ECONOMICS TRIPOS PART I

Friday 17 June 2011 9.00 to 12.00

Paper 3

QUANTITATIVE METHODS IN ECONOMICS

This exam comprises four sections:
Sections A and B are Mathematics; Sections C and D are Statistics.

You should do the appropriate number of questions from each section. The number of questions to be attempted is at the beginning of each section. Answer all parts of the question.

Answers from the Mathematics and the Statistics Sections must be tied up in separate bundles with the letter of the Section written on each cover sheet.

This written exam carries 80% of the marks for Paper 3.

Section A carries 24% of the marks;
Section B carries 16% of the marks;
Section C carries 24% of the marks and
Section D carries 16% of the marks.

STATIONERY REQUIREMENTS	SPECIAL REQUIREMENTS
20 Page booklet x 2	List of statistical formulae
Metric graph paper	New Cambridge Elementary Statistical Tables
Rough work pads	Approved calculators allowed
Tags	

You may not start to read the questions
printed on the subsequent pages of this question paper
until instructed that you may do so by the Invigilator

SECTION A
Answer **ALL FOUR** questions

1. In probability theory, an important function is the so-called standard normal density:
$$f(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}$$
   (i) Find the range of values of x where $f(x)$ is concave.
   (ii) Find an expression for the value of the maximum of the first-order derivative of $f(x)$ in terms of π and e.

2. Suppose that production in the economy is determined by the production function
$$\begin{aligned} Y &= F(K, L) \\ &= K^{\alpha} \ln(L + \beta), \end{aligned}$$
   where $K \geq 0$ and $L \geq 0$.
   (i) Compute the Hessian matrix of second derivatives of $F(K, L)$
   (ii) Suppose that $0 < \alpha < 1$ and $\ln(\beta) > \frac{\alpha}{1-\alpha}$.Using your result from part i) show that $F(K, L)$ is a concave function of K and L.

3. (i) Let A be the square matrix given by
$$A = \begin{bmatrix} 3 & \beta \\ \alpha & 1 \end{bmatrix}$$
   Compute the determinant of A. What is the condition which the variables α and β must satisfy to ensure that A is a nonsingular matrix? Assuming that this condition is satisfied, compute A^{-1}.
   (ii) Let B be the square matrix defined by
$$B = \begin{bmatrix} 1 & \theta \\ \theta & 2 \end{bmatrix}$$
   What is the condition which θ must satisfy to ensure that B is a positive definite matrix?
   (iii) Find the extreme point of the function
$$x^2 + 2y^2 - 2xy$$
   By using your result from part ii), or otherwise, show that this extreme point is a minimum.

4. Let $y(t)$ be the reserves of oil in an oil pool at time t. Suppose that extraction reduces reserves at a constant proportionate rate α. If initial reserves at $t = 0$ were 500 million barrels,

   (i) Solve for the expression showing reserves as a function of time.

   (ii) Now assume that $\alpha = 0.1$. Find the time at which 50% of oil reserves have been used up.

SECTION B

Answer **ONE** question

5. A consumer derives utility from consuming two goods. The value of the utility from consuming x units of the first good and y units of the second good is:

$$u(x,y) = \left(\sqrt{x} + 2\sqrt{y}\right)^2$$

The price of the first good is £3, and the price of the second good is £6. The consumer would like to spent exactly £30 on his total purchase of the two goods. He also wants to maximize his utility from consuming the goods.

(i) Derive the marginal utility functions $\frac{\partial}{\partial x}u(x,y)$ and $\frac{\partial}{\partial y}u(x,y)$.

(ii) Derive the optimality conditions for the consumer's constrained utility maximization problem using the Lagrange method.

(iii) Find the optimal combination of the two goods.

(iv) Check that the combination that you have found in iii) indeed provides the consumer with maximum utility rather than with minimum utility.

6. Consider the following simple macroeconomic model of a closed economy

$$\begin{aligned} Y &= C + I + G \\ C &= \bar{C} + c(1-t)Y \\ I &= \bar{I} + aY - br \\ G &= \bar{G} \\ M_d &= \alpha Y - \beta r \\ M_d &= M_s \end{aligned}$$

where Y, C, I and G respectively denote national income, consumption, investment and government expenditure. t denotes a proportionate tax rate on income. M_d, M_s and r denote respectively money demand, money supply and the interest rate. $\bar{C}, \bar{I}, \bar{G}$ and the parameters c, a, b, α and β are positive constants. Assume $a + c < 1$.

(i) Derive a system of two equations, one giving the relationship between r and Y consistent with goods market equlibrium (the IS curve), the other giving the relationship between r and Y consistent with money market equlibrium (the LM curve). Write this system of equations using matrix notation.

(ii) Derive the determinant associated with this system of equations.

(iii) Hence, or otherwise, solve for the values of r and Y consistent with macroeconomic equilibrium. Evaluate the effect on the interest rate of a fall in the autonomous component of investment $\bar{I}$.

(iv) Suppose that, in response to a 1 unit fall in $\bar{I}$, the Government expands monetary policy to maintain the initial level of output (while maintaining government expenditure $\bar{G}$ and the tax rate t at their original levels). Compute the necessary change in the money supply M_s. Are there values of the key parameters for which such a policy will be ineffective?

SECTION C
Answer **ALL FOUR** questions

7. (a) In an experiment on extra-sensory perception (ESP), a subject in one room is asked to state the colour (red or blue) of a card chosen from a deck of well-shuffled cards by an individual in another room. Half of the cards are red and half are blue, and each card is replaced randomly in the pack after being turned over. If, after 50 cards are turned over, the subject has identified 32 cards correctly, determine whether the results are significant at the 5% level. Comment on your decision to use a 1 or 2 tailed test.

   (b) It is suggested that believing in the existence of ESP increases the likelihood of doing well in the experiment in (a). In order to test this, a sample of 100 subjects who believe in ESP is taken along with a sample of 100 subjects who do not believe in ESP and the experiment from part (a) is undertaken again to see how many subjects score above average (where 'above average' is understood to mean that the candidate guessed more than 50% of the cards correctly). In the first group 75 obtain above average results and in the second group 65 obtain above average results. Is the claim supported at the 5% significance level or the 10% significance level?

8. (a) If $z \sim N(0, 1)$ and z_1 is such that $P(z < z_1) = 0.0228$, find z_1.

   (b) A population with standard deviation known to be of size 100 is hypothesised to have a mean of 400. Although the researcher knows the mean can be no less than 400, she is unaware that the actual value is 440.5. A sample of N observations is drawn from the population to test the hypothesis at the 5% level of significance. If it turns out that the power of the test is in fact 0.9772 determine the size of the sample N.

9. When complete, Table 1 below contains a summary of the data for the (total) population of a 100 people that constitute the members of a UK village at a given point in time. Each column (vertical array) of Table 1 gives the distribution of consumption expenditure Y corresponding to a **fixed** level of income X; that is, it *gives the conditional distribution of* Y conditional upon the given values of X. Various values though have been removed.

Table 1: Annual family income X, (£000s)

Y ↓ **X** →	**10**	**15**	**20**	**25**	**30**	**35**	**40**	**45**	**50**	**55**
Annual	13		15			24				
family	**14**		**17**			**28**				
consumption	15		23			30				
expenditure	**16**		**25**			**30**				
Y,	17					32				
£(000s)						**36**				
Number of families receiving annual income X	5	8	5	20	20	6	7	9	?	10

You are told by a researcher with access to the total data set, that the underlying population regression function takes the form:

$$E(Y|X_i) = \alpha + \beta X_i$$

(a) Determine α and β using the information available to you, and interpret your results.

(b) Determine the missing expenditure entry for $X = 20$.

(c) Determine total expenditure of the group of families that each receive an income of £50K.

10. Discuss some uses of dummy variables in regression analysis. What are the strengths and limitations of such a device?

SECTION D
Answer **ONE** question

11. (a) The following regression results were obtained using UK data for the years 1973 to 2009:

$$\widehat{\pi}_t = \underset{(2.71)}{9.49} - \underset{(0.36)}{0.45U_t} \quad R^2 = .041 \tag{1}$$

where 'π_t' is price inflation (measured by the rate of change of the index of consumer prices), and 'U_t' is the unemployment rate (measured by the percentage of claimants in the working population) at time t. Numbers in brackets are standard errors.

Interpret these results.

(b) In response to the perceived problems of the specification in (1), it is suggested that the relationship between inflation and unemployment takes the following form:

$$\pi_t - \pi_t^e = \beta(U_t - \mu_0) + v_t \tag{2}$$

Interpret the two terms '$\pi_t - \pi_t^e$ ' and '$U_t - \mu_0$', where the superscript e denotes an expected value, v_t denotes a stochastic error term and μ_0 is understood as the 'natural rate' of unemployment.

(c) In order to estimate (2), it is assumed that the expected rate of inflation is given by the following adaptive expectations formulation:

$$\pi_t^e = \pi_{t-1}^e + \lambda(\pi_{t-1} - \pi_{t-1}^e) \tag{3}$$

If expectations are not always fulfilled, what assumption is being made about λ, in (3), if the researcher estimates (2) as follows:

$$\Delta\pi_t = \beta_0 + \beta_1 U_t + v_t \tag{4}$$

(d) The following estimates of (4) were obtained using the same data as in part (a) above:

$$\Delta\widehat{\pi}_t = \underset{(1.38)}{3.04} - \underset{(0.18)}{0.44U_t} \quad R^2 = .15 \tag{5}$$

Interpret these results and obtain an estimate of the natural rate of unemployment μ_0.

(e) Another researcher suggests that although the assumptions about expectation formation are sound, unemployment is actually the dependent variable and that the following specification is the correct one:

$$U_t = \gamma_0 + \gamma_1 \Delta\pi_t + \omega_t \qquad (6)$$

where ω_t is a stochastic error term.

The researcher then suggests that it is possible to obtain estimates of γ_0 and γ_t in two ways. The first method involves deriving equation (6) from (4) and then simply using the estimates from (5) to estimate γ_0 and γ_1 directly. The second method involves running a new regression based on the specification given in (6), the results being as follows:

$$\begin{array}{lllll} \widehat{U}_t = & 7.23 & - \; .33\Delta\pi_t & & \\ & (.04) & \quad (.14) & R^2 = .15 & \widehat{\sigma}^2 = 2.3 \end{array} \qquad (7)$$

Obtain estimates of the natural rate of unemployment using each method and interpret your results. Comment on the researcher's assumption that these two methods are equivalent.

(f) You are told that in 1984 the change in inflation is -0.18. Given that the mean of $\Delta\pi$ in this sample is $-.14$ and that the sum of the squared deviations for the difference in inflation ($S_{\Delta\pi\Delta\pi}$) is 287, use the results from (7) to predict the value of the unemployment rate in 1984 and construct a confidence interval for your prediction. The actual unemployment rate in 1984 is 12% – how does this compare with your prediction?

12. Consider an experiment where a fair coin is tossed 3 times.

(a) The sample space consists of all possible outcomes. How many outcomes are there?

(b) We now define two random variables as follows: X is the number of heads obtained in the first 2 tosses (i.e. toss 1 and 2); Y is the number of heads obtained in the last two tosses (i.e. toss 2 and 3). Draw up a joint probability distribution table for X and Y (i.e. a two-dimensional table listing the different values of X and Y and the different probabilities associated with different combinations of these values). Check that the marginal probabilities for each variable add up to one.

(c) Calculate $E(X)$, $E(Y)$ and $E(XY)$, and use your results to show that X and Y are not independent random variables. What is the Covariance between X and Y?

[TURN OVER]

(d) Now draw up a second table showing the conditional probabilities of X given different values of Y (i.e. $p(X|Y)$ for each value of Y – e.g the first element in column 1, row 1 would be 1/2). Use this to find the conditional mean and variance for X given $Y = 2$ i.e. $E(X|Y = 2)$ and $V(X|Y = 2)$.

END OF PAPER