ECONOMICS TRIPOS Part I

Friday 11 June 2004   9 – 12

Paper 3

Quantitative Methods in Economics

*This exam comprises **four** sections. Sections A and B are on **Mathematics**; Sections C and D are on **Statistics**. You should do the appropriate number of questions from each section. The number of questions to be attempted is given at the beginning of each section.*

*Answers from the **Mathematics** and the **Statistics** Sections must be tied up in separate bundles, with the letter of the Section written on **each** cover sheet.*

This written exam carries 80% of the marks for Paper 3. Section A carries 24% of the marks, Section B carries 16% of the marks, Section C carries 24% of the marks and Section D carries 16% of the marks.

You are permitted to use your own calculator where it has been stamped as approved by the University.

Cambridge Elementary Statistical Tables, graph paper, and a list of statistical formulae are provided.

You may not start to read the questions printed in the subsequent pages of this question paper until instructed to do so by the invigilator

SECTION A – MATHEMATICS

Answer *four* questions

1 Consider the following macroeconomic model: Y and r denote output and the interest rate respectively. The consumption function is $C=0.5Y$, the investment function $I = 10 - 10r$, the money demand function is $M^d = 10 - 40r$ and the money supply is M^s.

(a) Compute the IS and LM curves of this economy. What is the equilibrium value of Y?

(b) Compute the effect on Y and r of a marginal increase in M^s, and show graphically the change of equilibrium from M_0^s to $M_1^s > M_0^s$.

2 Suppose that a firm, producing output Y from labour L and capital K according to the technology $Y = \left(K^\alpha + L^\alpha\right)^{1/\alpha} (\alpha > 0)$, wishes to minimise its costs given a production target $\bar{Y}$. The wage rate is w and the rental rate of capital is r.

(a) Derive the optimality conditions using the Lagrange method.

(b) Show that the demand functions of the firm are:

$$L^D = \frac{\bar{Y}}{\left(1 + \left(\frac{w}{r}\right)^{\alpha/(1-\alpha)}\right)^{1/\alpha}}, \quad K^D = \frac{\bar{Y}}{\left(1 + \left(\frac{r}{w}\right)^{\alpha/(1-\alpha)}\right)^{1/\alpha}}$$

3 Find expressions for the derivatives *dy/dx* of:

(a) (i)

$$y = \frac{\left(4x^2 - 2a + 3b\right)^2}{4x}$$

(ii)

$$y = \ln\left(\frac{e^x + 1}{x}\right)$$

And evaluate them at the point $x = 3, a = 1, b = -1$.

(b) Find the partial derivatives with respect to *x* and *y* of the function:

$$f(x, y) = \left(x^4 - 3xy + 2y\right)\left(2x - y\right)^2$$

4 Let:

$$A = \begin{bmatrix} 1 & 3 \\ 3 & 8 \end{bmatrix}, B = \begin{bmatrix} x \\ y \end{bmatrix}, C = \begin{bmatrix} 2 \\ 4 \end{bmatrix}$$

(a) Solve the system of equations *AB*=*C* for *x* and *y*.

Which of the following are well defined? Evaluate wherever possible.

(b) *AB*
(c) *BC*
(d) *CA*
(e) C^{-1}
(f) A^{-1}

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5 The level of per capita income in the economy at time t follows the differential equation:

$$\frac{dy}{dt} = 0.03y - 1.5$$

The value of income at time 0 is 60.

Derive a solution to this differential equation which gives the value of y as a function of time.

6 Production in the economy is given by the production function $Y = \sqrt{K^2 + L^2}$

(a) Show that this function is homogenous of degree 1.

(b) Show that this function satisfies Euler's theorem.

SECTION B - MATHEMATICS

Answer *one* question

1 A consumer has a utility function given by $U(x_1, x_2) = x_1 x_2 + 2x_1 + 5x_2$. The consumer has only $a>0$ units of good 1. The price of good i (i =1,2) is denoted p_i.

(a) Write down the budget constraint of the consumer, and derive the first order condition of this optimisation problem using the Lagrange method.

(b) Compute the optimal quantities x_1^* and x_2^* that the individual consumes.

(c) How much of good 1 does the consumer sell on the market? Check that the value of what he sells is equal to the value of what he buys.

(d) Setting $p_1 = 1$, compute the effect on x_1^*, x_2^* and $U(x_1^*, x_2^*)$ of a marginal increase in a.

2 Demand and supply in the market for potatoes are given by:

$$q_t^d = 50 - p_t$$
$$q_t^s = 0.5 p_{t-1} - 10$$

Where q_t^d, q_t^s and p_t denote quantity demanded, quantity supplied and the price in year t, respectively.

(a) Find the equilibrium price, p^*, such that if $p_{t-1} = p^*$, then $p_t = p^*$. What is the equilibrium quantity sold?

(b) Derive the difference equation and solve for the value of p as a function of time and the initial price p_0.

(c) Is p^* a stable equilibrium? Justify your answer mathematically.

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SECTION C – STATISTICS

Answer *four* questions

1 (a) Bag A contains 3 red balls and 2 blue balls. Bag B contains 2 red balls and 8 blue balls. A fair coin is tossed. If the coin turns up heads a ball is taken from bag A, while it is taken from bag B if the coin turns up tails. What is the probability of getting a red ball?

(b) For the problem in part (a), if a red ball is chosen, what is the probability that it was drawn from bag A?

2 (a) Under what conditions is the binomial distribution well approximated by the normal distribution?

(b) A University offers places to 20% of the students that apply to study there. A particular school sends 50 students for interview at this University. What is the probability that between 26% and 30% of these applicants will receive an offer of a place?

3 A travel company is making a study of whether holidaymakers from the South of England spend more per day while abroad than do holidaymakers from the North of England. In a sample of 11 tourists from the South of England, the average amount spent per day while on holiday is £43 with a standard deviation of £16. In a sample of 19 tourists from the North, the average amount spent per day while on holiday is £33 with a standard deviation of £11.

(a) Assess statistically (at the 5% level of significance) the hypothesis that tourists from the South of England spend more than do tourists from the North.

(b) Test (at the 5% level of significance) any assumptions about variances that you have made in answering (a).

(c) Comment upon the analysis and your results.

4 'It is generally agreed that the direction of causality between two variables cannot be proven using regression analysis, even when the results are statistically significant. In this case, it does not really matter which variable is regressed on which.' Discuss.

5 (a) What is the *expected value* or *mathematical expectation* of a random variable?

(b) Consider the probability distribution of the sum of the numbers obtained in a throw of two dice. Calculate the expected value for the sum of the two numbers. How does this change if it is discovered that the dice are biased and that, for both, the probability of getting a 6 is 2/7 while the probability of getting each of the other scores is 1/7?

6 An investigator is interested in finding out whether the proportion of people who do their shopping by bicycle differs between customers of Pescos, a City centre supermarket, and Lawsmart, an out-of-town supermarket. A survey of 150 Pescos shoppers found that 58 had cycled, while of the 120 Lawsmart shoppers sampled 28 had cycled. Using these results, test the hypothesis that the proportion of people who cycle to each shop is the same.

7 Two machines produce components with an identical mean size. However, the standard deviations of the size of the components are 40mm for the first machine and 60mm for the second machine. One unlabelled box of components is found. In order to find out which machine produced them, 12 components are taken from this box and the standard deviation is found to be 50mm. Test the hypothesis that the components in the unlabelled box were produced by the first machine.

What difference does it make to your results if the null hypothesis is rather that the unlabelled components were produced by the second machine?

What do your results suggest about the magnitude of the Type II error?

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SECTION D

Answer *one* question

1 (a) In the UK, the interquartile ratios for before tax and after tax income have been calculated to be 92 and 80, respectively, for the tax year 1979/80, and 103 and 88, respectively, for the tax year 1996/97. What do these ratios suggest about the changes in the distribution of personal incomes that took place in the UK between 1979/80 and 1996/97?

(b) Using the data given below, calculate interquartile ratios for both before tax and after tax income for the year 2003/04.

Distribution of total income before and after tax 2003/04

2003/04 Range of Income £ (lower limit)	No. of taxpayers % Before tax	After tax
4195	2.2	2.5
5000	11.5	13.2
7000	21.4	25.2
10000	24.3	26.6
15000	15.4	14.7
20000	23.0	16.8
50000	2.2	1.0

Source: Inland Revenue

(c) Using your results from parts (a) and (b), compare the changes in the distribution of personal incomes that occurred over the two periods.

(d) For before tax income only, convert your data to relative frequencies and calculate the mode. Using this and your working for (b), comment on the skewness of the distribution.

2 Ten MPhil students' grades were collected to assess how well two papers were taught.

Student	Score on Paper 1	Score on Paper 2
1	84	70
2	47	40
3	56	54
4	61	53
5	40	34
6	92	76
7	69	60
8	75	70
9	63	58
10	45	84

(a) Initially it is believed that a student's performance on one paper is not related to his or her performance on the other paper. Assuming this to be correct:

(i) test the hypothesis that the mean score is the same for both papers.

(ii) test the hypothesis that the proportion getting 60% or above (the pass mark for each paper) is the same.

(b) Calculate the correlation coefficient between the two sets of results and use this to assess the assumption made in part (a), i.e. that test scores are independent.

(c) It is now discovered that student 10 differs from all other students in that she has not done economics before. Repeat part (b) excluding student 10's scores and comment on the claim that the results in part (b) are not representative.

(d) What implications do your answers to parts (b) and (c) carry for the test results obtained in part (a)?

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3 An investigator collects the following data on household income and expenditure:

Household	Weekly Expenditure (£)	Weekly Income (£)
1	55	80
2	65	100
3	74	105
4	90	125
5	98	130
6	102	140
7	115	180
8	130	185
9	144	225
10	145	240
11	192	245
12	220	265

(a) Using these data, estimate the parameters of a consumption function of the following form

$$Y_i = \alpha + \beta X_i + \varepsilon_i$$

where

Y_i is weekly household expenditure in pounds
X_i is weekly household income in pounds

(b) Interpret your results.

(c) On the basis of your results, estimate the weekly income of a household that spent £100 a week.

(d) Using your results, estimate a 95% confidence interval for β.

(e) Using your estimates of α and β, calculate the residuals, $\hat{\varepsilon}_i$, and plot the square of these, $\hat{\varepsilon}_i^2$, against income X_i.

(f) What do the results of (e) suggest about the validity of your earlier findings?

END OF PAPER