ECONOMICS TRIPOS PART I

Friday 13 June 2008 9-12

Paper 3
QUANTITATIVE METHODS IN ECONOMICS

*This exam comprises **four** sections. Sections A and B are on **Mathematics**; Sections C and D are on **Statistics**. You should do the appropriate number of questions from each section. The number of questions to be attempted is given at the beginning of each section.*

*Answers from the **Mathematics** and the **Statistics** Sections must be tied up in separate bundles, with the letter of the Section written on **each** cover sheet.*

This written exam carries 80% of the marks for Paper 3. Section A carries 24% of the marks, Section B carries 16% of the marks, Section C carries 24% of the marks and Section D carries 16% of the marks.

STATIONERY REQUIREMENTS	SPECIAL REQUIREMENTS
2 x 20 Page booklet	List of statistical formulae
Metric Graph Paper	New Cambridge Elementary Statistical Tables
Rough Work pads	
Tags	Approved calculators allowed

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

SECTION A - MATHEMATICS

Answer **four** questions

1 λ is a positive constant with $0 < \lambda < 1$. By noting that $\lambda^h = exp(h\,ln(\lambda))$ or otherwise show that the function

$$f(h) = \frac{1-\lambda^h}{h}$$

is downward sloping for $h > 0$. [You may use without proof the fact that $ln(x) \leq x - 1$ for all x].

2 A firm's production function is $Q(L) = 12L^2 - \frac{1}{20}L^3$ where L denotes the number of workers with $L \in [0, 200]$

(a) Find the size of workforce (L^*) that maximises output $Q(L)$?

(b) Write down an expression for output per worker and find the size of the workforce (L^{**}) that maximises this?

(c) Explain why $Q'(L^{**}) = Q(L^{**})/L^{**}$.

3 Let I be the 2x2 identity matrix, and let i be the vector $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$. The symbol $'$ denotes transposition of a vector or matrix.

(a) Evaluate $i'i$ and ii'. Hence, or otherwise, show that the matrix $M = I - \frac{1}{2}ii'$ obeys the matrix equation

$$MM = M$$

(b) A square matrix which satisfies $M^2 = MM = M$ is called an *idempotent* matrix. Show that if M is idempotent, then the matrix $(I - M)$ is also idempotent, and evaluate the matrix $Q = M(I - M)$.

4 An agent has utility defined over consumption of two goods x and y given by $U(x,y) = x^a y^{1-a}$. Her budget constraint is

$$p_x x + p_y y = m$$

Calculate the optimal choices of x and y and the Lagrange multiplier λ. Sketch the relationship between λ/U and m. Give a brief interpretation of your graph in terms of the underlying optimisation problem.

5 A small macro model is described as follows

$$\begin{aligned} Y &= C + I + G \\ C &= \alpha + \beta Y \\ I &= \gamma - \delta R \\ G &= 20 \\ M^d &= \lambda Y - \mu R \\ M^s &= 360 \end{aligned}$$

where Y is output, C is consumption, I is investment, R is the interest rate, G is government spending and M^s and M^d are supply of and demand for money respectively.

(a) Solve for the equilibrium level of output and the interest rate when

$\alpha = 10, \beta = 0.8, \gamma = 50, \delta = 1, \lambda = 1, \mu = 3$

(b) Calculate the value of the government spending multiplier $\frac{dY}{dG}$.

6 Calculate the equation of the straight line tangent to the curve $y = ln(x)$ at the point $x = 1, y = 0$. Use your answer to show that for small ε we may approximate $ln(1 + \varepsilon)$ by ε. Now use this to show that if

$$y = \left(1 + \frac{x}{n}\right)^n$$

then for large enough n we may approximate y by e^x. Relate your result to the use of alternative formulae for calculating interest payments.

SECTION B - MATHEMATICS

Answer **one** question

1 The demand D_t for a good in period t is given by

$$D_t = a - bp_t$$

where p_t is the current market price and a and b are positive constants. The supply of the good S_t depends on the current expected price p_t^e as follows

$$S_t = -c + p_t^e$$

where c is a positive constant. Price expectations are formed as follows

$$p_t^e = p_{t-1}^e + \mu(p_{t-1} - p_{t-1}^e)$$

where $0 < \mu \leq 1$.

(a) If market equilibrium determines prices write down an equation linking p_t and p_t^e

(b) Use your answer to (a) and the price expectation mechanism to obtain a difference equation for p_t.

(c) Determine the equilibrium price p^* such that if $p_{t-1} = p^*$ then $p_t = p^*$

(d) What restrictions must be placed on a, b, c and μ for p^* to be a stable equilibrum?

(e) Describe what you would observe if $\mu = 0.8$ and $b = 0.5$.

2 A monopolist operates in two isolated geographic areas (so that goods sold in one area cannot easily be resold into the other) and faces the demand curves

$$P_1 = 100 - Q_1, \;\; P_2 = 80 - Q_2$$

where $Q_1$and Q_2 are the quantities sold in market areas 1 and 2 respectively. The monopolist's cost function is $C = 6(Q_1 + Q_2)$

(a) How much should be sold in each of the two markets in order to maximise profits? What would the two prices be?

(b) How much profit would be lost if it becomes illegal to price discriminate?

(c) The government imposes a tax of t per unit sold in market 1. Show that the discriminating monopolist's profits fall by $t^2/4$ more than the government gains in tax revenue.

(d) Sketch the relationship between the tax rate t and government revenue and calculate the revenue maximising tax rate.

(e) The government decides not to bother with the tax but now a budget airline appears that allows people to travel between the two areas at a return cost of only 8 (and no restrictions on the amount of this good carried). Analyse what the discriminating

monopolist should do now.

TURN OVER

SECTION C - STATISTICS

Answer **four** questions

1 A mobile phone company's records indicate that private customers pay on average £17.10 per month. A random sample of 10 customers' bills during a given month produced a sample mean of £22.10 and a sample variance of 45. A 5% significance test is to be performed to determine if the mean level of billing is in excess of £17.10. Set out carefully the hypothesis to be tested, calculate the test statistic and the appropriate critical value. What is your conclusion?

2 A researcher is interested in the relationship between body weight and income. She calculates the correlation between three categories of body weight (using the numerical scale underweight=0, normal=1 and overweight=2) and income (in £ per week) for 100 working adults and finds $r = -0.4$. Comment carefully on whether this provides an appropriate test of her hypothesis.

3 Two fair dice are thrown.

(a) What is the probability that the total score achieved is 2 or 6?

(b) If their total is not 2 or 6 they are thrown again. What is the probability that the final total is even?

(c) If both dice are thrown a total of 5 times, what is the probability that on precisely 3 occasions the total score is 7?

4

(a) Weekly Earnings data for a large manufacturing firm were obtained for all employees. The mean, median and mode Weekly Earnings are found to be £450, £430 and £420. Do the results indicate that the distribution of earnings is positively or negatively skewed? Do these results seem realistic? Explain your answer.

(b) It is decided to calculate a measure of spread of the data. Explain carefully under what circumstances you would consider each of the following to be an appropriate measure of spread:

(i) interquartile range

(ii) root mean square deviation

(iii) standard deviation

5 X is a non-negative random variable with $E(X^2) = 12.1$. In addition $P(X = 2) = 0.4$ and $P(X = 5) = 0.1$. The random variable can take on one other value besides 2 and 5.

(a) What is the other value that X takes?

(b) What is the variance of X?

6 A random sample of 200 city households is taken to estimate the proportion using double-glazing on their windows. Of these 80 said they had double-glazing.

(a) Determine a 95% confidence interval for the proportion of this population using double glazing

(b) A similar sample, this time of size 270, is taken of rural households. It was found that 90 had double-glazing. Test the hypothesis that city dwellers and country folk are equally

likely to have double-glazing.

TURN OVER

SECTION D - STATISTICS

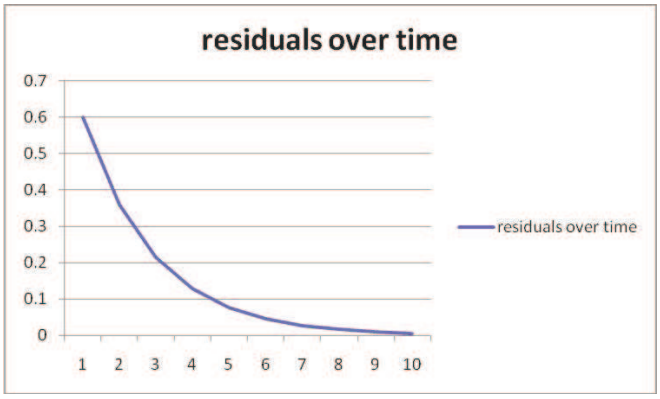
Answer **one** question

1

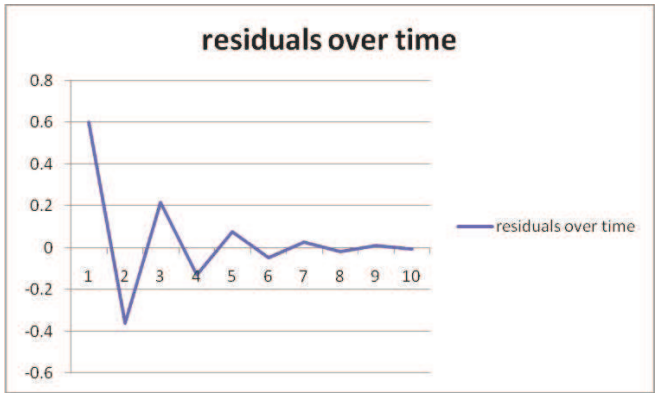
(a) When carrying out an ordinary least squares regression analysis using time series data what assumptions do you need to make concerning the disturbance terms?

(b) How would you interpret each of the following plots of residuals against time from a time series regression analysis?

(i)



(ii)



(c) A regression analysis of earnings on age is carried out for a sample of a cohort of employees in a particular institution, using the following data.

Observation	Age A (years)	Earnings E (£'000s)
1	20	20
2	25	30
3	30	39
4	35	47
5	40	50
6	45	52
7	50	54
8	55	55
9	60	56
10	65	56

and the following results are obtained (standard errors are in parentheses):

$$\widehat{E} = 14.53 + 0.738A \quad R^2 = 0.815$$
$$(5.58)\ (0.124)$$

(i) Interpret these results.
(ii) Give a 95% confidence interval for your estimate of the coefficient on age.
(iii) What earnings would you predict for someone aged 67?
(iv) Compare your answer to (iii) with the earnings of those aged 60 and 65, and comment.
(v) Determine the missing entries in the following table

Observation	Predicted Dependent Variable	Residual
1		
2		
3	36.67	2.33
4	40.36	6.64
5	44.05	5.95
6	47.75	4.25
7	51.44	2.56
8		
9		
10		

(vi) Interpret the residual pattern, and discuss how the exercise might have been improved upon (but do not carry out such an exercise).

TURN OVER

2 Two Cambridge Colleges A and B each have seven first year students studying Economics. The average marks achieved in the end of year exams are shown in the following table.

College A	College B
Marks	Marks
80	72
65	50
65	63
52	61
41	58
33	60
56	63

A first class mark is awarded for marks equal to or greater than 65.

(a) For each college calculate the mean, mode and median mark as well as the sample variance and standard deviation.
(b) Given the information in the table and your answer to (a), which College would you say has performed the best. Explain your answer.

(c) On the assumption that the students examined are representative of students entering, and the teaching found in, the two Colleges test the hypothesis that the mean mark is the same for both Colleges. State clearly any further assumptions that you make and test where possible.
(d) Test the hypothesis that the proportion achieving first class marks is the same for both Colleges.
(e) Describe how, if at all, your analysis would change if you learnt that all these students had studied economics at school except the two receiving lowest marks in College A. Which College would you say has performed best now?
(f) Redo step (d) but dropping the observations on the two who have not previously studied economics. Interpret your results.