UNIVERSITY OF CAMBRIDGE

ECT1
ECONOMICS TRIPOS PART I

Friday 10 June 2016 9:00-12:00

Paper 3

QUANTITATIVE METHODS IN ECONOMICS

Answer **ALL FOUR** questions from Section A, **ONE** question from Section B, **ALL FOUR** questions from Section C and **ONE** question from Section D.

Answers from Sections A and B (Mathematics) and from Sections C and D (Statistics) must be written in separate booklets with the letter of the Sections written on each cover sheet.

Sections A and C each carry 30% of the marks. Section B and D each carry 20% of the marks.

Each question within each section will carry equal weight.

Write you **candidate number** (not your name) on the cover of each booklet. If you identify an error in this paper, please alert the **Invigilator**, who will notify the **Examiner**. A **general** announcement will be made if the error is validated.

Write legibly.

STATIONERY REQUIREMENTS
20 Page booklet x 2
Rough work pads
Tags

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAMINATION
Calculator - students are permitted to bring an approved calculator
New Cambridge Elementary Statistical Tables

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

SECTION A Answer ALL FOUR questions from this Section

1. (a) Provide a Taylor series expansion for $f(x) = ln(1+x)$ at $x_0 = 0$ and use it to explain why $f(x)$ is approximately equal to x when x is close to zero.

   (b) Provide a first order Taylor series approximation for $f(x) = (9+x)^{0.5}$ around $x = 0$. Use this approximation to estimate:

   (i) $(9.01)^{0.5}$

   (ii) the absolute value of the remainder in part (i).

2. (a) Evaluate $\int_1^2 x\sqrt{(x-1)^3}dx$

   (b) Let $A = \frac{4(4x-3)}{(2x-3)(2x+1)}$
   Find B and C such that $A = \frac{B}{(2x-3)} + \frac{C}{(2x+1)}$

   (c) Evaluate $\int \frac{16x-12}{(2x-3)(2x+1)}dx$

3. Determine whether the functions below are convex or concave

   (a) $f(x) = -4ln\left(x\sqrt{x^3}\right) + 6lnx^{\frac{5}{2}} + 4$

   (b) $f(x,y) = x^4 - 12x^3 + 54x^2 + 10000x + \alpha y^2 + 4y + 79431$
   where

   (i) $\alpha \geq 0$

   (ii) $\alpha < 0$

4. A macro model of a closed economy consists of the following equations

$$\begin{aligned} Y &= C + I + G \\ M &= M^D \end{aligned}$$

where Y is income, C is consumption expenditure, I is investment, G is net government spending, M is the money supply, and M^D is the demand for money. In addition, you are told that consumption expenditure is a function of income, investment is a function of the interest rate r, and the demand for money is a function of both income and the interest rate.

   (a) Derive a matrix representation of how, when the system is very close to equilibrium, changes in the endogenous variables Y and r are related to changes in the exogenous variables G and M.

   (b) Use Cramer's Rule to solve for dY (changes to Y) in terms of changes in G and, second, in terms of changes in M.

SECTION B Answer ONE question from this Section

5. A student takes on some holiday work for a few weeks. Each day (of 24 hours) is divided into leisure time (including sleep) L and hours in employment E. The student is paid a wage of w per hour and saves a fixed amount S each day (to be spent after the holiday). The student buys an amount of consumption goods C each day at unit price p. Each day the amount spent on consumption plus the amount saved equals the amount earned exactly. The student's utility function is given by:

$$U(C,L) = C^{\beta} L^{1-\beta}$$

where $0 < \beta < 1$

(a) Write down the student's daily budget constraint in terms of p, w, C, L and S (i.e., eliminate E from the equation) and derive expressions for the optimal choices of C and L.

Now let $w = 10$; $p = 2$; $\beta = 0.5$; $S = 20$.

(b) Given these parameter values calculate the optimal values of C, L and $U(C,L)$ and find the change in optimised utility if the student were to increase slightly the amount saved daily.

(c) If the government decides to impose a 20% tax on the wage income of student workers, calculate the optimal values of C, L and $U(C,L)$ and determine the government's tax take from each student.

(d) If the government decides to impose a lump sum tax designed to raise precisely the amount achieved under part (c) calculate the optimal values of C, L and $U(C,L)$. Under which tax system does the student derive most utility?

6. A monopolist has a total cost function of the form $TC = kQ + \lambda$, where Q is the level of output, and k and λ are constants. The monopolist has the possibility of splitting customers into two distinct markets with the following demand equations:

$$\text{Market 1:} \quad P_1 = \alpha + \beta Q_1$$
$$\text{Market 2:} \quad P_2 = \delta + \gamma Q_2$$

where $Q = Q_1 + Q_2$, P_i is the price in market i ($i = 1, 2$), and α, β, γ and δ are constant parameters with $\beta, \gamma < 0$.

(a) Determine expressions for the prices (in terms of the constant parameters) that will be charged if the profit maximising monopolist is allowed to charge different prices in each market.

Now let $\alpha = 180$; $\beta = -10$; $\delta = 80$; $\gamma = -2.5$;$\lambda = 40$; and $k = 40$

(b) Check that the conditions for maximised profits do indeed hold, and determine the optimised values of prices, output and profits.

(c) The monopolist is now forced to charge the same price in both markets. What does the total demand curve look like?

(d) Determine the optimised values of prices, output and profits, if the monopolist is forced to charge the same price in both markets. Check how the optimised profits compare with those of part (b) and comment.

SECTION C Answer ALL FOUR questions from this Section

7. The discrete random variable X takes the values $1,2$ and 3 and the discrete random variable Y takes the values 1 and 2. Their joint probability mass function is such that both marginal distributions are uniform and $P(X=1,Y=1)=\frac{1}{12}$ and $P(X=3,Y=2)=\frac{1}{6}$.

   (a) Calculate and write out the joint probability mass function.

   (b) Calculate the conditional distributions $P(X \mid Y=y)$ for $y=1,2$ and $P(Y \mid X=x)$ for $x=1,2,3$.

   (c) Are X and Y independent? Justify your answer.

   (d) Calculate $P(X=2 \,\text{or}\, Y=2)$.

8. The discrete Poisson distribution has probability mass function $P(X=k)=\frac{e^{-\lambda}\lambda^k}{k!}$ for $k=0,1,2,\ldots$ where $\lambda>0$ is some parameter.

   (a) Show that $\sum_{k=0}^{\infty} P(X=k)=1$

   (b) Show that $E(X)=\lambda$

   (c) Show that $E\left(e^X\right)=e^{\lambda(e-1)}$ and hence that $E\left(e^X\right)>e^{E(X)}$

   You may use without proof the fact that $e^t=\sum_{i=0}^{\infty}\frac{t^i}{i!}$.

9. The index t takes on the values $t=1,2,3,\ldots,T$. X_t is a random variable with $E(X_t)=\mu$ and $Var(X_t)=1$ for all t. The random variables ε_t are distributed independently of the X_t with $E(\varepsilon_t)=0$ and $Var(\varepsilon_t)=\sigma^2$. The random variables Y_t are defined as

$$Y_t = tX_t + \varepsilon_t \qquad t=1,2,\ldots,T$$

   (a) What is $E(Y_t)$ and $Var(Y_t)$?

   (b) If $\bar{Y}=\frac{1}{T}\sum_{t=1}^{T} Y_t$ what is $E(\bar{Y})$ and $Var(\bar{Y})$?

   (c) Suppose we have observations (X_t,Y_t) for $t=1,2,\ldots T$. Give an unbiased estimator for μ and calculate the variance of your estimator.

   You may use without proof the fact that $\sum_{t=1}^{T} t=\frac{T(T+1)}{2}$ and $\sum_{t=1}^{T} t^2=\frac{T(T+1)(2T+1)}{6}$

10. (a) An economist has observations (x_i, y_i) for $i = 1, 2, \ldots N$ and estimates the relationship

$$y_i = \beta_0 + \beta_1 x_i + u_i$$

by OLS and obtains $\hat{\beta}_0 = 1, \hat{\beta}_1 = 2$. She calculates $\frac{1}{n}\sum(x_i - \bar{x})^2 = 10$, and $\frac{1}{n}\sum(y_i - \bar{y})^2 = 50$.

Let $\hat{y}_i = \hat{\beta}_0 + \hat{\beta}_1 x_i$ and $e_i = y_i - \hat{y}_i$. What is $\frac{1}{n}\sum(e_i)^2$?

(b) In the simple linear regression model $y_i = \beta_0 + \beta_1 x_i + u_i$, suppose that $E[u_i] = \mu \neq 0$. Show that the model can always be written with the same slope, but with a new intercept and error, where the new error has expected value zero.

(c) Which of the following can cause a least squares estimator to be biased? Explain.

- Heteroscedasticity
- Omitting an important variable
- A linear transformation of the X variable

SECTION D Answer ONE question from this Section

11. In this question you may assume that a t-distribution may be approximated by a normal distribution if the degrees of freedom are greater than 40.

(a) For a statistical test of a hypothesis H_0 against the alternative H_A define what is meant by Type I and Type II errors and the size and power of a test.

(b) A random sample of size $n = 50$ of a random variable X distributed $N(\mu, \sigma^2)$ has sample mean $\bar{x} = 4.3$ and $s^2 = \frac{1}{n-1}\sum(x_i - \bar{x})^2 = 9$.
Perform a test of the hypothesis $H_0 : \mu = 4$ against the alternative $H_A : \mu > 4$ at the 5% level.

(c) What is the power of the test in part (b) against the alternative $H_A : \mu = 5$

(d) Let $\Phi(z)$ be the cumulative density function of a standard normal random variable. For the test in part (b) if the sample size was just given as $n > 40$ (and the values of $\bar{x}$ and s^2 are unchanged) write an expression for the power against the alternative $H_A : \mu = 5$ in terms of n and the function $\Phi(.)$

(e) Show that the derivative with respect to n of the function you provide in part (d) is positive. How would you interpret this result?

(f) Hence or otherwise show that if $n \geq 100$ then the power of this test against the alternative $H_A : \mu = 5$ exceeds 95%

12. You have a large dataset that records information on earnings, schooling and gender for over 30,000 individuals. Your dataset reports the following variables:

- inc, monthly income in \$
- w, hourly wage in \$
- ed, years of education
- col, a dummy variable indicating completion of a university degree; 1= completed, 0=not completed
- $female$, a dummy variable indicating gender; 1=female, 0=male

(a) You estimate the least squares regression of monthly income on years of education and obtain the following parameter estimates with standard errors reported in parentheses:

$$\hat{inc} = \underset{(26.25)}{137.08} + \underset{(2.123)}{122.85}ed \tag{1}$$

Interpret the coefficient on ed and perform a hypothesis test, at the 1% significance level, that the coefficient is larger than zero. (Assume that all assumptions of the Gauss-Markov Theorem are satisfied).

(b) You next estimate two alternative specifications of the relationship between monthly income and schooling. Interpret the coefficients on ed in (2) and on $ln(ed)$ in (3). (Assume that all assumptions of the Gauss-Markov Theorem are satisfied).

$$ln(\hat{inc}) = \underset{(0.012)}{6.488} + \underset{(.001)}{.064}ed \tag{2}$$

$$ln(\hat{inc}) = \underset{(.0287)}{5.557} + \underset{(0.011)}{0.691}ln(ed) \tag{3}$$

(c) An econometrics classmate is interested in hourly wages rather than monthly income and specifies the following regression

$$ln(w) = \beta_0 + \beta_1 col + \beta_2 female + u \tag{4}$$

STATA produces the following from estimating this relationship. Some numbers in the output have been replaced by letters.

```
. reg lnw col female

      Source |       SS       df       MS              Number of obs =   31237
-------------+------------------------------           F(  2, 31234) = 2176.66
       Model |  615.110687     2  307.555344           Prob > F      =  0.0000
    Residual |  4413.26126 31234  .141296704           R-squared     =  0.1223
-------------+------------------------------           Adj R-squared =  0.1223
       Total |  5028.37194 31236  .160980021           Root MSE      =  .37589

------------------------------------------------------------------------------
         lnw |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
         col |   .2665278   .0043917    60.69   0.000     .2579199    .2751357
      female |  -.1369684     AAAA     -32.02   BBBB        CCCC        DDDD
       _cons |   2.204257   .0033226   663.41   0.000     2.197745    2.210769
------------------------------------------------------------------------------
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Calculate the values of AAAA, CCCC, and DDDD in the table above. Suggest a likely value for BBBB. Interpret this p-value.

(d) Suppose your classmate then estimates

$$ln(w) = \delta_0 + \delta_1 nocol + \delta_2 male + u \tag{5}$$

where $nocol = 1 - col$, $female = 1 - male$. What will be the least squares estimates $\hat{\delta}_0, \hat{\delta}_1$ and $\hat{\delta}_2$?

(e) The STATA output reports the R^2 for equation (4). What does the R^2 tell you? Can you say anything about the magnitude of R^2 for equation (5) relative to that for equation (4)? If you need additional information in order to determine the relative magnitude, state what information you need.

(f) As it turns out, females are more likely to go to college. Given this information, if the econometrician uses least squares to estimate

$$ln(w) = \gamma_0 + \gamma_1 col + v$$

which assumptions of the Gauss-Markov Theorem would be violated? What would this imply for your results?

END OF PAPER