UNIVERSITY OF CAMBRIDGE

ECT1
ECONOMICS TRIPOS PART I

Friday 13 June 2014 9:00-12:00

Paper 3

QUANTITATIVE METHODS IN ECONOMICS

There will be a 15 minute reading time prior to the beginning of the examination.

This paper is divided into four Sections:
Sections A and B are Mathematics; Sections C and D are Statistics.

You should do the appropriate number of questions from each Section.
The number of questions to be attempted is at the beginning of each Section.
Answer all parts of the question.

Answers from the Mathematics and the Statistics Sections must be written in separate booklets with the letter of the Section written on each cover sheet.

Section A carries 30% of the marks
Section B carries 20% of the marks
Section C carries 30% of the marks
Section D carries 20% of the marks

Write legibly.

STATIONERY REQUIREMENTS
20 Page booklet x 2
Rough work pads
Tags

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAMINATION
Calculator - students are permitted to bring an approved calculator
New Cambridge Elementary Statistical Tables

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

SECTION A - Answer **ALL FOUR** questions from this Section.

1 For each of the functions below, check whether it is continuous and differentiable at $x = -1$. If it is differentiable, find the derivative at $x = -1$.

(a)

$$y = \begin{cases} x^2 + 2x + 1 & \text{for } x > -1 \\ 0 & \text{for } x \leq -1 \end{cases}$$

(b)

$$y = \begin{cases} \ln(x + 5) & \text{for } x > -1 \\ \ln(2x^2 + 2) & \text{for } x \leq -1 \end{cases}$$

2 A consumer has the following ('Stone-Geary') utility function

$$U(x_1, x_2) = b_1 \ln(x_1 - a_1) + b_2 \ln(x_2 - a_2)$$

where x_1 and x_2 represent his consumption of good 1 and good 2 respectively, and b_1, b_2, a_1, a_2 are positive non-zero constants. Assume that $x_1 > a_1$ and $x_2 > a_2$, which also implies that the consumer has at least enough income, m, to be able to buy a_1 units of good 1, and a_2 units of good 2, i.e.,

$$m > p_1 a_1 + p_2 a_2$$

where p_1 and p_2 are the prices of the respective goods.

(a) Solve the consumer's utility maximization problem, i.e., solve for the optimal values of x_1 and x_2, given prices and a fixed income m.

(b) Interpret these demand functions, and describe how they differ from the case when the consumer's utility function is of the Cobb-Douglas form.

3 (a) Suppose that an individual initially invests £100, and then an additional £50 each month, at an annual rate of 6%, compounded monthly. Setting up this calculation as a difference equation, find the general solution and the particular solution, given the initial value of £100. How much money would have been accrued after five years?

(b) Compute the third-order Taylor series expansion of the function $y = \sqrt{1 + x}$ around $x = 0$.

4 Consider the function

$$f(x, y) = \alpha^2 x^2 - 6\alpha xy + 9y^2 + \ln \alpha$$

(a) Find a value of α for which $f(x, y)$ is a positive semidefinite quadratic function. Show how you obtain your answer.

(b) Assume that $\alpha = 2$. Is the function $f(x, y)$ concave, convex, or neither? Show how you obtain your answer.

SECTION B - Answer **ONE** question from this Section.

5 Assume that

$$\begin{aligned} Y &= \ln w - \alpha \ln w - \beta \ln c \\ Z &= -\gamma \ln w + \ln c - \delta \ln c \end{aligned}$$

where $\alpha, \beta, \gamma, \delta > 0$ and $c, w, Y, Z > 1$.

(a) Rewrite the system of equations using vectors and matrices, and solve for $\ln w$ and $\ln c$. What assumption(s) do you need to make?

(b) Using your result in part (a), or by otherwise solving the system, find an expression for $\frac{dY}{dw}$.

(c) Assume that $Y = \ln A$ and $Z = \ln B$, and that the functions $U(A, w) = \frac{w}{A}$ and $V(B, c) = \frac{c}{B}$ are both homogeneous of degree 1. For the values of α, β, γ and δ associated with these assumptions, solve for w and c. Comment on your results.

6 Consider the following production function, where output, Q, depends on the amounts used of the factors of production, K and L:

$$Q(K, L) = A[\delta K^{-\rho} + (1 - \delta)L^{-\rho}]^{-\frac{1}{\rho}}$$

where $A > 0$, $0 < \delta < 1, -1 < \rho$ and $\rho \neq 0$.

(a) Is this function homogeneous? If so, then to what degree?

(b) Find an expression for the slope of the isoquants generated by this production function. Are the isoquants convex?

(c) Set the isoquant slope equal to the ratio of the factor prices, r and w, for the factors K and L respectively, and find the elasticity of substitution.

(d) Set $\rho = -1$ in the expression for the production function above, and for the resulting function, remark on the shape of the isoquants and provide an intuitive interpretation in terms of the substitutability of the factors of production.

Section C - Answer **ALL FOUR** questions from this Section.

7 Random variables X and Y both take on only the discrete values 1 and 2. You are given that

$$\begin{aligned} P(X=1, Y=1) &= 0.3 \\ P(X=2, Y=1) &= 0.1 \\ P(X=1, Y=2) &= 0.1 \end{aligned}$$

(a) Calculate $P(X=2, Y=2)$.
(b) Calculate the marginal probability mass functions for X and for Y.
(c) Calculate the conditional probability mass function $P(X \mid Y=1)$.
(d) Are X and Y independent? Justify your answer.

8 Estimates are that 85% of all email traffic is spam. An email spam filter uses a frequency count of suspicious words to flag messages that might be spam and stores these messages in a special folder. Examination of this folder shows that 99% of the messages that have been flagged are indeed spam. Experiments show that if a message is spam it is flagged by the filter 90% of the time. What is the false positive rate for this filter, i.e., the probability that a non-spam message is flagged as spam?

9 Using a dataset on 528 individuals i, with Y_i = hourly wage and X_i = years of education, we can calculate the following Sample Statistics:

- $n = 528$
- $\sum_i X_i = 6910$
- $\sum_i X_i^2 = 93698$
- $\sum_i Y_i = 4777.1$
- $\sum_i Y_i^2 = 57166.245$
- $\sum_i X_i Y_i = 65176.83$
- $S_X^2 = 6.197$
- $S_Y^2 = 26.462$

(a) Calculate the sample covariance of X and Y using the information provided above.

(b) Typically, economists run the regression of Y on X. Suppose that instead of running the least-squares linear regression of Y on X, we ran the least-squares linear regression of X on Y. What are the formulas for the intercept and slope of this *reverse regression* line?

(c) Use the Summary Statistics above to calculate the least-squares linear regression of education on wage (i.e., the slope and intercept of this *reverse regression* line).

10 Please answer all of the following short questions about the relationship between random variables X and Y.

(a) 'If there is no relationship between X and Y, then $C(X, Y) = 0$'. True or False? Explain the reasoning behind your answer.

(b) In a certain bivariate probability distribution, the variables X and Y are not independent, and the conditional expectations of Y given X are not the same for all values of X. Is it possible that $C(X, Y) = 0$? Explain the reasoning behind your answer.

(c) 'The regression of Y on X has the same R^2 as the regression of X on Y'. True or False? Explain the reasoning behind your answer.

(TURN OVER)

Section D - Answer **ONE** question from this Section.

11 A set of observations y_i is generated by the following model

$$y_i = x_i\beta + \varepsilon_i \quad i = 1, \ldots, N$$

with x_i a sequence of fixed numbers, and ε_i a set of independent and identically distributed random variables with $E(\varepsilon_i) = 0$ and $E(\varepsilon_i^2) = \sigma^2$, and β is an unknown parameter.

Using the observations $\{y_i, x_i\}$ three estimators for β are considered

$$\widehat{\beta}_1 = \frac{\sum_{i=1}^N x_i y_i}{\sum_{i=1}^N x_i^2}, \quad \widehat{\beta}_2 = \frac{\sum_{i=1}^N y_i}{\sum_{i=1}^N x_i}, \quad \widehat{\beta}_3 = \sum_{i=1}^N \frac{y_i}{x_i}$$

Calculate $E\left(\widehat{\beta}_j\right)$ and $Variance\left(\widehat{\beta}_j\right)$ for each estimator $j = 1, 2, 3$. Based on these calculations, which estimator is most preferred? Explain why.

12 Consider the relationship between wages, experience, and union status during the 1980s. Our dataset contains 528 observations. Each observation corresponds to a person, for whom we observe the hourly wage, union status (1 if the worker is in a union, 0 otherwise), and years of work experience. In what follows, denote these variables by *wage*, *union*, and *exper* respectively.

(a) Consider the regression of wages on union status.

$$\widehat{wage} = \underset{(0.245)}{8.659} + \underset{(0.573)}{2.140} union$$

What does the coefficient on *union* mean?

(b) Test the null hypothesis that on average, the wages of unionized and non-unionized workers are the same, against the alternative that they are not.

(c) Next, consider the regression of wages on union status and experience below:

$$\widehat{wage} = \underset{(.394)}{8.130} + \underset{(0.577)}{2.015} union + \underset{(0.018)}{0.031} exper$$

Based on this regression, write the expression for the fitted least squares line for unionized workers.

(d) Similarly, based on the regression of wages on union status and experience, write the expression for the fitted least squares line for non-unionized workers.

(e) If two people have the same labor market experience, but one is unionized and the other is not, what is their predicted average wage difference?

(f) List the assumptions under which the least squares estimators are unbiased. Do you think the coefficient estimate on the union variable in part (a) is unbiased? Explain why or why not.

END OF PAPER