The NAIRU: Still ›Not An Interesting Rate of Unemployment‹

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This paper assesses the Layard et al. (1991) NAIRU framework for explaining unemployment. Their approach is distinct from the natural rate of unemployment framework in that it postulates a short-run NAIRU influenced by ›hysteresis‹. It is pointed out that this is not hysteresis in the meaning employed elsewhere, so an outline of what hysteresis actually implies for unemployment is offered. The main implication is that unemployment does not revert to a long-run ›natural rate‹ equilibrium, as claimed by Layard et al., but instead is shaped by the past extrema of dominant exogenous shocks. It is argued that this is a more useful approach to the explanation of equilibrium unemployment than the NAIRU, which, for its analytical and empirical flaws, can be considered to be ›not an interesting rate of unemployment‹. The hysteretic alternative to the natural rate hypothesis can be called the DESIRU (dominant extrema, steady inflation, rate of unemployment).

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1. Introduction

The NAIRU, or non-accelerating inflation rate of unemployment, was the hallmark of the Centre for Labour Economics (CLE) at LSE. Its most popular version appears in the book by Richard Layard, Stephen Nickell and Richard Jackman (LNJ hereafter), *Unemployment:*

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Macroeconomic Performance and the Labour Market (1991). Since the beginning of the 2000's, this version of the NAIRU has been included in the so-called ›New Macroeconomic Consensus‹ model, a model used by most of the central banks in the world (see Arestis [2007] for a complete account of the NCM model). This model has often been criticized as one of the causes of the 2007 crisis (see for example King [2009]). This paper proposes to examine one of the most popular versions of the NAIRU – the one framed by LNJ – and to assess its relevance for the current economic policies.

The discussion is structured as follows: Section 2 discusses what LNJ mean by the NAIRU, and how this relates to the natural rate concept of equilibrium unemployment. Section 3 traces how the original single NAIRU bifurcated into medium- and long-run NAIRUs: the LNJ medium-run NAIRU is influenced by ›hysteresis‹, the long-run NAIRU is not. Section 4 discusses the interpretation placed on hysteresis by LNJ, and compares this to the way hysteresis is employed elsewhere. Section 5 discusses the analytical and empirical arguments used by LNJ to support their claim that unemployment always reverts to a long-run NAIRU corresponding to the natural rate of unemployment. Section 6 details the main conclusions to emerge from the paper.

2. The NAIRU

An obvious question is how the NAIRU relates to the natural rate of unemployment concept framed by Phelps (1967) and Friedman (1968). A common characteristic is that both the natural rate and the NAIRU will be accompanied by correct inflation expectations and/or by a constant rate of inflation. Given this characteristic, shared by lots of versions of the concept but especially by the one of LNJ, ›NAIRU‹ is clearly the wrong acronym. Let P stand for the log of the price level: thus $\dot{P}$ is the rate of inflation, $\ddot{P}$ is the rate of change of the rate of inflation, and $\dddot{P}$ is the rate of acceleration of the rate of inflation, where dots indicate time derivatives. Using this terminology, the NAIRU implies $\dddot{P} \leq 0$. But the NAIRU is clearly meant to imply that $\dddot{P} = 0$, as it is defined as »the level of unemployment at which inflation stabilises« (LNJ: 8). This suggests that correct acronyms would involve dropping a time derivative and the inequality sign. Thus NANDPLRU (non-accelerating, non-decelerating price level rate of unemployment) or, more succinctly, CIRU (constant inflation rate of unemployment) would convey the meaning intended. Footnotes to the main text display an awareness of this problem:

> »NAIRU stands for non-accelerating inflation rate of unemployment […] as is also well known this description is incorrect having slipped a derivative […] it is the price level that is non-accelerating« (LNJ: 396, fn.14); »a more accurate term would be the non-increasing inflation rate of unemployment« (LNJ: 77, fn. 5).

These corrections remove the problem of the slipped derivative, but still imply a weak inequality $\dddot{P} \leq 0$. The question begged is why, if the misleading nature of the acronym was ›well known‹, it was used. The natural rate of unemployment concept is invariably attributed to Friedman (1968), and the associated symbol u^* to Phelps (1967): it is odd that the NAIRU mis-acronym is not attributed to a particular author.

The substantive issue concerns the relationship between the NAIRU and the natural rate of unemployment. The authors have at least three reasons for not using the ›well known‹ natural rate terminology to describe their concept of equilibrium unemployment:

> »the long-run equilibrium rate of unemployment is also often called the ›natural‹ rate of unemployment (Friedman 1968) […] we avoid this usage which smacks of inevitability« (LNJ: 77, fn.3); »there is no point trying to label this theory as Keynesian or classical […] it has classical elements (the NAIRU) and it has Keynesian elements (the role of demand and persistence) […] so it is best to avoid those terms, which mean something different to every reader« (LNJ: 11);

and

> »the level of unemployment at which inflation stabilises is the equilibrium rate of unemployment […] this concept of equilibrium has nothing to do with the concept of ›market-clearing‹, any more than the equilibrium of a system of pulleys has to do with market-clearing. […] it simply represents the state to which the system will return after a disturbance« (LNJ: 8–10).

As far as market-clearing is concerned, the LNJ microfoundations are concerned with price and wage setting behaviours, hence with imperfectly competitive product and labour markets (LNJ, chapters 2–7). Friedman's definition of the natural rate, however, included

> »the actual structural characteristics of labour and commodity markets, including market imperfections, stochastic variability in demands and supplies, the cost of gathering information about job vacancies and labour availabilities, the costs of labour immobility, and so on« (Friedman 1968: 8),

so market imperfections are not a point of departure. The Phelps account of the microfoundations of the natural rate is formulated in terms of price-setting rather than price-taking behaviour (Phelps, 1970), so this also is not a point of departure. The distinction is clear in terms of the Lucas (1972) account of market-clearing, price-taking equilibrium, as pointed out by LNJ (20–21), but Lucas did not coin the natural rate terminology.

It is also not clear that the natural rate ›smacks of inevitability‹. Friedman stressed that this was not intended:

> »to avoid misunderstanding, let me emphasise that by using the term ›natural‹ rate of unemployment, I do not mean to suggest that it is immutable and unchangeable, on the contrary, many of the market characteristics that determine its level are man-made and policy-made« (Friedman 1968: 9)

In fact LNJ themselves make it clear that their long-run NAIRU is the same thing as the natural rate:

> »past events affect the current short-run NAIRU […] but there is no long-term ›hysteresis‹ […] there is a unique long-run NAIRU […] in the end the unemployment rate always reverts« (LNJ: 10); »in the long run unemployment is determined by the long-run supply factors« (LNJ: 16); and »a key feature of the model is that it is fundamentally of the ›natural rate‹ type: that is, exogenous demand-side factors do not influence the equilibrium« (LNJ: 369).

Thus the long-run NAIRU is a classical, natural rate equilibrium, leaving the short-run NAIRU as the equilibrium concept which might be worth the ungainly and inaccurate acronym.

3. The short-run NAIRU

The natural rate parallel

Friedman originally postulated two types of Phillips relationships: the short-run Phillips curve for unanticipated inflation, and natural rate loci for anticipated inflation. The problem confronting this hypothesis in the 1970s was that

> »in recent years higher unemployment has often been accompanied by higher, not lower, unemployment, especially for periods of several years in length […] a simple statistical Phillips curve for such periods seems to be positively sloped, not vertical […]« (Friedman 1977: 459–460).

With a view to »accommodating this apparent empirical phenomenon« Friedman postulated a second type of natural rate relationship for the medium run, the positively sloped Phillips curve. This medium-run relationship is characterised by correctly anticipated inflation, but conditioned by the previous experience of a different rate of anticipated inflation. The requirements for quick adjustment to a higher anticipated rate of inflation are

> »that inflation is […] no more variable at a high rate than at a low […] that relative price adjustments are the same with a 20 per cent inflation as with a zero inflation […] and that there are no obstacles to the indexation of contracts […] when a country initially moves to higher rates of inflation these requirements will be systematically departed from […] such a transitional period may well extend over decades« (Friedman 1977: 465).

The parallel is that the NAIRU was initially proposed as a single concept of equilibrium unemployment, and spawned distinct short- and long-run counterparts, as the original NAIRU was perceived to be inconsistent with the evidence. In both cases the original concept of equilibrium unemployment was retained but postulated to be relevant to some longer-run domain than initially postulated.

The single NAIRU

The first version of the NAIRU did not distinguish between short- and long-run equilibria (Layard/Nickell 1985). The idea that the long-term unemployed »exert less downward pressure on wages than the short-term unemployed« (Layard/Nickell 1985: 71) is considered, but in the preferred wage equation is captured by using log *u* rather *u* (male unemployment rate in per cent) as an explanatory variable:

> »as unemployment rises, the short-term proportion falls, and the unemployment effect has the concave shape characteristic of the log function« (Layard/Nickell 1985: 72).

Thus even though the long-term unemployment proportion depends on past unemployment (equations 36 and 40), this potential source of ›hysteresis‹ is captured by solving the model for log NAIRU rather than by distinguishing between short- and long-run equilibria. The conclusion regarding the effect of lagged unemployment in the wage equation, estimated on annual data 1954–83, was:

> »we have experimented with simple lagged unemployment and with various measures of earlier unemployment in no case did this show up with significant effect« (Layard/Nickell 1985: 79–80).

This single NAIRU structure is retained in the next version of the model (Layard/Nickell 1986), though a hint of possible ›hysteresis‹ effects had appeared in the quarterly results: »we have some evidence that past unemployment tends to raise unemployment today, but only in our quarterly results« (Layard/Nickell 1986: 165).

The short-run NAIRU

Although this is not made clear in LNJ, the short-run NAIRU emerged as an attempt to rescue the original NAIRU from an embarrassingly early empirical refutation. Actual unemployment in the UK continued to rise until 1987, yet the wage pressure index (based on a replacement ratio for unemployment benefits, union power, employers' labour tax rates, mismatch and relative import prices) determining the NAIRU turned downwards around 1982. Hence the gap between actual and estimated equilibrium unemployment ($U - U^*$) increased over 1982–87 (Jenkinson 1987: fig. 1). The natural rate-NAIRU hypothesis implies that $\ddot{P} < 0$ when $U > U^*$. Over the 1982–1987 period, however, $\ddot{P}$ was approximately zero for various measures of inflation, so the NAIRU encountered a major empirical refutation during the period in which it was constructed.

The short-run NAIRU was unveiled in Layard and Nickell (1987) and Nickell (1987). The point of departure is the observation that

> »since 1982, wage inflation has not fallen to any significant extent yet male unemployment is now a staggering 10 percentage points higher than its 1979 level […] must we then conclude that the natural rate of unemployment has risen by an

> equally staggering amount? Such evidence as we possess appears totally against this conclusion […] AU 1

Layard and Nickell (1985 and 1986) indicate that by 1983 the natural rate of unemployment was 9–11 per cent,

> »the evidence suggests that the key wage pressure variables underlying this estimate have, if anything, tended to move in a favourable direction since that time and there seem few reasons for believing that wage pressure has significantly increased in the last 3 years« (Nickell 1987: 256, 280 from the 1988). AU 2

The distinction between the short- and long-run NAIRU is illustrated by the following simplified equations presented by LNJ (378), with the capital-labour ratio terms omitted:

$$p - w = \beta_0 - \beta_1 U - \beta_{11}\Delta U - \beta_2 \Delta^2 p \ , \tag{1}$$

$$w - p = \gamma_0 - \gamma_1 U - \gamma_{11}\Delta U - \gamma_2 \Delta^2 p + Z_w \ , \tag{2}$$

Where p and w are the logs of the price and wage levels, U is percentage unemployment, $\Delta^2 p$ is the LNJ proxy for the $(p-p^e)$ and $(w-w^e)$ unexpected price and wage terms in the price and wage equations, and Z_w is the index of wage pressure. The innovation in relation to the original NAIRU is the presence of the ΔU terms, the coefficients β_{11} and γ_{11} picking up the effects of ›hysteresis‹ in price and wage setting behaviour, respectively.

A NAIRU equilibrium is defined as occurring when $(p-p^e) = (w-w^e) = \Delta^2 p = 0$, i.e. price and wage level expectations are correct, and the rate of inflation is constant. Without the ›hysteresis‹ term, i.e. setting $\beta_{11} = \gamma_{11}$, or $\Delta U = 0$, the equilibrium condition implies a long-run natural rate NAIRU of:

$$U^* = \frac{\beta_0 + \gamma_0 + Z_w}{\beta_1 + \gamma_1} \ , \tag{3}$$

The short-run NAIRU is termed U_S^* and is defined as »that level of unemployment which is consistent with stable inflation *during the current period*« (LNJ: 382, emphasis in original). The emphasis is presumably there to stress that the short-run NAIRU is a temporary equilibrium, i.e. one that is not sustainable because of the ›hysteresis‹ effects arising from $\Delta U \neq 0$. Using this last condition and $\Delta^2 p = 0$, implies:

$$U_S^* = \frac{\beta_0 + \gamma_0 - (\beta_{11} + \gamma_{11})\Delta U + Z_w}{\beta_1 + \gamma_1} \tag{4}$$

$$\text{or} \quad U_S^* = U^* - \frac{(\beta_{11} + \gamma_{11})}{\beta_1 + \gamma_1}\Delta U \tag{5}$$

Thus U_S^* can be interpreted as the long-run natural rate NAIRU adjusted for the change in (or more generally, dynamics in) unemployment: rising unemployment implies $U_S^* < U^*$ and falling unemployment $U_S^* > U^*$.

Alternatively, equation (4) can be re-written, using $U_S^* > U^*$, as (LNJ: 382):

$$U_S^* = \frac{(\beta_1 + \gamma_1)\, U^* + (\beta_{11} + \gamma_{11})\, U_{-1}}{\beta_1 + \gamma_1 + \beta_{11} + \gamma_{11}} \quad . \tag{6}$$

Thus

> »the short-run NAIRU is a weighted average of last period's unemployment rate and the actual NAIRU. If the hysteresis coefficients are dominant $(\beta_1 + \gamma_1) + (\beta_{11} + \gamma_{11})$ then U_S^* is much closer to U_{-1} than to U^*, and during the current period the economy behaves as if its equilibrium is close to last period's actual rate, whatever that might be« (LNJ: 382).

4. Hysteresis and the NAIRU

Basically, the LNJ short-run NAIRU captures ›hysteresis‹ by having the change in, or more generally the dynamics in, unemployment (or the level of economic activity) influence price and wage-setting behaviour. In the price equation, this is captured by the ΔU term in equation (1) above:

> »the hysteresis effect is generated by employment adjustment costs […] short-run marginal costs increase more rapidly than long-run marginal costs because of the incomplete adjustment of employment in the short-run […] this generates an upward pressure on prices, in response to increases in demand, which is greater in the short run than in the long run […] hence we have a positive effect arising from *changes* in demand, as well as the level *effect*« (LNJ: 344).

In the wage equation, the $\gamma_{11}\Delta U$ ›hysteresis‹ effect in equation (2) arises because »long-term unemployment reduces the effectiveness of the unemployed as potential fillers of vacancies« (LNJ: 4):

> »long-term unemployment both demoralises the individual and is also used by employers as a (biased) screening device […] thus if the average level of unemployment rises, we can expect the average level of […] ›effectiveness‹ of the average unemployed job seeker to fall« (LNJ: 39).

The link between the proportion of the long-term unemployed (LTU) and unemployment is given by:

$$LTU = \alpha_0 + \alpha_1 U - \alpha_2 \Delta U \quad , \tag{7}$$

> »when unemployment rises, the inflow of new entrants naturally tends to reduce the long-term proportion […] however, in the long run higher unemployment tends to be associated with a high long-term proportion« (LNJ: 203).

The authors postulate that ›hysteresis‹ influences only the short-run NAIRU:

> »there is short-term ›hysteresis‹, in the sense that past events affect the current short-run NAIRU […] but there is no long-term ›hysteresis‹: there is a unique long-run NAIRU […] in the end the unemployment rate always reverts« (LNJ: 10).

This reflects a distinction drawn between *pure* and *partial* hysteresis:

> »if there are effects from the rate of change of economic activity but not from its level […] then any exogenous shock to economic activity will tend to have a permanent impact because, once the shock is no longer in force, the changed level of economic activity will have no impact on wages and prices […] there is, therefore, no tendency for the economy to move back to its original state […] this may be thought of as *pure* hysteresis […] any tendency in this direction, where there are short-run change effects but long-run level effects are also present may thus be thought of as *partial* hysteresis« (LNJ: 336).

Apart from this, the authors offer little by way of formal analysis of what they understand by ›hysteresis‹, and do not address the issue of whether ›partial hysteresis‹ in the short-run NAIRU is consistent with a non-hysteretic long-run NAIRU. For this reason, and because the invocation of hysteresis has been central to the LNJ attempt to save their original NAIRU hypothesis from empirical refutation, it is of interest to ask how the LNJ interpretation of ›hysteresis‹ relates to usage elsewhere.

Hysteresis in linear systems

In terms of formal analysis, economists have tended to view hysteresis as a special case of linear systems of equations. In continuous time models this is the special case of a zero root solution:

> »in a system of linear differential equation with constant coefficients of the form $\dot{x} = Ax - z$ hysteresis occurs when the transition matrix is singular so that the solution to $A\bar{x} - \bar{z} = \mathbf{0}$ is indeterminate […] this indeterminacy however is only apparent […] if the stability conditions are satisfied, for any set of initial conditions there exists a unique stationary equilibrium which may be directly computed« (Giavazzi/Wyplosz 1985: 353).

In discrete time models, hysteresis has been defined as the unit root case: »formally, hysteresis occurs […] when the system of difference equation possesses a unit root« (Wyplosz 1987: 124). Again the indeterminacy is more apparent than real: if $b = 1$ in $x_t = a + bx_{t-1} + z_t$, $\bar{x} = (a + \bar{z})(1-b)^{-1}$ admits an infinity of solutions, but at any time T the solution will be $x_T = x_\mathbf{0} + aT + \sum z_{T-i}$.

If this linear equation interpretation of hysteresis is what LNJ mean by ›pure hysteresis‹, there is a basic asymmetry in their NAIRU analysis. As far as the rate of inflation is concerned, the authors »assume […] that the inflation process has a unit root« (LNJ: 396). This assumption plays a key role in their empirical work, where it is assumed that $w-w^e = p-p^e = \Delta^2 p$. The rationale for this assumption is that »over the last two decades inflation processes in most OECD countries have had a root very close to unity« (LNJ: 378). Thus the rate of inflation follows a process $\Delta P = \Delta P_{-1} + v$, where v is white noise, and inflation expectations are assumed to be geared to this process, so that $P^e = P_{-1} + \Delta P_{-1}$ and hence $P - P^e = \Delta P - \Delta P_{-1} = \Delta^2 P$ (LNJ: 377).

The asymmetry is that LNJ do not apply similar reasoning to unemployment. It is reasonably clear from the LNJ (2) time series that, in the 1970s as well as in the 1990s, unemployment rates in OECD countries have had roots ›very close to unity‹. This is confirmed by the regressions reported by LNJ for the UK and USA, albeit for 1900–89:

> »when unemployment is regressed on lagged unemployment, the coefficient on the latter is close to unity […] for Britain, $U_t = 0.0041 + 0.943\ U_{t-1}$ (0.039), and for the USA, $U_t = 0.0080 + 0.877\ U_{t-1}$ (0.051) (s.e. in brackets)« (LNJ: 77).

This evidence, however, is disregarded by assumption: »this kind of exercise assumes an unvarying stochastic ›unemployment process‹, which we do not« (LNJ: 77). This begs the question of why a unit root process ›over the last two decades‹ can be a safe and central assumption for the rate of inflation, but not for the rate of unemployment. Such an assumption for unemployment is implicit in the cointegration tests reported in Nickell (1988: 382–383), so it is odd that this avenue of ›pure hysteresis‹ is ignored in the present volume. AU 3

Hysteresis in Non-Linear Systems

The term ›hysteresis‹ was first coined to describe the effects of stress on the thermoelectric properties of metals (Ewing 1881). The celebrated application was to the hysteresis loops described by electromagnetic fields in ferric metals in response to variations in magnetising force:

> »I have […] found it convenient and even necessary to employ a new term […] hysteresis (occurs) when there are two qualities M and N such that cyclic variation of N cause cyclic variations of M (and) the changes of M lag behind those of N […] the value of M at any point of the operation depends not only on the actual value of N, but on all the preceding changes of N« (Ewing 1885: 524–526).

Non-linearities in the form of hysteresis loops were thus a key feature when the term was first coined for application to scientific phenomena.[1]

This is important for the analysis of the NAIRU because LNJ lay great stress on the non-linear nature of the unemployment effect in the wage equation:

> »until now we have supposed that the impact of unemployment on (log) wages is linear […] there are, however, a number of reasons for believing this relationship may be concave […] as unemployment rises, the downward pressure which it exerts on

1 For a complete presentation of hysteresis models and their empirical implementation, see Lang (2009).

(log) wages is decreasing at the margin […] this notion has a long history exemplified by the fact that the relevant line is known as the Phillips *curve* rather than the Phillips *line* […] recently unemployed persons are both more active in looking for work and more attractive to employers […] so the higher the proportion of such individuals in the unemployment pool, the more effective are the unemployed as a whole in exerting downward pressure on wages […] since, in the long run, this proportion will tend to be lower at a higher level of unemployment, this will automatically introduce concavity into the wage-unemployment relationship« (LNJ: 381).

This concavity is represented by log U in the LNJ wage equation for the UK, 1956–85, though it is not clear why the LTU variable is included as well:

$$w - p = \underset{[4.8]}{8.62 - 0.091} \log U + \underset{[2.7]}{0.193\, LTU} + Z_w \qquad (8)$$

$$LTU = \underset{[2.8]}{0.074} + \underset{[3.7]}{0.496\, LTU_{-1}} - 2.15\, U + \underset{[5.3]}{5.65\, U_{-1}} - \underset{[2.3]}{2.39\, U_{-2}} \qquad (9)$$

(LNJ: 441, with t-ratios in brackets and the factors determining the natural rate or long-run NAIRU represented by the wage pressure index Z_w). This non-linearity is the more obvious starting point for the analysis of ›hysteresis‹ in the LNJ NAIRU model. The concavity of the Phillips curve, for example, implies that an increase in the size of demand or supply shocks will, by increasing the variation of actual unemployment, increase the natural rate or NAIRU. The question is then one of whether permanent hysteresis effects will arise from the duration composition variation underlying the LNJ interpretation of the concavity of the Phillips curve.

The general mathematical analysis of the hysteresis effects arising from such a non-linearity has been developed by the Krasnosel'skii research group (Krasnosel'skii/Pokrovskii 1989), and can be traced back to the Preisach model of 1935. The central result arising from this analysis is that non linearities at the micro level generate macro level systems with non-local memory when exposed to input shocks. This hysteresis in systems behaviour is a general characteristic of non-linearities, and can be demonstrated for both static and dynamic non-linearities, for scalar and vector inputs and outputs, for piecewise monotonic functions and for complete function spaces, and for both deterministic and stochastic processes (Krasnosel'skii/Pokrovskii 1989). Chaotic behaviour, for example, arises as special case (Krasnosel'skii/Pokrovskii 1989: Part 6).

The most straightforward way to illustrate the non-local memory property of hysteretic systems is the case of static piecewise monotonic functions (Mayergoyz 1985 and 1991). The hysteresis non-linearity appears as what is termed a ›hysteron transducer‹ (HT), which describes the way the input shocks, $i(t)$, are transformed into the output of the system $f(t)$: schematically $i(t) \rightarrow \text{HT} \rightarrow f(t)$. A basic distinction is between what can be termed ›weak hysteresis‹, where the system has only local memory, and ›strong hysteresis‹, characterised by non-local memory. In the case of *weak hysteresis*,

»the value of output $f(t_0)$ at some instant of time t_0 and the values of input $i(t)$ at all subsequent instants of time $t > t_0$ uniquely predetermine the value of output $f(t)$ for all $t > t_0$ […] in other words the past exerts its influence upon the future through the current value of output« (Mayergoyz 1991: xiv).

With *strong hysteresis*, »future values of output $f(t)$, $t > t_0$, depend not only on the current value of output $f(t_0)$ but also on the past extremum values of input as well« (Mayergoyz 1991: xv).

The distinction is illustrated in Figures 1 and 2, with regard to the duration composition effect on equilibrium unemployment. Contractionary and expansionary shocks to the system are measured along the horizontal axis, and are labelled $\varepsilon-$ and $\varepsilon+$ respectively, corresponding to the input $i(t)$ above. The hysteresis transducer, HT above, appears in the form of the LNJ non-linear wage (equation [8] above), and the induced effects on long-term unemployment (equation [9] above) and the rest of the system. A contractionary shock, for example, will raise unemployment, and as unemployment rises the downward pressure on real wages will fall as long-term unemployment eventually rises, so raising the equilibrium unemployment rate. The vertical axis measures unemployment, corresponding to the $f(t)$ output above, with unemployment being at its various NAIRU or U^* values along the vertical axis, i.e. when $\varepsilon = 0$ and the rate of inflation is constant.

Figure 1 illustrates the case of *weak hysteresis*. In the upper panel, the outer loop describes the maximum variation of (U, ε) feasible. The inner loops describe how unemployment responds to shocks within the feasible range. The characteristic of the inner curves for this case of *weak hysteresis* is that each feasible unemployment rate has only two possible trajectories that are uniquely defined, one for an expansionary shock, and one for a contractionary shock. The uniquely defined trajectories are illustrated in the lower panel of Figure 1.

Figure 2 illustrates the case of *strong hysteresis*, which arises when the impact of the shocks is determined by the interaction with the rest of the system. An initial contractionary shock to nominal demand, for example, will have a contractionary impact which will be partly determined by the way the rate of inflation responds, and the consequent real balance effects. In this case, the minor loops in the upper panel of the diagram cross, indicating that feasible points within the major loop are not uniquely defined. Instead, as illustrated in the lower panel, there is an infinity of curves representing how unemployment can react to contractionary or expansionary shocks: »each of these curves depends on a particular past history, namely, on a particular sequence of past extremum values of input« (Mayergoyz 1991: xvi). The implication is that the equilibrium unemployment rates in the range $U^*_{MIN} - U^*_{MAX}$ are not uniquely defined by the trajectories describing how current input shocks affect unemployment, but are pinned down instead by the sequence of past input extrema.

The way past input shock extrema affect unemployment can be illustrated most easily for the case of pairwise shocks to economic systems (see Mayergoyz [1985 and 1991] for the mathematics, and Cross [1990] and Amable et. al. [1991] for applications to economic systems). The number α represents an expansionary shock, and $\boldsymbol{\beta}$ a contractionary shock, where $\alpha \geq \boldsymbol{\beta}$.

*Figure 1: Weak hysteresis and U**

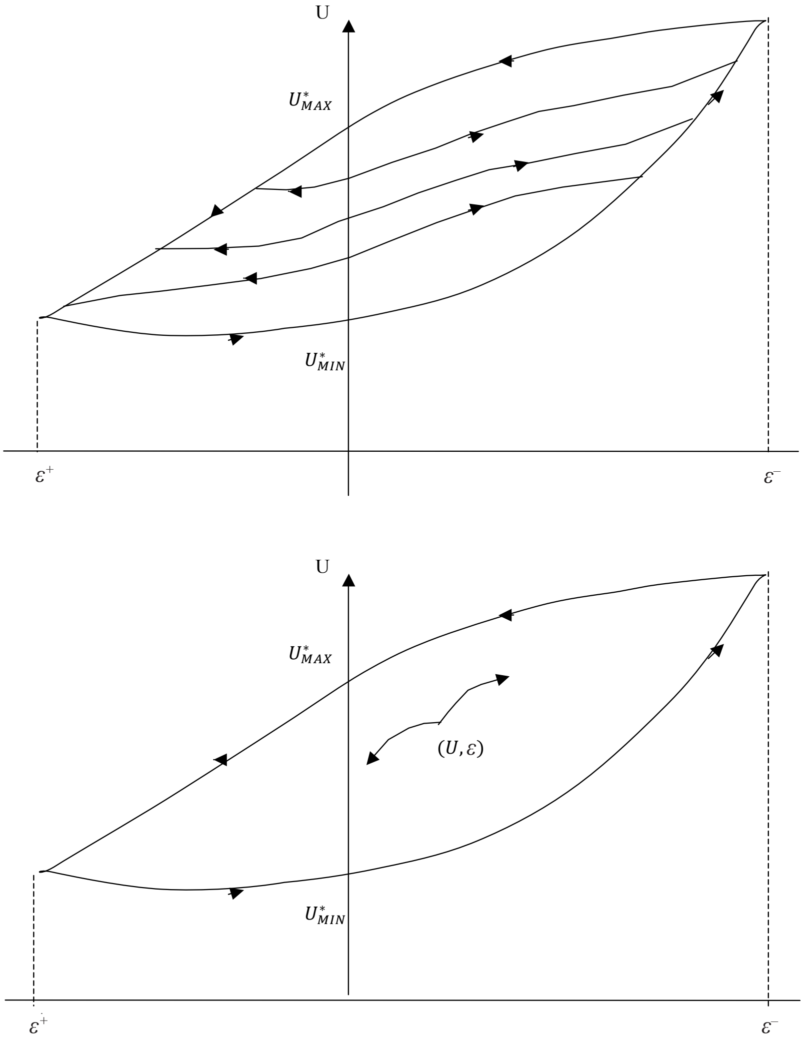
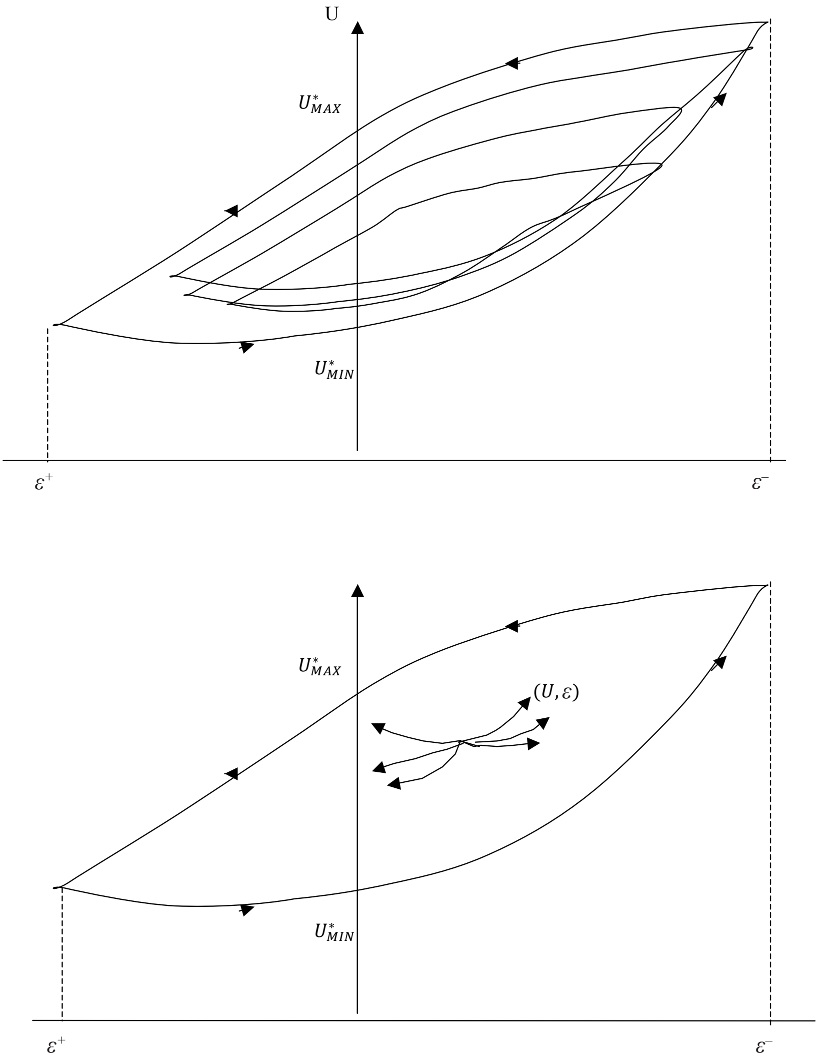


Figure 2: Strong hysteresis and U^*



These two types of input shock form an infinite set of hysteresis operators $\gamma\alpha\boldsymbol{\beta}$ defined over the set of heterogeneous individual markets in the system. Figure 3 (Mayergoyz 1991: 2) describes the workings of the hysteresis operators. When an expansionary shock occurs, the ascending branch ABCDE is followed, with the hysteresis operator carrying a positive value of +1. A contractionary shock leads to the descending branch EDFBA, with the hysteresis operator carrying a value of -1. A similar process operates in a multiplicity of labour and product markets, yielding the set up illustrated in Figure 4. The effects of exogenous shocks ε_t are propagated in the individual markets by way of the hysteresis operators $\gamma\alpha\boldsymbol{\beta}$, the unemployment outcomes are then multiplied by the weight function $w(\alpha, \boldsymbol{\beta})$ and aggregated via $\iint$ □ (see below) to yield an aggregate unemployment outcome U_t. Imposing the $\varepsilon_t = 0$ condition then yields the steady inflation U_S^*. AU 4

Algebraically, this model can be written as:

$$U_t = \iint_{\alpha \geq \beta} \gamma\alpha\beta\varepsilon_t w(\alpha, \beta) d\alpha d\beta \quad . \tag{10}$$

Note here that, because expansionary shocks reduce unemployment, and contractionary shocks increase unemployment, the weight function is negative in α, and positive in $\boldsymbol{\beta}$.

Figure 3: Hysteresis operators

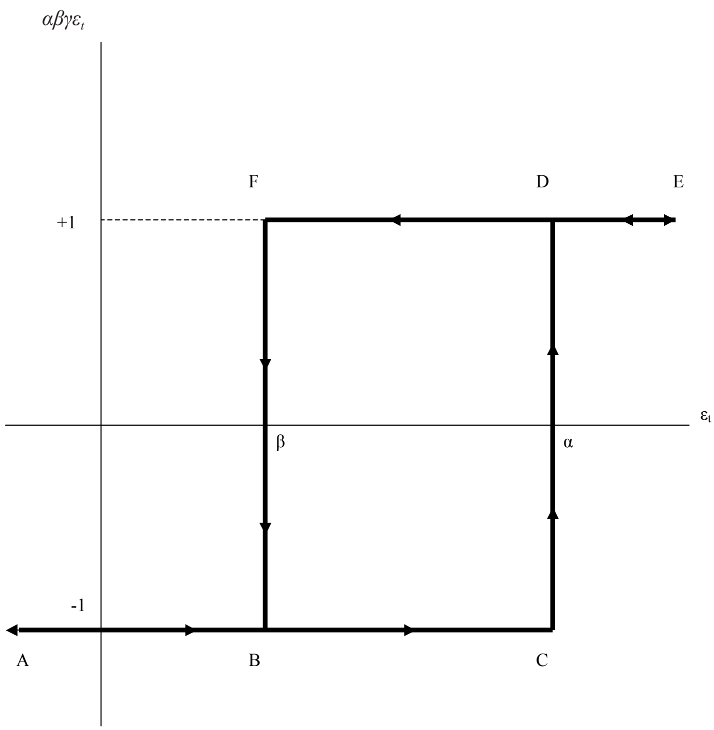
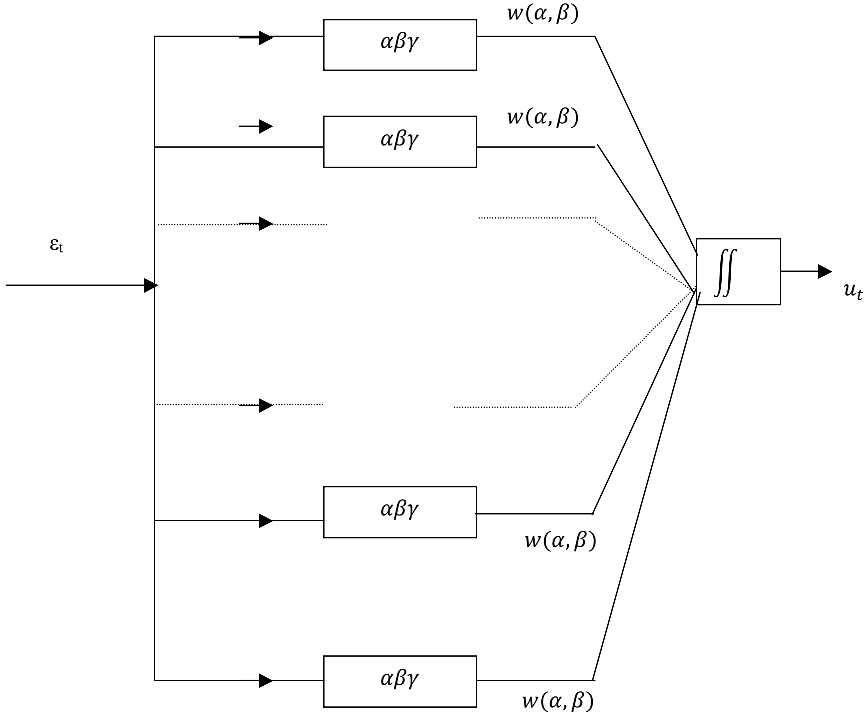


Figure 4: Hysteresis operators in a multiplicity of markets



The Mayergoyz (1985 and 1991) geometric interpretation of how past input shock extrema affect current output is reproduced in Figure 5. Each point in the half-plane $\alpha \geq \beta$ has a one-to-one correspondence with the input shock carried by the hysteresis operator $\gamma\alpha\boldsymbol{\beta}$. The weight function $w(\alpha, \boldsymbol{\beta})$ is assumed to be finite and equal to zero outside the right-angled triangle with right-angle vertex $(\alpha_0, \boldsymbol{\beta_0})$. The starting point for the analysis is the case where the initial input ε_0 is less than β_0, so the economic system has been experiencing a contractionary shock such that all the $\gamma\alpha\boldsymbol{\beta}$ operators in the right-angled triangle have been carrying contractionary effects of -1.

Consider an expansionary shock that increases the input value to ε_1 in Figure 6. Some local markets will now have α switching values less than the macro input shock ε_1, so their hysteresis operators will carry an expansionary value of +1. Other markets will have α switching values greater than ε_1, so will continue to carry a contractionary value of -1. This serves to sub-divide the right-angled triangle into two sets: S^+ for the markets with $(\alpha, \boldsymbol{\beta})$ values, which are experiencing the expansionary mode, and S^- for markets with $(\alpha, \boldsymbol{\beta})$ values in the contractionary mode.

Figure 5: The geometry of hysteresis

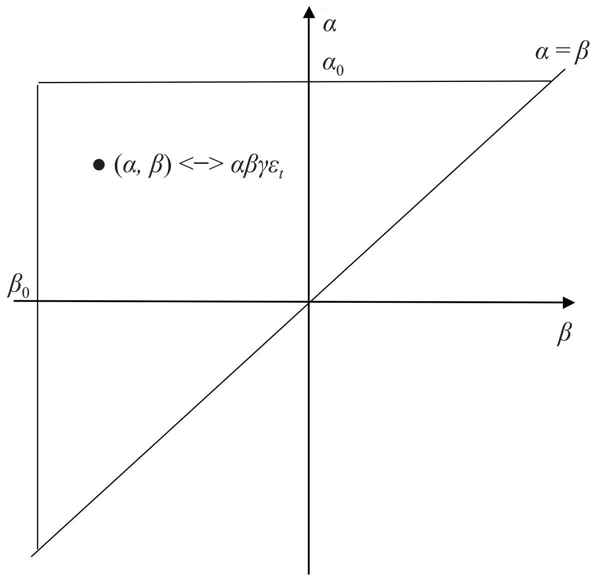
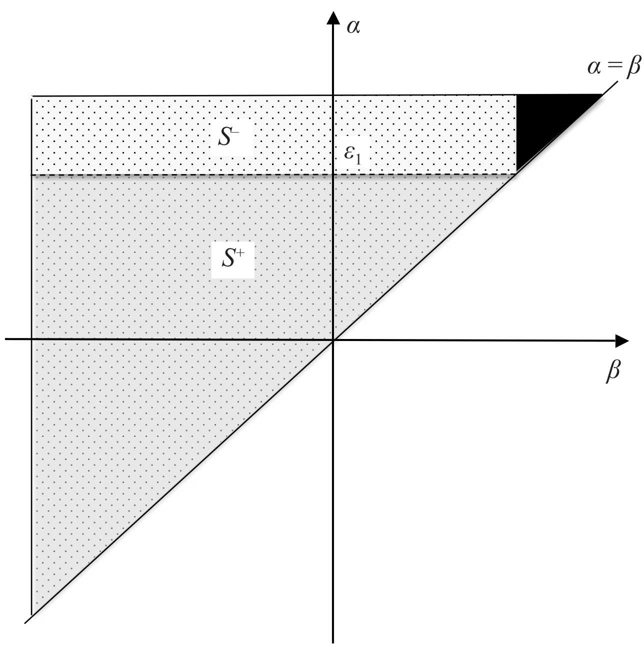
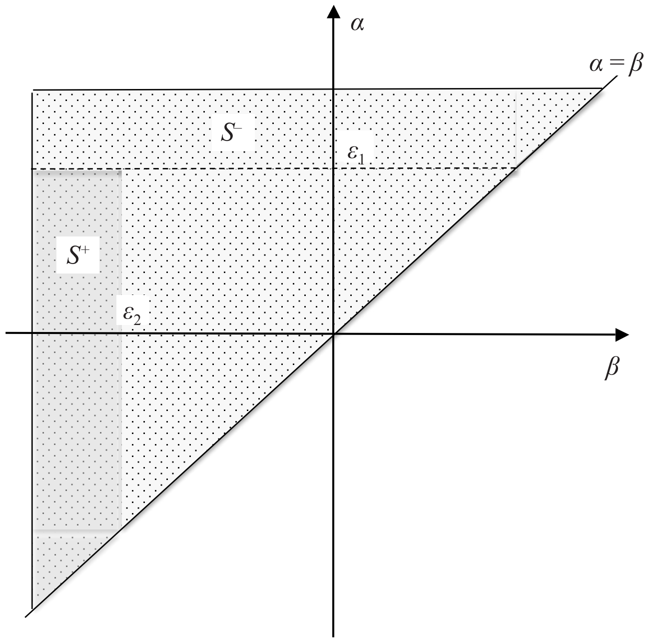


Figure 6: An expansionary shock



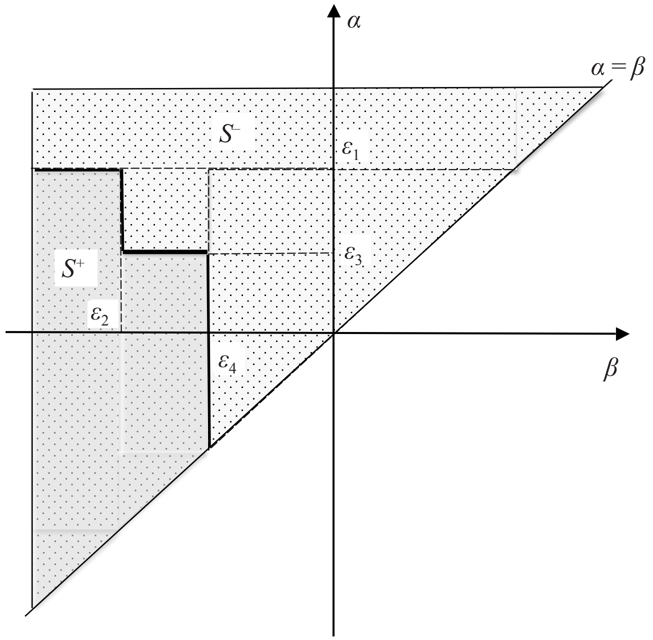
The effects of a subsequent contractionary shock are illustrated in Figure 7. The input is monotonically decreased until a value of ε_2 is reached. This leads the local markets with $\boldsymbol{\beta}$ switching values greater than ε_2 to move into a contractionary mode. The previous horizontal line subdividing markets in the contractionary mode, S^-, and expansionary mode, S^+, now becomes wiped by the contractionary shock for markets with $\boldsymbol{\beta} > \varepsilon_2$, being replaced by the vertical line through $\boldsymbol{\beta} = \varepsilon_2$. The input shock maximum ε_1 and minimum ε_2 have hence determined the vertex of the interface between S^- and S^+.

Figure 7: A subsequent contractionary shock



A second expansionary shock which reaches a maximum at $\varepsilon_3 < \varepsilon_1$ will place the markets with $\alpha < \varepsilon_1$ into an expansionary mode; a second contractionary shock which reaches a minimum at $\varepsilon_4 > \varepsilon_2$ will place the markets with $\beta > \varepsilon_4$ into a contractionary mode. Figure 8 illustrates how this second round of expansionary and contractionary shocks has reshaped the interface between S^- and S^+. The second round of maxima and minima serve to create a new vertex (ε_3, ε_4), in what is now a step function distinguishing markets in the contractionary mode, S^-, from the expansionary mode, S^+. In the context of equilibrium unemployment the S^- markets will obviously be characterised by higher unemployment, and the S^+ markets by lower unemployment, and consequently by eventually higher and lower long-term unemployment ratios respectively.

Figure 8: A second round of expansionary and contractionary shocks



Generalising, the interface between the contractionary S^- and expansionary S^+ operators is a step function whose vertices correspond to the sequence of past local maxima and minima in input shocks. The final step in this function is the link with the $\alpha = \boldsymbol{\beta}$ line: this is horizontal for expansionary shocks, moving upwards as the shock is increased; and vertical for contractionary shocks, moving to the left as the contractionary shock is increased. The algebraic implication of the foregoing geometric analysis is that equation (10) can be rewritten in terms of the expanding and contracting markets distinguished by the step function:

$$U_t = \iint_{S_t^+} \gamma\alpha\beta\varepsilon_t w(\alpha,\beta)d\alpha d\beta + \iint_{S_t^-} \gamma\alpha\beta\varepsilon_t w(\alpha,\beta)d\alpha d\beta \ . \quad (11)$$

Given that $\gamma\alpha\beta\varepsilon_t = +1$ for (α, β) in S_t^+ and $\gamma\alpha\beta\varepsilon_t = -1$ for (α, β) in S_t^-, (11) becomes AU 5

$$U_t = \iint_{S_t^+} w(\alpha,\beta)d\alpha d\beta - \iint_{S_t^-} w(\alpha,\beta)d\alpha d\beta \ . \quad (12)$$

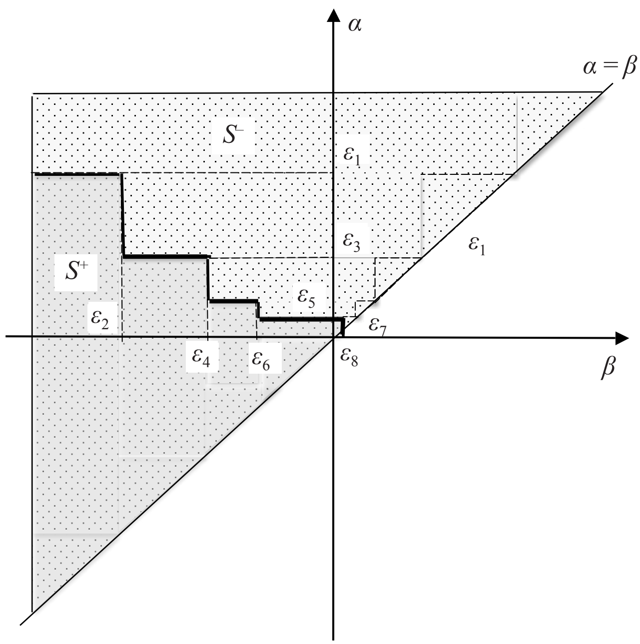
Thus the unemployment rate depends on the subdivision of markets between the expansionary and contractionary modes, and this subdivision is determined by the past extremum values of input shocks.

So far the discussion has concerned monotonically decreasing sequences of local input maxima, such as $(\varepsilon_1, \varepsilon_3, \varepsilon_5, \varepsilon_7)$ in Figure 9 (taken from Mayergoyz [1991: 13]), and monotonically increasing sequences of local input minima, such as $(\varepsilon_2, \varepsilon_4, \varepsilon_6, \varepsilon_8)$. Now consider the effect of an expansionary shock which reaches a maximum at ε_9, such that $\varepsilon_3 < \varepsilon_9 < \varepsilon_1$. It is clear that this shock will bring all the markets with α values less than ε_9 into the expansionary mode. The consequence is to wipe out the vertices of the interface between S^- and S^+ with α coordinates below ε_9. Thus, only the original vertex at $(\varepsilon_1, \varepsilon_2)$ is retained, the rest of the original vertices being wiped out. The new step function has two vertices corresponding to the non-dominated local extrema at $(\varepsilon_1, \varepsilon_2)$ and $(\varepsilon_9, \varepsilon_2)$. This means that the hysteresis memory of input extrema is selective:

> »each local input maximum wipes out the vertices […] whose α-coordinates are below this maximum, and each local minimum wipes out the vertices whose β-coordinates are above this minimum […] only the alternating series of dominant input extrema are stored […] all other input extrema are wiped out« (Mayergoyz 1991: 13–15).

The implication for unemployment is that it is only the dominant extrema of expansionary and contractionary shocks that are stored in the memory system for the current unemployment rate, U_t.

Figure 9: The wiping out of input extrema



The point of this section has been to investigate how LNJ employ hysteresis in their short-run NAIRU model, and to contrast this with the analysis of hysteresis elsewhere. The conclusion must be that the ›partial hysteresis‹ which is the distinguishing feature of the short-run

NAIRU is not really hysteresis at all. The hallmark of hysteresis is the presence of some nonlinearity in the way shocks affects the system under consideration. The implication of such hysteresis nonlinearities is that the output emerging from the system depends inextricably on the sequence of non-dominated extremum values of shocks. The LNJ version of hysteresis does not capture this selective memory process, nor does it capture the fact that the memory phenomenon does not disappear over time. A sketch of how such a process works in relation to unemployment has been offered in order to show how the mathematical analysis of the original Preisach model developed by Krasnosel'skii can be applied to the problem of the determination of unemployment in economic systems. This suggestion of an interesting »application beyond the conventional area of hysteresis modelling« (Mayergoyz : 18) is appropriate because, as LNJ stress, the Phillips curve was always a *curve*, and hence provides the basic non-linearity required for hysteresis to be relevant. Several interpretations of the non-linearity of the Phillips curve have been offered, including the aggregation over local labour markets approach of Lipsey (1960), and the duration composition of unemployment effect stressed by LNJ. If such nonlinearities are basic features of economic systems, hysteresis implies a permanent though selective memory of input shocks, not the short-run effects that disappear in the long run initially proposed by LNJ and still embedded today in the so-called ›New Consensus‹ models. AU 6

5. Convergence to the natural rate

LNJ state that »there is no long-term ›hysteresis‹: there is a unique long-run NAIRU […] in the end the unemployment rate always reverts« (LNJ: 10). This is a strong claim, which, if true, would mean that the analysis of permanent hysteresis effects in the preceding section is not relevant to equilibrium unemployment. Thus it is worth assessing the arguments used by LNJ to support this strong claim.

Analytically, the claim that ›the unemployment rate always reverts‹ is equivalent to saying that the actual rate of unemployment always converges on the natural rate or long-run NAIRU, i.e. that the natural rate is a stable equilibrium. Formal analysis of the dynamics of long-run implication of ›hysteresis‹ in wage-setting is provided in Annex 1.5 (LNJ: 525). The time path of actual unemployment is set to minimise :

$$\int_0^\infty \frac{1}{2(U^2 + k\dot{P}^2)e^{-rt}dt} \quad , \tag{14}$$ AU 7

$$s.t. \;\; \ddot{P} = \lambda_1(U^* - U) - \gamma_{11}\dot{U}$$

where r is the discount rate, k, γ_1 and γ_{11} are positive constants, $\dot{P}$ is the rate of inflation and U^* and U are the ›natural‹ and actual rates of unemployment. The solution path for unemployment is

$$\ddot{U} + b_1\dot{U} + b_2(U - U^*) = 0 \quad , \tag{15}$$

where $b_1 = -r < 0$ and $b_2 = \frac{-(\gamma_1 + \gamma_{11} r) k \gamma_1}{1 + \lambda_{11}^2 k} < 0$.

The roots of this equation are

$$\lambda_1, \lambda_2 = \frac{1}{2\left[-b_1 \pm \sqrt{b_1^2 - 4b_2}\right]} \quad . \tag{16}$$

Given that $b_1 < 0$, the dominant root λ_1 is clearly

$$\lambda_1 = \frac{1}{2\left[-b_1 + \sqrt{b_1^2 - 4b_2}\right]} \quad . \tag{17}$$

Hence the dominant root $\lambda_1 > 0$, so the differential equation is clearly unstable and therefore the actual rate does not converge on the natural rate over time.

The authors, however, proceed *as if* equation (15) *were* stable: »the stable solution to this equation is« (LNJ: 525). The non-dominant root $\lambda_2 = \frac{1}{2\left[-b_1 - \sqrt{b_1^2 - 4b_2}\right]}$ is clearly < 0, so in this respect the LNJ analysis is correct, but this is irrelevant: if Modulus $\lambda_1 >$ Modulus λ_2, clearly the λ_1 root will dominate the evolution of $(U - U^*)$ over time. Hence, in the absence of reasons as to why the dominant root can be ignored, the LNJ analysis implies the opposite of the strong claim that ›the unemployment rate always reverts‹ to the ›natural‹ rate.

The implication that the long-run natural rate equilibrium is irrelevant to the dynamics of actual unemployment is confirmed by the LNJ estimates of how the long-term unemployment ratio, the source of ›hysteresis‹, is determined by actual unemployment. The empirical relationship reported (LNJ: 44) is equation (9) above. In the long-run natural rate equilibrium presumably $LTU = LTU_{-1}$, there being no change in the ›hysteresis‹ effects arising from long-term unemployment. Imposing this condition implies:

$$U - 2.64\ U_{-1} + 1.12\ U_{-2} = 0.04 - 0.12\ LTU\ . \tag{18}$$

The roots of this equation are real, and the dominant root $\lambda_2 = 2.1$. Given that $\lambda_1 > 1$, the equation is clearly unstable, the dynamics of actual unemployment being inconsistent with convergence on some LTU^* component of a natural rate equilibrium. Similarly, the first-order approximation for the unemployment process determining LTU (LNJ: 440) is unstable.

An obvious way to test the claim that ›the unemployment rate always reverts‹ is to test whether the unemployment rate is cointegrated with the Z_w wage pressure variables which determine the natural rate in equation (3) above. Such cointegration tests are notable for their absence in LNJ. No reference is made to evidence reported in Jenkinson (1988: 373) that unemployment is *not* cointegrated with the Z_w variables. The response confirmed the lack of cointegration of U and Z_w, but produced evidence that log U and Z_w *were* cointegrated (Nickell 1988: 383). Given that cointegration tests had already become a standard method of checking the equilibrium claims made by economic theories when LJN wrote their paper, it is quite odd that LNJ did not produce cointegration evidence for their claim that

›there is a unique long-run NAIRU‹. If non-linear effects captured by log *U* are important, the relevant analysis, as reported in Section 4 of this paper, suggests that the natural rate is irrelevant, unemployment equilibria being shaped by the dominant extrema of past shocks to economic systems.

6. Conclusion

This paper has dealt with the short- and long-run NAIRU concepts which are the hallmark of the unemployment analysis reported in LNJ, concepts which are still alive and well in the so-called ›New Consensus‹ models. The principal conclusion to emerge is that the short-run NAIRU does not adequately capture hysteresis, and that the presence of hysteresis implies, *contra* LNJ, that the past has permanent effects on unemployment and equilibrium unemployment. This influence is selective, unemployment depending on the non-dominated sequence of extremum values of exogenous shocks. This is interesting, especially given that, considered on its own terms, the LNJ analysis and evidence does not support their claim that unemployment always reverts to a long-run natural rate equilibrium. It is noteworthy that, more than 15 years after the publication of LNJ's very influential papers, there is still no evidence that unemployment would revert to any long-run natural rate equilibrium.

Undeniably, the LNJ NAIRU framework has been constructed with great ingenuity, and their empirical work has added much to the understanding of wage-price-unemployment dynamics. The framework, however, has not proved sound enough to provide a coherent explanation of equilibrium unemployment. This is apparent in the LNJ estimates of equilibrium unemployment in the UK for the 1980s:

> »in the last two periods (1974–80 and 1981–87), but particularly in the 1980s we are unable to provide complete explanations for the rise in equilibrium unemployment as estimated by our earlier method of removing the inflation, trade balance and hysteresis (ΔU) effects from the actual unemployment rate« (LNJ: 446).

The authors go on to say that »this is surely the result of our inability to capture all the relevant exogenous factors at work« (LNJ: 446). Any theory can be defended against empirical refutation along such lines. The defence is circular: poor empirical performance occurs when all the relevant exogenous factors are not captured, and this occurs when there is poor empirical performance. The question begged is why it was possible to ›capture the relevant factors‹ during periods of satisfactory empirical performance, but not otherwise.

Given all this, the conclusion must be that the NAIRU is really ›Not An Interesting Rate of Unemployment‹. An interesting alternative is offered by the Krasnosel'skii methods of analysing systems with hysteresis. If labour and product markets have non-linear relationships whose effects can only be analysed in the context of system-wide interaction, the implication is strong hysteresis. This, in turn, implies that unemployment, and its equilibrium state, will have a permanent memory of the dominant extrema of the exogenous shocks to which the individual markets in the economic system have been subjected. This approach is clearly distinct from the non-hysteretic natural rate hypothesis retained for the long run by LNJ. If there must be another acronym, DESIRU (dominant extrema, steady inflation, rates of unemployment) might be a lesser evil.

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