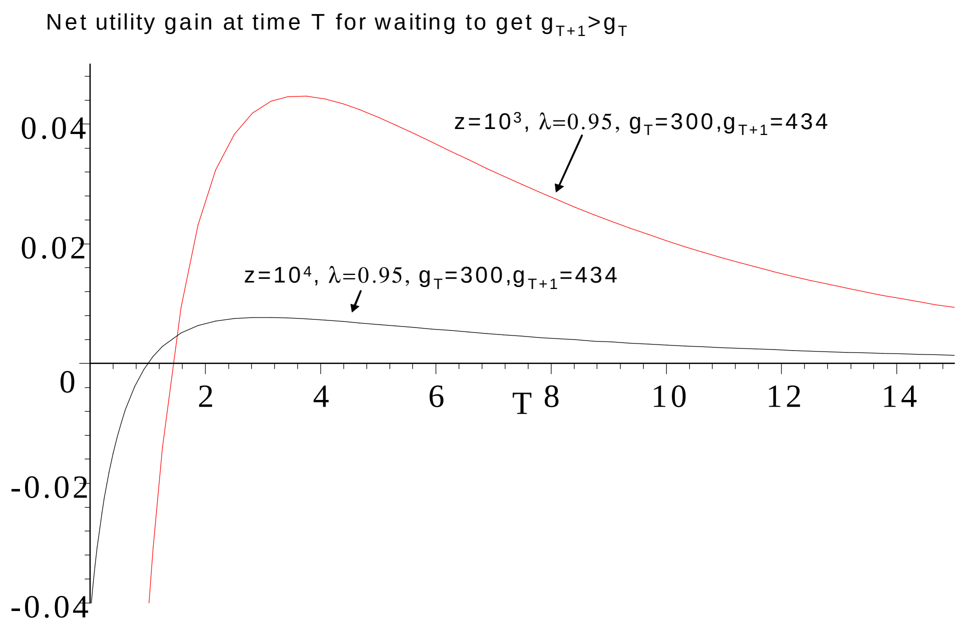

Fig. 9. The impatient poor

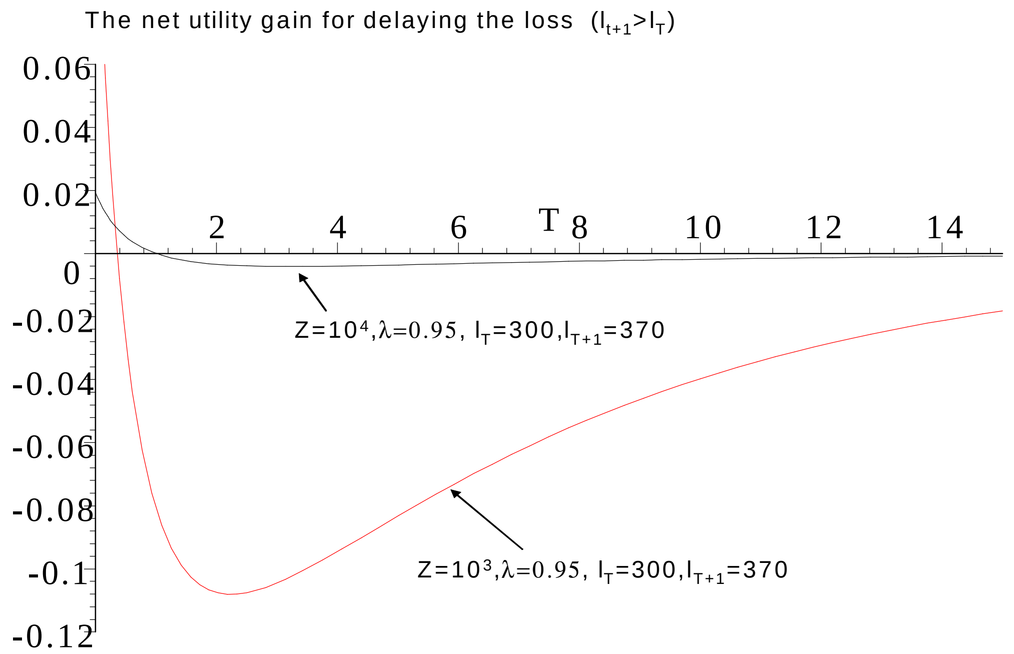

Fig. 10. The impatient poor again

shift the loss one period ahead than the poor. If losses/gains relative to wealth are the same for both subjects, these behavioral differences will disappear.

6 Summary

The present paper argues that S-shaped probability weighting (caused by disappointment aversion and elation loving behavior) and observed hyperbolic anomalies in intertemporal decision making are intimately related phenomena: Subjects revealing S-shaped probability weighting will also be more inclined to discount hyperbolically. Hyperbolic preferences appear if (1) the future realization of gains and losses is recognized as slightly uncertain by probands and (2) the subjective probability of non-realization rises with the length of the resolution lag. Furthermore, the same set of assumptions implies quite naturally the famous 'sign-effect' (gains are discounted more than losses). While the 'magnitude effect' is definitely present with regard to losses (larger losses are discounted at lower rates than smaller ones), the model predicts that large gains will be discounted at a *higher* rate than small gains. A corollary to this prediction is that poor subjects will show higher (lower) imputed discount rates for gains (losses) than rich ones. The 'magnitude effect' for gains observed in experimental settings can be rationalized, however, by restricting the domain of outcomes to small ('lab-like') ones burdened by small transaction costs. It is argued that the common cause behind all those phenomena seems to be emotional probability weighting under conditions of uncertainty.

Appendix

Emotional reactions are anticipated to arise if and only if the flow of utility of wealth, $u(w)$, happens to fall outside the barriers of $|u(s)+\sigma^-, u(s)+\sigma^+|$, where s is the certainty equivalent wealth and σ^+, σ^- are expected standard deviations of utility (see below for a formal definition). Only 'exceptional' ex post realizations will induce emotional reactions ('such an incredible luck!').

Let $u(w)$ be the instantaneous flow of (well-behaved) utility from wealth ($u'(w) > 0, u''(w) < 0, u'(0) = +\infty, u'(+\infty) = 0$). A binary lottery ticket, $L = (p, w_1, w_2)$, is offered[10], where $w_1 < w_2$ realizes with probability p. The moment of decision and the moment at which uncertainty is resolved are both set equal to be zero. Let θ be the exponential speed of adjustment towards a neutral state of 'emotional equilibrium' after a euphoric shock. The instantaneous flow of utility from elation, u_e, is defined as

$$u_e = e^{-\theta t}\alpha[u(w_2) - (u(s) + \sigma^+)] \tag{13}$$

$$\sigma^+ = (1-p)(u(w_2) - u(s)) \tag{14}$$

[10] For different methods of generalization to multivalued and continuous prospects, see *Walther (2003).*

where α reflects the 'impact' effect of an emotional shock. Similarly, the flow of utility from disappointment, u_d, can be defined

$$u_d = e^{-\rho t}\beta[u(w_1) - (u(s) + \sigma^-)] \tag{15}$$
$$\sigma^- = p(u(w_1) - u(s)) \tag{16}$$

where β reflects the 'impact' effect and ρ the rate of decay. Note that $u_e > 0$ and $u_d > 0$ is always fulfilled for our binary prospect and $0 < p < 1$. Let δ be the rate of time preference. Substituting (14) into (13) and (16) into (15), the discounted flow of expected utility of wealth, expected elation and disappointment becomes

$$\begin{aligned} E(U) = & \int_0^{+\infty} e^{-\delta t}\left[pu(w_1) + (1-p)u(w_2)\right] dt \\ & + (1-p)\int_0^{+\infty} e^{-\delta t}\left[e^{-\theta t}\alpha p(u(w_2) - u(s))\right] dt \\ & + p\int_0^{+\infty} e^{-\delta t}\left[e^{-\rho t}\beta(1-p)(u(w_1) - u(s))\right] dt \end{aligned} \tag{17}$$

Let $U(s)$ be equal to the discounted flow of utility from the certainty equivalent wealth, s.

$$U(s) = \int_0^{+\infty} e^{-\delta t}u(s)dt = \frac{u(s)}{\delta} \tag{18}$$

If we set the r.h.s. of (18) equal to the r.h.s. of (17) and solve for $u(s)$, the following results:

$$u(s) = q(p)u(w_1) + (1 - q(p))u(w_2) \tag{19}$$
$$q(p) \equiv p\frac{1 + (1-p)\mu}{1 + (1-p)p(\gamma + \mu)} \tag{20}$$
$$\gamma \equiv \frac{\delta\alpha}{\delta + \theta} \tag{21}$$
$$\mu \equiv \frac{\delta\beta}{\delta + \rho} \tag{22}$$

Note that probability weights are additive and preferences monotonic.

The simplest way to apply this approach to multivalued and/or continuous prospects is the 'dual decision hypothesis' presented in Walther (2003). In a first step, a binary ('standard') prospect is identified inducing the same pure expected utility of wealth like the multivalued or continuous prospect. In a second step, the probabilities of the binary standard prospect are transformed by the weighting procedure just presented. Alternative approaches are discussed in Walther (2003).

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