# ECONOMICS TRIPOS Part I

Friday 17 June 2005 9 – 12

## Paper 3

## Quantitative Methods in Economics

*This exam comprises* ***four*** *sections. Sections A and B are on* ***Mathematics****; Sections C and D are on* ***Statistics****. You should do the appropriate number of questions from each section. The number of questions to be attempted is given at the beginning of each section.*

*Answers from the* ***Mathematics*** *and the* ***Statistics*** *Sections must be tied up in separate bundles, with the letter of the Section written on* ***each*** *cover sheet.*

*This written exam carries 80% of the marks for Paper 3. Section A carries 24% of the marks, Section B carries 16% of the marks, Section C carries 24% of the marks and Section D carries 16% of the marks.*

| STATIONERY REQUIREMENTS | SPECIAL REQUIREMENTS |
|---|---|
| *20 Page booklet* | *List of statistical formulae* |
| *Metric graph paper* | *New Cambridge Elementary Statistical Tables* |
| *Rough Work Pad* | *Approved calculators allowed* |
| *Tags* | |

You may not start to read the questions printed in the subsequent pages of this question paper until instructed to do so by the invigilator

# SECTION A – MATHEMATICS

Answer *four* questions

1 A consumer has the utility function $u(x_1, x_2) = x_1(x_2 + 2), x_1, x_2 \geq 0.$ If $p_1 = 2, p_2 = 1,$ and income $I = 10,$ what are the optimal consumption levels $x_1^*$ and $x_2^*$? (Use the Lagrange method to answer).

2 Suppose that a firm, producing output $Y$ from labour $L$ and capital $K$ according to the technology $Y = L^{1/3}(K-2)^{1/3},$ wishes to minimise its costs given a production target $\bar{Y}$. The wage rate is $w = 2$ and the rental rate of capital is $r = 2.$

(a) Derive the conditional factor demands by using the Lagrange method.

(b) Derive the long run total cost function.

(c) Derive the long run average cost function and marginal cost function.

3 Consider the following equation system:

$$\begin{aligned} 5x_1 - x_2 - x_3 &= 0 \\ x_1 - 2x_2 + x_3 &= 8 \\ 3x_1 + x_2 - 2x_3 &= 6 \end{aligned}$$

(a) Write the system in matrix notation. Is the matrix of the coefficients non-singular?

(b) Solve the system by using Cramer's rule.

4 Solve the following integrals:

(a) $\int \sqrt{1/x^3}\,dx, x \neq 0$

(b) $\int 4x(x+2)^{1/2}\,dx$

(c) $\int_1^3 (ax^2 + bx + c)dx$

5 Find the derivatives $\frac{dy}{dx}$ of:

(a) $y = \ln(4x^2) + 8e^{-3x}$

(b) $y = (3x+1)^{-1/3}(x-1)^{2/3}$

6 Find the partial derivatives of: $z = f(x,y) = (x^3 + y - 2)(x^2 y^{1/3})$

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# SECTION B - MATHEMATICS

Answer *one* question

1 A consumer has a utility function given by $U(x, y) = xy^3$, an income $I$ and faces prices $p_x$ and $p_y$ respectively.

(a) Write down the consumer maximization problem by using the Lagrange method. Find the demand functions for the goods $x$ and $y$ as functions of prices and income.

(b) Is the demand function of good $x$ homogeneous of degree zero in all prices and income? What about the demand of good $y$? Explain your answers.

(c) What quantities will the consumer choose in the case when $p_x = 4, p_y = 3,$ and $I = 160$?

(d) Assume now that the price of good $y$ increases to 4. What are the quantities now chosen by the consumer?

(e) Calculate the income and the substitution effects.

2 An economy is described by the following equations:

$$
\begin{aligned}
C &= C_0 + cY^d \\
Y^d &= (1-\tau)Y \\
I &= I_0 - br \\
T &= \tau Y \\
G &= \overline{G} \\
M_S &= \overline{m} \\
M_D &= aY - dr
\end{aligned}
$$

Where $Y$ is aggregate output, $C$ is consumption, $I$ investment, $T$ taxes, $G$ government spending, $M_S$ money supply, $X_D$ money demand and $r$ the interest rate.

(a) Determine the equation of the IS curve.

(b) Determine the equation of the LM curve.

(c) Compute the equilibrium of this economy and give a graphical analysis of it.

(d) Write down the economy in matrix form and check your previous answer.

(e) Suppose that the government decides to adopt an expansionary fiscal policy by increasing $\overline{G}$ , and, in order to avoid the "crowding-out effect" increases $M_S$ to keep the interest rate constant. By how much would income go up? How would your answer change if an income tax is used to finance the increase in $\overline{G}$ ?

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# SECTION C – STATISTICS

## Answer *four* questions

1

(a) The following set of scores is obtained on a statistics test:

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| 8 | 12 | 16 | 18 | 22 | 26 | 32 | 48 | 48 | 48 | 52 |

The examiner computes all of the descriptive indices of central tendency and variability on these data, and then discovers that an error was made, and one of the 48's is actually a 34. Which of the following indices will be changed from the original computation?

(i) Median
(ii) Mode
(iii) Range
(iv) Standard deviation
(v) None of the above

(b) The following data shows the distribution of scores in a calculus test taken by 50 students.

| Score | Frequency |
|---|---|
| 90-100 | 2 |
| 80-89 | 3 |
| 70-79 | 22 |
| 60-69 | 16 |
| 50-59 | 6 |
| 40-49 | 1 |

Compute the sample mean, sample median and sample variance of this data.

2 The change in value (expressed as a percentage) of 10 residential properties in Cambridge during 2004 was found to be:

12.0 5.6 -2.3 8.5 16.2 0.6 4.4 -1.9 9.2 5.2

(a) Calculate

(i) Point estimates, and
(ii) 90% confidence intervals,

for the mean and variance of the change in value of all residential properties in Cambridge during 2004.

(b) At the start of the year, housing market experts forecast that annual house price inflation in Cambridge during 2004 would be 9%. Assess statistically (at the 5% level of significance) the hypothesis that house price inflation in Cambridge was lower than that forecast by the experts.

3 A researcher interested in the link between child illiteracy and public library provision obtains the following data for 12 countries.

| Country | Library Provision (Libraries per 10000 children) | Child Illiteracy (% of population failing basic literacy tests) |
|---|---|---|
| 1 | 6.1 | 8.6 |
| 2 | 0.5 | 32.0 |
| 3 | 1.2 | 28.3 |
| 4 | 3.3 | 13.0 |
| 5 | 5.0 | 8.1 |
| 6 | 2.0 | 20.7 |
| 7 | 3.9 | 22.0 |
| 8 | 2.3 | 26.9 |
| 9 | 1.5 | 21.6 |
| 10 | 5.5 | 23.1 |
| 11 | 5.8 | 11.9 |
| 12 | 4.7 | 17.7 |

Using the data given in the table above calculate a 95% confidence interval for the population correlation coefficient between library provision and child illiteracy.

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4 An investigator has annual aggregate time-series data on per capita consumption of petrol $Q_t$ (in gallons per year), the real price of petrol $P_t$ (an index number), and real per capita disposable income $Y_t$ (an index number). The data cover the 42 years from 1960 to 2001. A demand curve for per capita petrol consumption is specified in terms of logarithms of consumption, prices and income as follows

$$q_t = \gamma_0 + \gamma_y y_t - \gamma_p p_t + \varepsilon_t$$

where $q_t = \log(Q_t)$, $y_t = \log(Y_t)$ and $p_t = \log(P_t)$. $\gamma_0$, $\gamma_y$ and $\gamma_p$ are constant coefficients and $\varepsilon_t$ is a random error.

The investigator has a strong prior belief that $\gamma_y = 1$, and reformulates the demand curve in terms of petrol expenditure as a share $S_t$ of disposable income, where $S_t = P_t Q_t / Y_t$. Two specifications of the regression equation are considered, one for the log share $s_t = \log(S_t)$ on the log price $p_t = \log(P_t)$, and the other for the share $S_t$, without any transformation, on the untransformed price $P_t$ :

$$\text{Specification (A)}: \; s_t = \alpha + \beta p_t + \varepsilon_t$$
$$\text{Specification (B)}: \; S_t = a + bP_t + \eta_t$$

(a) Show how specification (A) can be obtained from the original demand curve by imposing the investigator's belief that $\gamma_y = 1$.

(b) Find an expression for the price elasticity of demand for each specification.

5

| UK Total employment: 2000 Quarter 1 to 2003 Quarter 4 | | | |
|---|---|---|---|
| **Quarter** | **Total Employment (thousands)** | **Quarter** | **Total Employment (thousands)** |
| 2000Q1 | 27346 | 2002Q1 | 27765 |
| 2000Q2 | 27440 | 2002Q2 | 27850 |
| 2000Q3 | 27517 | 2002Q3 | 27846 |
| 2000Q4 | 27497 | 2002Q4 | 28000 |
| 2001Q1 | 27604 | 2003Q1 | 28049 |
| 2001Q2 | 27662 | 2003Q2 | 28112 |
| 2001Q3 | 27670 | 2003Q3 | 28130 |
| 2001Q4 | 27735 | 2003Q4 | 28152 |
| Source: Economic Trends Annual Supplement 2004 | | | |

An economist decomposes the time series data in the table above according to the following additive model:

$$D = T + C + S + R$$

Where D refers to the data series, T to the trend component, C to the cyclical component, S to the seasonal factor, and R to the random residual.

Using the method of moving average, calculate the four seasonal factors, S, found by the economist.

6

(a) The probability distribution function of a random variable $x$ is

$$P(x) = x^2 / k, \qquad x = 0,1,2,3,4,5 \text{ otherwise } P(x) = 0$$

(i) Find $k$.

(ii) Calculate the mean and variance of $x$.

(b) Exactly two taxi companies operate in Small Town. The Red Company has red taxis, and the Orange Company has orange taxis. 85% of the taxis are red and the other 15% are orange. A taxi was involved in a hit-and-run accident at night. A witness identified the taxi as orange. Careful tests were done to ascertain peoples' ability to distinguish between red and orange taxis at night. The tests showed that people were able to identify the colour correctly 80% of the time, but they were wrong 20% of the time. What is the probability that the taxi involved in the accident was indeed an orange cab?

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# SECTION D – STATISTICS

## Answer *one* question

1

(a) Describe the method of ordinary least squares estimation, making clear the conditions under which its application is appropriate.

(b) An economist interested in the relationship between the price of overnight accommodation in a seaside town and the proximity of that accommodation to the beach collects the following data for 12 hotels:

| Hotel | Distance to nearest beach (Km) | Price of standard room (£) |
|---|---|---|
| 1 | 0.2 | 99 |
| 2 | 0.9 | 95 |
| 3 | 1.3 | 83 |
| 4 | 2.2 | 65 |
| 5 | 2.4 | 70 |
| 6 | 3.3 | 60 |
| 7 | 3.9 | 65 |
| 8 | 5.0 | 59 |
| 9 | 5.2 | 52 |
| 10 | 6.2 | 48 |
| 11 | 6.8 | 61 |
| 12 | 8.0 | 65 |

Using these data, determine the parameters of the following regression model:

$$Y = \alpha + \beta X + \varepsilon$$

where

$Y$ is the price of hotel accommodation in pounds
$X$ is the distance of the hotel from the nearest beach

(c) Using your results assess the claim that the proximity of a hotel to a beach is a significant factor in the price charged by hotels for a room.

2 Monthly data on prices and dividends are collected for two assets (A and B). At the beginning of month *t* the prices of assets A and B are $P_t^A$ and $P_t^B$ respectively. If an asset is held during month *t-1* it pays a dividend at the beginning of month *t*, which is denoted by $D_t^A$ and $D_t^B$ respectively for assets A and B. The monthly percentage returns on holding each asset from the beginning of month *t-1* to the beginning of month *t* are denoted by $R_t^A$ and $R_t^B$ .

(a) Describe how the returns $R_t^A$ and $R_t^B$ are calculated.

(b) Monthly returns for assets A and B are calculated for a period of 25 months. You are given the following information:

$$\overline{R}^A = 0.517 \qquad \frac{1}{T}\Sigma_{t=1}^T\left(R_t^A - \overline{R}^A\right)^2 = 13.624$$

$$\frac{1}{T}\Sigma_{t=1}^T\left(R_t^A - \overline{R}^A\right)^3 = -26.198 \qquad \frac{1}{T}\Sigma_{t=1}^T\left(R_t^A - \overline{R}^A\right)^4 = 1230.894$$

$$\overline{R}^B = 0.388 \qquad \frac{1}{T}\Sigma_{t=1}^T\left(R_t^B - \overline{R}^B\right)^2 = 4.757$$

$$\frac{1}{T}\Sigma_{t=1}^T\left(R_t^B - \overline{R}^B\right)^3 = -1.847 \qquad \frac{1}{T}\Sigma_{t=1}^T\left(R_t^B - \overline{R}^B\right)^4 = 79.488$$

Calculate the standard deviation, the skewness, and the kurtosis of returns for both assets, using the following formulae for the skewness and kurtosis of a variable $X_t$ :

$$\text{Skewness} = m_3 / m_2^{3/2} \qquad \text{Kurtosis} = m_4 / m_2^{2}$$

$$\text{where } m_j = \frac{1}{T}\sum_{t=1}^{T}\left(X_t - \overline{X}\right)^j$$

Give a short description of what the measures of skewness and kurtosis represent.

(c) If the returns were normally distributed, what values would you expect for the skewness and kurtosis measures? Comment on the appropriateness of the normality assumption for the two assets' returns.

(d) Test at the 5% level of significance whether the standard deviation of the monthly return is higher for asset A than asset B.

(e) 'Asset A has a higher average return than B and so is clearly a superior investment to asset B.' Comment on this statement in the light of your answer to part (d).

END OF PAPER