UNIVERSITY OF CAMBRIDGE

ECT1
ECONOMICS TRIPOS PART 1

Thursday 17 June 2021 11.00am – 2.30pm + 30 minutes upload

Paper 3

QUANTITATIVE METHODS IN ECONOMICS

Answer **ALL FOUR** questions from Section A. **ONE** questions from Section B. **ALL FOUR** questions from Section C and **ONE** question from Section D.

Answers from Section A and B (Mathematics) and from Section C and D (Statistics) must be completed on separate pages with the letter of the Section written at the top of the page.

Section A and C will each carry 30% of the marks for this paper. Section B and D each carry 20% of the marks.

Each question within each section carries equal weight.

Write your **Blind Grade Number** (not your name) on your answers.

Candidates are asked to note that there may be a reduction in marks for scripts with illegible handwriting.

Please note that there is no mechanism available to raise a query during the assessment. If you have queries regarding your assessment or the question paper, you should continue to the best of your ability and raise these with your College Tutor after the assessment.

SPECIAL REQUIREMENTS ATTACHED TO THIS EXAM
Standard normal table
t distribution table

Submitting your assignment to Moodle
You must submit your answers within the assessment window by uploading **one single** PDF file to Moodle. This file must include the completed cover sheet and **all** your typed and hand-written pages with your answer.

Assessment window
You have **3 hours and 30minutes** to complete the exam + 30minutes to upload your answers.

SECTION A - Answer **ALL FOUR** questions from this Section.

1. Give an example of:
   (a) A function $f(x)$ which is differentiable for all $x \in \mathbb{R}$, but not twice differentiable for some $x \in \mathbb{R}$.
   (b) A function $g(x)$ which is continuous and differentiable for all $x \in \mathbb{R}$, strictly concave for $x < 0$, and simultaneously concave and convex for $x > 0$.

2. Answer the following questions:
   (a) Consider the function

$$g(x) = \int_{1/x}^{x} \ln u \mathrm{d}u$$

   with $x > 1$. For which values of $a > 1$ is this function increasing over $(1, a)$? For which values of $a > 1$ is this function concave over $(1, a)$? Justify your answers.
   (b) Find the value of $\int_0^{100} f(x)\mathrm{d}x$, where

$$f(x) = \begin{cases} 2^{-n}(x - 2n) & \text{for } x \in [2n, 2n+1] \\ -2^{-n}(x - 2n - 2) & \text{for } x \in [2n+1, 2n+2] \end{cases}$$

   with $n = 0, 1, 2, ...$ Show your work!

3. Suppose that $f(\mathbf{x})$ is a strictly concave continuous function defined on a compact convex subset D of $\mathbb{R}^2$.
   (a) Show that $f(\mathbf{x})$ does achieve its maximum on D.
   (b) Suppose further that D_0 is a compact but not necessarily convex subset of D. Give an example of $f(\mathbf{x})$, D and D_0 that satisfy the above requirements and are such that the maximum of $f(\mathbf{x})$ on D_0 is achieved at two different points.

4. Consider a 2×2 matrix $A = \begin{pmatrix} x & y \\ y & x \end{pmatrix}$
   (a) Describe the set of pairs $(x, y) \in \mathbb{R}^2$ such that $x \in [-1, 1]$, $y \in [-1, 1]$, and the corresponding A is positive definite. Is this set convex? Is this set compact? Justify your answers.
   (b) Find the maximum value of the determinant of A if it is known that $x \in [-1, 1]$, $y \in [-1, 1]$, that A is positive definite, and $2x = y + 1$. Explain why you think this is indeed the maximum.

SECTION B - Answer **ONE** question from this Section.

5. Suppose that a consumer has utility function $U(x, y) = x^\alpha + y^\alpha$ with constant $\alpha \in (0, 1)$, where $x \geq 0$ and $y \geq 0$ are quantities of two goods available for consumption.
   (a) Using the Lagrange multiplier method, find the bundle (x^*, y^*) that maximises the utility subject to the budget constraint $px + y = 1$, where $p > 0$ is the relative price of good x.
   (b) Show that your answer in (a) indeed delivers the maximum utility as opposed to, say, the minimum.
   (c) Express the value of the Lagrange multiplier at the maximum as a function of α and p. Interpret this value and comment on how it depends on p in light of this interpretation.
   (d) Let $U^*(p, \alpha) = U(x^*, y^*)$ denote the indirect utility function. Find $\frac{\partial}{\partial p} U^*(p, \alpha)$ as a function of p and α. How does this quantity behave as $\alpha \to 0$?

6. A country's population at time t equals $P(t)$ million people. This function satisfies the differential equation

$$\frac{\mathrm{d}}{\mathrm{d}t} P(t) = bP(t) + I(t),$$

where b is a positive constant, t is measured in years, and $I(t)$ equals the rate of immigration to the country from the rest of the world.
   (a) Suppose that the rate of immigration is constant $I(t) = a$. Then all solutions to the above differential equation have the form $P(t) = Ae^{bt} - a/b$, where A is some constant. Explain why this does not imply that the higher immigration rate $a_1 > a$ would lead to a lower population size $P(t)$.
   (b) Let $a = 1, b = 0.005$, and $P(2000) = 50$. Find the value of $P(2020)$.
   (c) You would like to approximate $P(t)$ by its Taylor polynomial of second degree around $t = 2000$. Suppose that all assumptions made in (a) and (b) hold. Using Taylor's remainder theorem, estimate an upper bound on the maximum of the absolute value of the approximation error for $t \in [2000, 2020]$.
   (d) Now suppose that $b = 0.005$ and $P(2000) = 50$, as above, but that the immigration was stopped at 2010 so that $I(t) = 1$ for $t \leq 2010$ but $I(t) = 0$ for $t > 2010$. Find the value of $P(2020)$. Show your work!

Section C - Answer **ALL FOUR** questions from this Section.

7. A large number of people, N, are subjected to a test for a disease. This can be administered in two ways: (1) each person can be tested separately, in this case N tests are required, (2) the samples of k persons can be pooled and analysed together. If this test is negative, this one test suffices for the k people. If the test is positive, each of the k persons must be tested separately, and, in all, $k+1$ tests are required for the k people. Assume that the probability p that a test for a single person is positive is the same for all people and that these events are independent.
   (a) Find the probability that the test for a pooled sample of k people will be positive.
   (b) What is the expected value of the number X of tests necessary under plan (2)? (Assume that N is divisible by k.)
   (c) Show that the value of k which will minimise the expected number of tests under the second plan when p is small is approximately $1/\sqrt{p}$.

       You may use, if necessary, the approximation $(1-p)^k \approx 1 - kp$ for small p.
   (d) For an airborne infectious disease if we pool by households the assumption of independence is unlikely to be met. What might be the consequence on the number of tests required under pooled testing?

8. Let X and Y be independent random variables with density functions $f_X(x)$ and $f_Y(y)$ respectively. Denote their sum by $Z = X + Y$. The density of Z is then given by the formula

$$f_Z(z) = \int_{-\infty}^{+\infty} f_X(z-y)\, f_Y(y)\, dy$$

An exponentially distributed random variable has density function

$$f_X(x) = \begin{cases} \lambda e^{-\lambda x} & x \geq 0 \\ 0 & elsewhere \end{cases}$$

where λ is a positive parameter.

(a) Sketch the density function of an exponentially distributed random variable X for the case $\lambda = 1$.

(b) Show that if X and Y are independent exponentially distributed random variables with the same λ then the density function of their sum Z is given by

$$f_Z(z) = \begin{cases} \lambda^2 z e^{-\lambda z} & z \geq 0 \\ 0 & elsewhere \end{cases}$$

Sketch the density function of Z in the case $\lambda = 1$.

Hint: In part (b) pay careful attention to the limits and variable that is integrated over.

9. A researcher has data $X_1, ..., X_N$ with $N = 50$, where X_i are independent and identically distributed random variables with unknown population mean μ and known population variance $\sigma^2 = 1600$. The researcher uses these data to run a standard one-tail test of the null hypothesis $H_0 : \mu = 170$ against the alternative $H_1 : \mu = 190$ at a significance level of $\alpha = 0.05$. Find the power of this test. State clearly any assumptions you make.

(TURN OVER)

10. Figure 1 summarises the results of a test for Covid-19 applied to a small sample of individuals. Predicted outcomes based on test results are cross-classified with a final diagnosis.

		Final Diagnosis	
		Covid	**Not Covid**
Test Result	**Covid**	TP = 82	FP = 13
	Not Covid	FN = 71	TN = 301

Figure 1

True positive (TP): the numbers with Covid-19 who had a *positive* test result.
False positive (FP): the numbers without Covid-19 who had a *positive* test result.
True negative (TN): the numbers without Covid-19 who had a *negative* test result.
False negative (FN): the numbers with Covid-19 who had a *negative* test result.

(a) Based on the data in Figure 1 interpret the probabilities

$$\tau_1 = TP/\left(TP + FN\right) \text{ and } \tau_2 = TN/\left(TN + FP\right)$$

Comment on the numerical values of τ_1 and τ_2.

(b) Using a large sample, the Covid-19 testing programme in the UK has been verified by Public Health England, with both τ_1 and τ_2 over 95%.

i. What does this tell us about the extent of false positives and false negatives?

ii. What is the link between τ_1 and τ_2 and Type I and Type II errors?

(c) An analyst computes the ratio

$$LR = \left(\frac{TP}{TP + FN}\right) / \left(\frac{FP}{FP + TN}\right)$$

i. Use the data in Figure 1 to compute a value for LR and interpret your findings.

ii. Use the data in Figure 1 to compute the pre-test odds ratio

$$\Pr\left(COVID\right) / \left(1 - \Pr\left(COVID\right)\right)$$

In what sense does the information in LR update the pre-test odds of having Covid-19?

Section D - Answer **ONE** question from this Section.

11. For two events A, B in an outcome space Ω we say event A *attracts* event B if $Prob\,(B \mid A) > Prob\,(B)$ and event A *repels* event B if $Prob\,(B \mid A) < Prob\,(B)$.
   (a) Prove that A attracts B if and only if B attracts A. Hence we can say that A and B are *mutually attractive* if A attracts B.
   (b) Prove that A neither attracts nor repels B if and only if A and B are independent.
   (c) Prove that A and B are mutually attractive if and only if $Prob\,(B \mid A) > Prob\left(B \mid \tilde{A}\right)$ where $\tilde{A}$ is the complement of A (that is those elements of Ω that are not in A so that $\Omega = A \cup \tilde{A}$ and A and $\tilde{A}$ are disjoint).
   (d) Prove that if A attracts B, then A repels $\tilde{B}$, where $\tilde{B}$ is the complement of B.
   (e) A standard pack of cards contains 52 cards divided into four suits. Let R_i be the event that the i^{th} player in a poker game is dealt a royal flush (A,K,Q,J,10 of one suit). Show that a royal flush attracts another royal flush, that is $Prob\,(R_2 \mid R_1) > Prob\,(R_2)$. Assume that, first, five cards are dealt to the first player, and then five cards from what remains in the pack are dealt to the second player.

12. Consider the regression model

$$Y_i = \beta_1 + \beta_2 X_i + u_i \tag{1}$$

with $i = 1, 2, ..., n$, where β_1 and β_2 are unknown parameters, $E(u_i) = 0$ and $Cov\,(X_i, u_i) = 0$. Let $\bar{Y}, \bar{X}$, and $\bar{u}$ denote the respective sample means.
   (a) Express β_2 in terms of $Cov(Y_i, X_i)$ and $Var\,(X_i)$. How is this expression related to $\widehat{\beta_2}$, the OLS estimate of β_2?
   (b) We now demean X_i to obtain $X_i^* = X_i - \bar{X}$.
      i. Demonstrate that the OLS estimate of the intercept in the regression of Y_i on X_i^* will be equal to $\bar{Y}$.
      ii. Show that the OLS estimate of the slope coefficient in the regression of Y_i on X_i^* will be the same as in regression (1).
   (c) Equation (1) implies

$$\bar{Y} = \beta_1 + \beta_2 \bar{X} + \bar{u}$$

   and therefore

$$Y_i^* = \beta_2 X_i^* + u_i^* \tag{2}$$

   for $i = 1, 2, ..., n$, where $Y_i^* = Y_i - \bar{Y}$, $X_i^* = X_i - \bar{X}$, and $u_i^* = u_i - \bar{u}$.
      i. Demonstrate that regression (2) will yield the same OLS estimate of β_2 as regression (1).
      ii. Determine the OLS estimate of the intercept if Y_i^* were regressed on X_i^* with an intercept included in the regression specification.

END OF PAPER

STANDARD NORMAL DISTRIBUTION: Table Values Represent AREA to the LEFT of the Z score.

Z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
-3.9	.00005	.00005	.00004	.00004	.00004	.00004	.00004	.00004	.00003	.00003
-3.8	.00007	.00007	.00007	.00006	.00006	.00006	.00006	.00005	.00005	.00005
-3.7	.00011	.00010	.00010	.00010	.00009	.00009	.00008	.00008	.00008	.00008
-3.6	.00016	.00015	.00015	.00014	.00014	.00013	.00013	.00012	.00012	.00011
-3.5	.00023	.00022	.00022	.00021	.00020	.00019	.00019	.00018	.00017	.00017
-3.4	.00034	.00032	.00031	.00030	.00029	.00028	.00027	.00026	.00025	.00024
-3.3	.00048	.00047	.00045	.00043	.00042	.00040	.00039	.00038	.00036	.00035
-3.2	.00069	.00066	.00064	.00062	.00060	.00058	.00056	.00054	.00052	.00050
-3.1	.00097	.00094	.00090	.00087	.00084	.00082	.00079	.00076	.00074	.00071
-3.0	.00135	.00131	.00126	.00122	.00118	.00114	.00111	.00107	.00104	.00100
-2.9	.00187	.00181	.00175	.00169	.00164	.00159	.00154	.00149	.00144	.00139
-2.8	.00256	.00248	.00240	.00233	.00226	.00219	.00212	.00205	.00199	.00193
-2.7	.00347	.00336	.00326	.00317	.00307	.00298	.00289	.00280	.00272	.00264
-2.6	.00466	.00453	.00440	.00427	.00415	.00402	.00391	.00379	.00368	.00357
-2.5	.00621	.00604	.00587	.00570	.00554	.00539	.00523	.00508	.00494	.00480
-2.4	.00820	.00798	.00776	.00755	.00734	.00714	.00695	.00676	.00657	.00639
-2.3	.01072	.01044	.01017	.00990	.00964	.00939	.00914	.00889	.00866	.00842
-2.2	.01390	.01355	.01321	.01287	.01255	.01222	.01191	.01160	.01130	.01101
-2.1	.01786	.01743	.01700	.01659	.01618	.01578	.01539	.01500	.01463	.01426
-2.0	.02275	.02222	.02169	.02118	.02068	.02018	.01970	.01923	.01876	.01831
-1.9	.02872	.02807	.02743	.02680	.02619	.02559	.02500	.02442	.02385	.02330
-1.8	.03593	.03515	.03438	.03362	.03288	.03216	.03144	.03074	.03005	.02938
-1.7	.04457	.04363	.04272	.04182	.04093	.04006	.03920	.03836	.03754	.03673
-1.6	.05480	.05370	.05262	.05155	.05050	.04947	.04846	.04746	.04648	.04551
-1.5	.06681	.06552	.06426	.06301	.06178	.06057	.05938	.05821	.05705	.05592
-1.4	.08076	.07927	.07780	.07636	.07493	.07353	.07215	.07078	.06944	.06811
-1.3	.09680	.09510	.09342	.09176	.09012	.08851	.08691	.08534	.08379	.08226
-1.2	.11507	.11314	.11123	.10935	.10749	.10565	.10383	.10204	.10027	.09853
-1.1	.13567	.13350	.13136	.12924	.12714	.12507	.12302	.12100	.11900	.11702
-1.0	.15866	.15625	.15386	.15151	.14917	.14686	.14457	.14231	.14007	.13786
-0.9	.18406	.18141	.17879	.17619	.17361	.17106	.16853	.16602	.16354	.16109
-0.8	.21186	.20897	.20611	.20327	.20045	.19766	.19489	.19215	.18943	.18673
-0.7	.24196	.23885	.23576	.23270	.22965	.22663	.22363	.22065	.21770	.21476
-0.6	.27425	.27093	.26763	.26435	.26109	.25785	.25463	.25143	.24825	.24510
-0.5	.30854	.30503	.30153	.29806	.29460	.29116	.28774	.28434	.28096	.27760
-0.4	.34458	.34090	.33724	.33360	.32997	.32636	.32276	.31918	.31561	.31207
-0.3	.38209	.37828	.37448	.37070	.36693	.36317	.35942	.35569	.35197	.34827
-0.2	.42074	.41683	.41294	.40905	.40517	.40129	.39743	.39358	.38974	.38591
-0.1	.46017	.45620	.45224	.44828	.44433	.44038	.43644	.43251	.42858	.42465
-0.0	.50000	.49601	.49202	.48803	.48405	.48006	.47608	.47210	.46812	.46414

STANDARD NORMAL DISTRIBUTION: Table Values Represent AREA to the LEFT of the Z score.

Z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.50000	.50399	.50798	.51197	.51595	.51994	.52392	.52790	.53188	.53586
0.1	.53983	.54380	.54776	.55172	.55567	.55962	.56356	.56749	.57142	.57535
0.2	.57926	.58317	.58706	.59095	.59483	.59871	.60257	.60642	.61026	.61409
0.3	.61791	.62172	.62552	.62930	.63307	.63683	.64058	.64431	.64803	.65173
0.4	.65542	.65910	.66276	.66640	.67003	.67364	.67724	.68082	.68439	.68793
0.5	.69146	.69497	.69847	.70194	.70540	.70884	.71226	.71566	.71904	.72240
0.6	.72575	.72907	.73237	.73565	.73891	.74215	.74537	.74857	.75175	.75490
0.7	.75804	.76115	.76424	.76730	.77035	.77337	.77637	.77935	.78230	.78524
0.8	.78814	.79103	.79389	.79673	.79955	.80234	.80511	.80785	.81057	.81327
0.9	.81594	.81859	.82121	.82381	.82639	.82894	.83147	.83398	.83646	.83891
1.0	.84134	.84375	.84614	.84849	.85083	.85314	.85543	.85769	.85993	.86214
1.1	.86433	.86650	.86864	.87076	.87286	.87493	.87698	.87900	.88100	.88298
1.2	.88493	.88686	.88877	.89065	.89251	.89435	.89617	.89796	.89973	.90147
1.3	.90320	.90490	.90658	.90824	.90988	.91149	.91309	.91466	.91621	.91774
1.4	.91924	.92073	.92220	.92364	.92507	.92647	.92785	.92922	.93056	.93189
1.5	.93319	.93448	.93574	.93699	.93822	.93943	.94062	.94179	.94295	.94408
1.6	.94520	.94630	.94738	.94845	.94950	.95053	.95154	.95254	.95352	.95449
1.7	.95543	.95637	.95728	.95818	.95907	.95994	.96080	.96164	.96246	.96327
1.8	.96407	.96485	.96562	.96638	.96712	.96784	.96856	.96926	.96995	.97062
1.9	.97128	.97193	.97257	.97320	.97381	.97441	.97500	.97558	.97615	.97670
2.0	.97725	.97778	.97831	.97882	.97932	.97982	.98030	.98077	.98124	.98169
2.1	.98214	.98257	.98300	.98341	.98382	.98422	.98461	.98500	.98537	.98574
2.2	.98610	.98645	.98679	.98713	.98745	.98778	.98809	.98840	.98870	.98899
2.3	.98928	.98956	.98983	.99010	.99036	.99061	.99086	.99111	.99134	.99158
2.4	.99180	.99202	.99224	.99245	.99266	.99286	.99305	.99324	.99343	.99361
2.5	.99379	.99396	.99413	.99430	.99446	.99461	.99477	.99492	.99506	.99520
2.6	.99534	.99547	.99560	.99573	.99585	.99598	.99609	.99621	.99632	.99643
2.7	.99653	.99664	.99674	.99683	.99693	.99702	.99711	.99720	.99728	.99736
2.8	.99744	.99752	.99760	.99767	.99774	.99781	.99788	.99795	.99801	.99807
2.9	.99813	.99819	.99825	.99831	.99836	.99841	.99846	.99851	.99856	.99861
3.0	.99865	.99869	.99874	.99878	.99882	.99886	.99889	.99893	.99896	.99900
3.1	.99903	.99906	.99910	.99913	.99916	.99918	.99921	.99924	.99926	.99929
3.2	.99931	.99934	.99936	.99938	.99940	.99942	.99944	.99946	.99948	.99950
3.3	.99952	.99953	.99955	.99957	.99958	.99960	.99961	.99962	.99964	.99965
3.4	.99966	.99968	.99969	.99970	.99971	.99972	.99973	.99974	.99975	.99976
3.5	.99977	.99978	.99978	.99979	.99980	.99981	.99981	.99982	.99983	.99983
3.6	.99984	.99985	.99985	.99986	.99986	.99987	.99987	.99988	.99988	.99989
3.7	.99989	.99990	.99990	.99990	.99991	.99991	.99992	.99992	.99992	.99992
3.8	.99993	.99993	.99993	.99994	.99994	.99994	.99994	.99995	.99995	.99995
3.9	.99995	.99995	.99996	.99996	.99996	.99996	.99996	.99996	.99997	.99997

Table entry for p and C is the critical value t^* with probability p lying to its right and probability C lying between $-t^*$ and t^*.

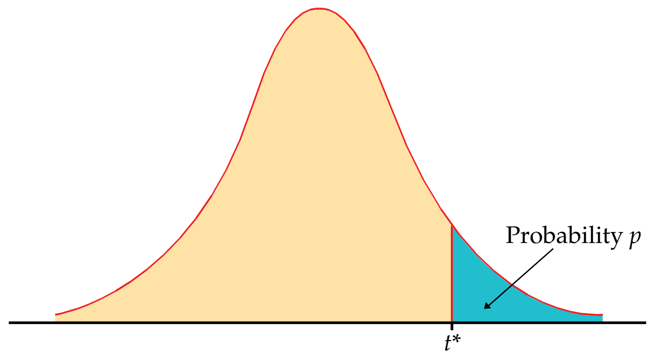

TABLE D

t distribution critical values

	Upper-tail probability p											
df	.25	.20	.15	.10	.05	.025	.02	.01	.005	.0025	.001	.0005
1	1.000	1.376	1.963	3.078	6.314	12.71	15.89	31.82	63.66	127.3	318.3	636.6
2	0.816	1.061	1.386	1.886	2.920	4.303	4.849	6.965	9.925	14.09	22.33	31.60
3	0.765	0.978	1.250	1.638	2.353	3.182	3.482	4.541	5.841	7.453	10.21	12.92
4	0.741	0.941	1.190	1.533	2.132	2.776	2.999	3.747	4.604	5.598	7.173	8.610
5	0.727	0.920	1.156	1.476	2.015	2.571	2.757	3.365	4.032	4.773	5.893	6.869
6	0.718	0.906	1.134	1.440	1.943	2.447	2.612	3.143	3.707	4.317	5.208	5.959
7	0.711	0.896	1.119	1.415	1.895	2.365	2.517	2.998	3.499	4.029	4.785	5.408
8	0.706	0.889	1.108	1.397	1.860	2.306	2.449	2.896	3.355	3.833	4.501	5.041
9	0.703	0.883	1.100	1.383	1.833	2.262	2.398	2.821	3.250	3.690	4.297	4.781
10	0.700	0.879	1.093	1.372	1.812	2.228	2.359	2.764	3.169	3.581	4.144	4.587
11	0.697	0.876	1.088	1.363	1.796	2.201	2.328	2.718	3.106	3.497	4.025	4.437
12	0.695	0.873	1.083	1.356	1.782	2.179	2.303	2.681	3.055	3.428	3.930	4.318
13	0.694	0.870	1.079	1.350	1.771	2.160	2.282	2.650	3.012	3.372	3.852	4.221
14	0.692	0.868	1.076	1.345	1.761	2.145	2.264	2.624	2.977	3.326	3.787	4.140
15	0.691	0.866	1.074	1.341	1.753	2.131	2.249	2.602	2.947	3.286	3.733	4.073
16	0.690	0.865	1.071	1.337	1.746	2.120	2.235	2.583	2.921	3.252	3.686	4.015
17	0.689	0.863	1.069	1.333	1.740	2.110	2.224	2.567	2.898	3.222	3.646	3.965
18	0.688	0.862	1.067	1.330	1.734	2.101	2.214	2.552	2.878	3.197	3.611	3.922
19	0.688	0.861	1.066	1.328	1.729	2.093	2.205	2.539	2.861	3.174	3.579	3.883
20	0.687	0.860	1.064	1.325	1.725	2.086	2.197	2.528	2.845	3.153	3.552	3.850
21	0.686	0.859	1.063	1.323	1.721	2.080	2.189	2.518	2.831	3.135	3.527	3.819
22	0.686	0.858	1.061	1.321	1.717	2.074	2.183	2.508	2.819	3.119	3.505	3.792
23	0.685	0.858	1.060	1.319	1.714	2.069	2.177	2.500	2.807	3.104	3.485	3.768
24	0.685	0.857	1.059	1.318	1.711	2.064	2.172	2.492	2.797	3.091	3.467	3.745
25	0.684	0.856	1.058	1.316	1.708	2.060	2.167	2.485	2.787	3.078	3.450	3.725
26	0.684	0.856	1.058	1.315	1.706	2.056	2.162	2.479	2.779	3.067	3.435	3.707
27	0.684	0.855	1.057	1.314	1.703	2.052	2.158	2.473	2.771	3.057	3.421	3.690
28	0.683	0.855	1.056	1.313	1.701	2.048	2.154	2.467	2.763	3.047	3.408	3.674
29	0.683	0.854	1.055	1.311	1.699	2.045	2.150	2.462	2.756	3.038	3.396	3.659
30	0.683	0.854	1.055	1.310	1.697	2.042	2.147	2.457	2.750	3.030	3.385	3.646
40	0.681	0.851	1.050	1.303	1.684	2.021	2.123	2.423	2.704	2.971	3.307	3.551
50	0.679	0.849	1.047	1.299	1.676	2.009	2.109	2.403	2.678	2.937	3.261	3.496
60	0.679	0.848	1.045	1.296	1.671	2.000	2.099	2.390	2.660	2.915	3.232	3.460
80	0.678	0.846	1.043	1.292	1.664	1.990	2.088	2.374	2.639	2.887	3.195	3.416
100	0.677	0.845	1.042	1.290	1.660	1.984	2.081	2.364	2.626	2.871	3.174	3.390
1000	0.675	0.842	1.037	1.282	1.646	1.962	2.056	2.330	2.581	2.813	3.098	3.300
z^*	0.674	0.841	1.036	1.282	1.645	1.960	2.054	2.326	2.576	2.807	3.091	3.291
	50%	60%	70%	80%	90%	95%	96%	98%	99%	99.5%	99.8%	99.9%
	Confidence level C											