The point of effective demand

A monetary economy … is essentially one in which changing views about the future are capable of influencing the quantity of employment and not merely its direction. But our method of analysing the economic behaviour of the present under the influence of changing ideas about the future is one that depends upon the interaction of supply and demand, and is in this way linked up with our fundamental theory of value. We are thus led to a more general theory, which includes the classical theory with which we are familiar, as a special case. (Keynes, 1936, pp. xxii-xxiii)

1. Introduction

The title of this paper is deliberately ambiguous. Firstly, what does Keynes's concept of effective demand add to the concept of income and why do we need it? Secondly, what is the nature of the equilibrium represented by the point of effective demand in *The General Theory* (Keynes, 1936, hereafter *GT* in chapter and page references)? This paper argues that, despite an extensive literature[1] including major monographs by Davidson & Smolensky (1964) and Amadeo (1989), these questions have not yet been answered conclusively. King (1994, p. 28) notes a decline in Post Keynesian interest in Keynes's principle of effective demand since 1989, which he attributes to its neoclassical flavour. Consistent with this view, the recent literature on effective demand has largely divided into two strands, one tending to follow Kalecki rather than Keynes (see Arestis, 1996), the other addressing Keynes's methodology (see Chick & Dow, 2001). Orthodoxy has long discounted the suggestion that Keynes offers a serious alternative to Walrasian general equilibrium analysis, and has followed Lucas (1981) in its negative assessment of Keynes's theoretical contribution.

The ultimate purpose of this paper, emerging from the methodological strand, is to help liberate equilibrium theory from the Walrasian citadel, and to recognise the principle of effective demand as the correct platform on which to develop any theory of the competitive equilibrium of the system as a whole that takes seriously the historical nature of time (see opening quotation) and treats equilibrium as observable, not merely ideal. The key contribution is a reconsideration of Keynes's use of

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[1] For a comprehensive account of early treatments from 1947 to 1964 see King (1994). On the literature between 1964 and 1989 see Section 2 below.

Marshall's equilibrium periods, leading to the conclusion that, for analytical purposes, in *The General Theory* Keynes treats employment as being *in equilibrium at all times*. This view contrasts both with the received interpretation of employment equilibrium in Keynesian[2] and Post Keynesian economics, as the position where short-term expectations of income and expenditure are fulfilled through a multiplier process; and with the Walrasian conception of general equilibrium, which includes preferred allocation, full employment and the clearing of factor markets, which for Keynes is only the limiting case.

There are several extant definitions of equilibrium (see Backhouse, 2004): this paper refers to more than one, and the reader must be sensitive to the different usages. The primary definition, here ascribed to Keynes, is equilibrium in the old-fashioned static 'mechanical' sense of the balancing of supply and demand through price competition in perfect markets: a state of rest, however temporary. The Marshallian market-, short- and long-period equilibrium positions need to be carefully distinguished from each other and from the Walrasian inter-temporal equilibrium. Keynes's concept of the competitive equilibrium of the system as a whole determining aggregate employment centres on the choices only of entrepreneurs, investors and consumers, and not of factors: production and employment decisions are reserved to entrepreneurs, by definition, based on their price expectations; unemployed factors as such cannot insist on employment (*GT,* p. 291) or set up firms.[3] Thus equilibrium as a state of rest at a point in time and as a position of choice coincide, although the equilibrium position may change from day to day. This concept differs markedly from equilibrium as either the convergence of expectations (involving a dynamic process

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[2] The unqualified term 'Keynesian' will be used in this paper to refer to the neoclassical synthesis of Hicks, Samuelson, Tobin and Modigliani rather than the ideas of Keynes himself.

[3] The term 'involuntary' unemployment intrinsically requires a non-Walrasian concept of equilibrium with non-clearing factor markets, to some extent found in Pigou as well as in Keynes (see Hayes, 2006). Keynes should not be confused with the French school of 'general disequilibrium' models based on quantity rather than price adjustment, in which disequilibrium remains defined with reference to the Walrasian equilibrium, since Keynes maintains the assumption of market clearing in all other than factor markets. Arrow & Hahn refer to Keynes's concept of general equilibrium as a 'quasi-equilibrium' (1971, p. 366).

over time) or as the allocation preferred by the owners of the factors of production (which assumes all factor markets clear).

Section 2 identifies three different treatments of the relation between income (*Y*), equilibrium income (*Y**), effective demand (*D**) and aggregate demand (*D*) in the Keynesian and Post Keynesian literature on the principle of effective demand, and argues that none fully represent the treatment in *The General Theory*. Here the claim is advanced that Keynes treats employment as being in equilibrium at all times and determined by the state of expectation, and the following three sections substantiate this. The point of departure in Section 3 is Keynes's radical use of Marshall's equilibrium periods, his definitions of units of calendar time in relation to production, and their relation to expectation, leading to a demonstration of the difference between income and effective demand. Section 4 takes as its heading the title of Keynes's Chapter 5, builds upon the definitions of equilibrium periods and units of time in Section 3 to identify how the state of expectation determines employment as a position of short-period equilibrium at any time, and resolves the vexed question of Keynes's reference to long-period employment. Having established in Section 4 that the point of effective demand involves neither the convergence of expectations nor the multiplier, Section 5 considers what kind of equilibrium process determines effective demand. This requires a conceptual model of the point of effective demand as a position of the continuous equilibrium of supply and demand in short-term forward markets, faithful to Keynes's definition of aggregate demand in terms of the expectations of entrepreneurs. This could easily be confused with the Walrasian model of general equilibrium, and for those who find it helpful or necessary, the formal difference between Keynes and Walras in their treatments of factor markets and the state of expectation is considered in Appendix A, and the nature of the multiplier in Appendix C. Section 6 concludes that this proposed understanding of Keynes's definitions of time and equilibrium periods is essential to discovering his meaning and resolving the paradoxical readings, and to revealing *The General Theory* as fully integrated with the Marshallian theory of value. Equilibrium is thus reclaimed as a description of the real world, useful within strict limits, rather than the ideal state of Walrasian general equilibrium.

2. The Received Interpretation of the Point of Effective Demand

In the standard text-book exposition the point of effective demand is the intersection of the Keynesian cross (Samuelson 1939, p. 790, Hansen 1953, p. 34). It represents the level of aggregate output where entrepreneurial expectations of income (*ex ante*, to use the Swedish term) are fulfilled (*ex post*). At the point of effective demand, i.e. in equilibrium, there is no conceptual difference between income (expected and realised) and effective demand. Since price changes are abstracted from, effective demand and aggregate demand also become equivalent in equilibrium. These propositions are captured by an equilibrium condition:

$$Y^* \equiv D^* = D \qquad\qquad (1)$$

where Y^* represents equilibrium income, D^* effective demand and D aggregate demand.

In the Post Keynesian tradition following Weintraub (1957) and Davidson & Smolensky (1964), effective demand D^* and aggregate demand D are more fully distinguished from each other, and Keynes's aggregate supply price (Z) is explicitly restored. The effective demand D^* is now given by the intersection of the Z and D curves and once again, in equilibrium, income corresponds to effective demand. So we can now write the equilibrium condition as:

$$Y^* \equiv D^* = D = Z \q\qquad\qquad (2)$$

The Post Keynesian treatment identifies that at the point of effective demand individual firms are maximising expected profits, since Z represents the Marshallian supply price (abstracting here from any degree of monopoly and user cost). In both Keynesian and Post Keynesian treatments it is possible for entrepreneurial expectations of income to depart from their equilibrium values, so that realised income does not equal expected income, and $Y \neq Y^*$. In a dynamic process involving the multiplier, firms adjust output and employment over time so that expected income converges on its equilibrium value $Y^* \equiv D^*$. In these treatments the levels of expected income and employment at any time can be *disequilibrium* values.

An extensive literature, prompted by the 1973 publication of Vols. XIII and XIV of Keynes's *Collected Writings* (hereafter CW), has grappled with the process of

convergence of expectations to equilibrium[4]. Without doing justice to the many nuances of this complex and subtle discussion, it stems from Keynes's definition (followed here) of the aggregate demand function *D* as 'the proceeds which *entrepreneurs* expect to receive from the employment' (*GT,* p. 25; emphasis added; see also *GT,* pp. 28-29, 89), rather than in terms of the expenditure of consumers and investors, the aggregate demand of Keynesian economics. The difference leads different authors to introduce a variety of additional functions to reflect expenditure plans, e.g. *E* for expenditure (Parinello, 1980), D^e for entrepreneurial expectations of aggregate demand *qua* expenditure *D* (Casarosa, 1981; Chick, 1983), and *A* for actual proceeds versus *X* for expected proceeds (Dutt, 1987). Here the two positions of equilibrium, 'chosen' and 'at rest' (Asimakopulos, 1989, p. 16), may not coincide, and it is unclear which of the two should be called the point of effective demand. The fact that Keynes appears to neglect this distinction in Chapter 3 of *The General Theory* is reconciled by attributing to him a tacit assumption that short-term expectations are fulfilled (e.g. Chick, 1983, pp. 64, 76; Hoover 1997, pp. 223-224).

The claim of this paper is that this established emphasis on the convergence of expectations and the multiplier is misplaced. I suggest that throughout *The General Theory* Keynes, with one practical exception (*GT*, p. 290), treats the system as at all times in equilibrium at the point of effective demand. As a corollary, the level of employment (but not the labour market) is for theoretical purposes treated as at all times in equilibrium[5]. The level of employment is determined solely by the state of expectation, and strictly, it is not relevant to employment whether expectations are fulfilled. The level of income *Y*, whether realised or expected, bears no simple relation to effective demand, and the equilibrium level of income *Y** is conceptually different from the point of effective demand *D**. In the earlier shorthand this may be written as the proposition:

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[4] See Kregel (1976), Patinkin (1976), Roberts (1978), Davidson (1978), Wells (1978), Parinello (1980), Casarosa (1981, 1984), Chick (1983), Torr (1984), Dutt (1987), Asimakopulos (1989) and Amadeo (1989).

[5] Roberts (1978, p. 371), in denying that 'any dynamic process is required … to bring an economy to a position of unemployment equilibrium', comes within a whisker of making the same claim, but draws back in his footnote 10.

$$Y \neq Y^* \neq D^* = D = Z \tag{3}$$

This means that while it is in theory possible that the values of income and effective demand coincide if expectations are fulfilled $(Y = Y^* = D^*)$, their coincidence does not define the equilibrium level of employment. Employment is *always* in equilibrium corresponding to D^*, and in general the value of D^* need correspond neither to realised income Y nor to the level of income Y^* corresponding to the fulfilment of expectations. The next three sections expand and clarify this claim.

3. Income and Effective Demand

Keynes defines income as the value of output (*GT,* p. 63), and the expectation of income relevant to employment, i.e. effective demand, is the expected value of the output resulting from the employment offered and accepted in any period. Expectation enters in because production takes time, as recognised by Marshall in his division between the market, short and long periods. Keynes uses Marshall's concepts of equilibrium periods and normal prices but, I suggest, radically changes their meaning.[6]

The production and employment decision involves two separate units of calendar time, which Keynes defines as the *day* and the *period of production*, which is a number of *days*. The day is Keynes's quantum unit of time, 'the shortest interval after which the firm is free to revise its decision as to how much employment to offer. It is, so to speak, the minimum effective unit of economic time' (*GT,* p. 47, n1); recall that the primary concern of *The General Theory* is the employment (hiring) decisions of firms.[7] The *period of production* is the number of *days* 'notice of changes in the

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[6] The consensus about Keynes's use of time periods, from which this paper departs, is that Keynes's day and production period coincide, and correspond to a Hicksian week (Chick, 1983, p. 20; Hansson, 1985, p. 334; Amadeo, 1989, pp. 12-16; Fontana, 2004, p. 80; Hartwig, 2004, p. 312), an equation which tacitly assumes a uniform production period for all goods. Daily employment thus differs from the short-period employment equilibrium in which expectations are fulfilled (Casarosa, 1984, p. 942 n1). Hicks takes the short period to be Marshall's 'few months or a year' (Hicks, 1980, p. 141; Marshall, 1920, p. 314).

[7] Keynes's 'day' relates to calendar time since it reflects employment practices; it is not simply the indefinite interval between changes in the opinion of an entrepreneur, but the

demand for [a product that] have to be given if it is to offer its maximum elasticity of employment' (*GT*, p. 287). This definition is the macroeconomic counterpart of the period between starting and finishing an individual production process (*GT*, p. 46) or *production period*,[8] and differs from it in taking into account the time to produce other goods, required as inputs. On any day in any firm a number of production periods of differing lengths overlap (CW XIV, p. 180); today's decision to begin or continue a process will, if uninterrupted, deliver an addition to finished output after a period of time determined by the particular good, technology and capital equipment (including here fixed, working and liquid capital-goods).

Keynes's day corresponds to *both* Marshall's market and short periods.[9] Whereas Marshall distinguishes between them in terms of the length of time over which production and employment can adjust so that market prices become equal to normal prices (Keynesian and Post Keynesian alike have followed Marshall in this), for Keynes the important difference between the market and the short periods is that between realised and expected prices, and between income and effective demand. Each day firms must decide, in a short-period equilibrium process Keynes calls 'the principle of effective demand', how much employment to offer today based on their expectations of the market prices they will receive for the heterogeneous finished

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period since the last employment decision after which any such change or sequence of changes can be acted upon. Bradford & Harcourt (1997, pp. 120-121) suggest otherwise, and so question the precision of Keynes's definition of the period of production as *n* days (*GT*, p. 287). Keynes's day need not be a terrestrial day, but it does no harm to think of it as such.

[8] The *production period* of an individual good in isolation is not explicitly defined in *The General Theory* but appears in a 1933 draft (CW XXIX*,* p. 88) as the time after which a decision to employ labour realises finished output, ready for use in production or sale, as at p. 46 of *The General Theory*. Keynes uses the terms 'accounting' and 'employment' period to mean this production period in early drafts (CW XXIX, p. 74). See also Marshall (1920, p. 315 n1) on the distinction between the macro and micro production period.

[9] It is the quantum nature of Keynes's short period that justifies treatment of the state of long-term expectation as strictly independent, and answers Kalecki's critique (1936).

output that will emerge at the end of the various production periods.[10] These expected prices may reflect, but are quite different from, the market prices which output finished today will fetch if sold in the spot market at once, and which determine the value of today's income.

Day Process	t-4	t-3	t-2	t-1	t	t+1	t+2	t+3	t+4
1	$-U_1^{t-4}$	$-U_1^{t-3}$	$-U_1^{t-2}$	$-U_1^{t-1}$	A_1^{t}				
2		$-U_2^{t-3}$	$-U_2^{t-2}$	$-U_2^{t-1}$	$-U_2^{t}$	A_2^{t+1}			
3			$-U_3^{t-2}$	$-U_3^{t-1}$	$-U_3^{t}$	$-U_3^{t+1}$	A_3^{t+2}		
4				$-U_4^{t-1}$	$-U_4^{t}$	$-U_4^{t+1}$	$-U_4^{t+2}$	A_4^{t+3}	
5					$-U_5^{t}$	$-U_5^{t+1}$	$-U_5^{t+2}$	$-U_5^{t+3}$	A_5^{t+4}

Figure 1: Income and effective demand

These ideas are more difficult to convey than those of the Keynesian cross (see Appendix A for a formal approach), but Figure 1 illustrates the central concepts on the assumption that there is one production method which takes five days and is started up each day, so that there are five processes running in parallel at any time, say the construction of five log-cabins. We also assume no producible capital-goods other than work-in-progress which is identifiable by date of production. The gross value of output on day t, before allowing for the value of work-in-progress depleted (user cost), is made up from the money-value A_1^t of the output of finished goods in process 1, together with the money-value of the addition to the work-in-progress in processes 2-5, represented by $-U_j^t$. The symbol U is a reference to user cost, which is the inverse of investment (the value of capital-goods produced), so that $I_j^t = -U_j^t$. The money-value of the outstanding work-in-progress, on which the production decisions of day t are partly based, is represented by all the cells for days prior to t, which can be written:

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[10] These production periods represent the optimal production plan (in the sense of Hicks, 1939, p. 193) in a given state of expectation, if more than one technical process, each with different delivery dates, is available to a given industry (*GT,* pp. 215-216).

$$ {}^{t}G^{t-1} = -\sum_{i=1}^{4} {}^{t}U_1^{t-i} - \sum_{i=1}^{3} {}^{t}U_2^{t-i} - \sum_{i=1}^{2} {}^{t}U_3^{t-i} - {}^{t}U_4^{t-1} \qquad (4) $$

where the prefix t means that work-in-progress brought forward is valued at market prices on day t (in this model we can abstract from the possibility of holding rather than using capital-goods, so that ${}^{t}G^{t-1} = (G' - B')$ in Keynes's terminology, *GT* p. 53). The level of income (Y^t) is given by the shaded area:

$$ Y^t = A_1^t - \sum_{i=1}^{4} {}^{t}U_1^{t-i} + \sum_{j=1}^{4} - U_{j+1}^t \qquad (5) $$

where the first two terms represent the value added in completing the first log-cabin after *deducting* the value already embodied in the work-in-progress, and so the value of consumption (one log-cabin delivered to consumers); and the third term represents the addition to the value of the work-in-progress on the other four cabins, being the value of investment. By contrast, the level of effective demand D^{*t} is represented by the entire grid:

$$ D^{*t} = \sum_{j=1}^{5} \left[{}^{t}A_j^{t+j-1} - \sum_{i=1}^{4} {}^{t}U_j^{t+j-1-i} \right] \qquad (6) $$

The level of employment N^t that it is worth offering today depends on the expected sales values ${}^{t}A_j^{t+n}$, the expected costs of future construction work ${}^{t}U_j^{t+n}$, and the value of the work-in-progress to date ${}^{t}G^{t-1}$. It is clear that formulae (5) and (6) for Y and D^* are quite different; income and effective demand at time t coincide only in the case of process 1 in isolation, and only if prices are unchanged. The aggregate expected income represented by effective demand, which motivates today's employment, does not correspond to the aggregate income expected on any one future day, but is spread over a number of days. The income of the factors of production hired by an entrepreneur is indeed fixed when they are employed, and this is important in allowing Keynes to switch from employment to income as the determinant of consumption (*GT*, p. 90), but here we are concerned with the expectations of the entrepreneur.

Keynes noted for his 1937 lectures:

> *Time* relationship between effective demand and income incapable of being made
> precise. In case of factors other than entrepreneurs and rentiers the two are more or less

simultaneous. For the latter income becomes [determinate] and is transferred at varying subsequent dates. No definite relationship between aggregate effective demand at one time and aggregate income at some later time. This does not matter. Employment is determined *solely* by effective demand which is influenced by realised results up to date irrespective of the date to which the decision relates. (CW XIV, pp. 179-180; 'determinate' replaces 'determinant').

These statements fit the perspective of this paper, while the denial of an exact correspondence in time between aggregate effective demand and aggregate income presents a conundrum for the Keynesian interpretation of the point of effective demand. Aggregate effective demand and income are not commensurable because they do not have the same dimensions in time, which in our simple model can be seen in the difference in their *shapes*. Given a system with heterogeneous output and production periods, finished capital-goods and (crucially) shifting expectations, aggregate income and expected income of the same time-period cannot be expected to define the stable 'aggregate excess demand function' necessary for the multiplier process of convergence usually illustrated with the Keynesian cross (or the Post Keynesian *Z* diagram). The Walrasian analysis of inter-temporal excess demand functions has shown how dubious is the global stability of its version of equilibrium (Arrow & Hahn, 1971, pp. 322, 366-369), and the New Classical practice is simply to assume continuous equilibrium through rational expectations, justified in turn by the assumption of ergodicity (Davidson, 1996). The methodological difference between Keynes and the New Classicals is not in the belief that (if we are to make progress with equilibrium theory) we have to assume the system is always in equilibrium, but in the definition of equilibrium; and not, as we shall see in the next sections, in the assumption of rational expectations, but in its application to the long term.

4. 'Expectation as Determining Output and Employment'

Having clarified Keynes's statement that 'employment is determined *solely* by effective demand', we must now consider the relationship between the state of expectation and effective demand, in order to understand the title of Keynes's Chapter 5 and this section. The state of short-term expectation which governs decisions to produce consumption- and capital-goods is best construed as the set of expected prices (later we shall call it a matrix, **Π**) corresponding to effective demand, where these prices and the quantities they determine, together making up effective demand,

are equilibrium values (a claim to be substantiated in the next section). Since the state of short-term expectation is determined partly by the state of long-term expectation and is a subset of the state of expectation as a whole, it is often convenient to follow Keynes in referring simply to the latter.[11]

The state of expectation and effective demand may change from day to day, and the level of employment may change with them, but will remain in short-period equilibrium. Nevertheless, a further aspect of production time must be taken into account, that the capital equipment cannot be adjusted as quickly as the state of expectation and employment may change. It is necessary to introduce a long period, in the technical sense of the time required to adjust the capital equipment to a new state of expectation, but Keynes's long period differs from Marshall's in so far as it relates to the state of short-term expectation, rather than to the long term of 'several years' (Marshall, 1920, p. 315). Keynes's short and long term relate to expectation and are not at all the same as his short and long periods, which relate to the equilibrium of employment and capital equipment respectively.[12]

Keynes denotes as the 'long-period employment' the equilibrium level and distribution of employment corresponding to a given state of expectation once the disequilibrium fluctuations, arising from the change that led to the current state of expectation, have fully worked themselves out (*GT,* p. 48). The source of these disequilibrium fluctuations is emphatically not expectational error but production time, which introduces lags into the dynamic process of convergence of short-period

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[11] By contrast, Kregel (1976, p. 210) associates 'short-period' with particular individual expectations and 'long-period' with the general state of expectation, rather than with production and investment decisions respectively (*GT,* p. 47). This association leads to a quite different reading, including the likelihood that employment is in disequilibrium (1976, p. 217), although the main conclusion remains that the disappointment of expectations is not the central issue.

[12] Yet more confusion is caused by the distinction often made in Keynesian economics between the short term, in which factor and product prices are sticky, and the long term in which they are flexible in the absence of permanent rigidities. The underlying concept of equilibrium is Walrasian (i.e. all markets clear in the long term), so that flexible prices would deliver full employment.

equilibrium employment on its long-period position (*GT,* pp. 47-51, 122-124, 287-288). Although the state of expectation will most likely change before employment has reached its long-period position (*GT,* p. 50), on any given day employment will be both in short-period equilibrium and on a traverse or convergence path towards the long-period position, in accordance with Hicks's dictum (1980, p. 151).[13] Appendix B restates Keynes's exposition in formal terms using a simple dynamic model.

Thus the title of Keynes's Chapter 5 and this section should be taken literally, that the state of expectation determines the daily equilibrium level of employment. On this reading, it is not relevant to today's decision whether today's state of expectation is correct; if tomorrow's market prices lead to a change in the state of expectation, employment will change accordingly, yet tomorrow's state of expectation is strictly independent of today's and of the day after tomorrow's. Furthermore, although expectational error is one cause for revision in the state of expectation, it is neither the only one (as implied by the mechanical 'adaptive expectations' hypothesis), nor necessarily the most important. The state of short-term expectation that defines effective demand depends partly on the state of long-term expectation, itself a function of the state of confidence; on the state of liquidity-preference; and on the

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[13] In contrast with the present argument that Keynes's long-period employment is an integral part of his theory of effective demand, Carvalho (1990) and Asimakopulos (1984, 1989) consider 'Keynes's discussion of [long-period employment] has the nature of an aside, without any consequence for the model presented in his book' (Carvalho, 1990, p. 286). For Carvalho, the long-period equilibrium may never be reached, even in principle. Nevertheless, these authors correctly identify that 'a long-period model "in the old sense" [of Marshall] … will not do.' (Carvalho, 1990, p. 289). Hansson argues that Keynes's long-period employment requires, as a theoretical fiction, the fulfilment of *long-term* expectations (1985, p. 336). Yet Keynes's long period is short enough that a constant state of expectation, in which *short-term* expectations are fulfilled, is at least conceivable in practice. From this perspective, Keynes did in fact consider the dynamic consequences of short-period investment for future productive capacity (Asimakopulos, 1989, p. 26), without which the theory of effective demand is incomplete. See also CW XXIX, pp. 221-222 on the distinction between long term (run) and long period. Hicks (1939, p. 206-212) analyses convergence from short- to long-period equilibrium consequent upon production time-lags in similar fashion to Keynes, and both - in certain respects - anticipate 'real business cycle' analysis.

propensity to consume. All three of these major psychological independent variables are capable of unpredictable shifts leading to the disappointment of previous expectations. In a key passage Keynes writes:

> Express reference to current long-term expectations can seldom be avoided. But it will often be safe to omit express reference to *short-term* expectation, in view of the fact that in practice the process of revision of short-term expectation is a gradual and continuous one, carried on largely in the light of realised results; so that expected and realised results run into and overlap one another in their influence. For, although output and employment are determined by the producer's short-term expectations and not by past results, the most recent results usually play a predominant part in determining what these expectations are. (*GT,* pp. 50-51)

This statement has usually been read, in line with a definition of employment equilibrium as a point where expectations are fulfilled, as evidence of an assumption in Chapter 3 of *The General Theory* that short-term expectations are fulfilled. The present argument (and Roberts, 1978, p. 372) takes the passage more literally as meaning, not that current expectations correctly anticipate future results, but that past results heavily influence current expectations; so that if change is at a gradual pace, relative to the shortness of the day, expectations often will be fulfilled, or 'overlap' with realised results. This passage is therefore part of Keynes's concern throughout *The General Theory* (*GT*, pp. 64, 117, 249-254) to reflect the normal empirical stability of the price system, and remains consistent with his statement quoted earlier that 'employment is determined *solely* by effective demand'. It also gives explicit notice of Keynes's switch in focus, in discussing aggregate demand, from entrepreneurial expectations in Book II of *The General Theory* to the expenditure decisions of Books III and IV.[14]

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[14] Amadeo (1989) interprets this switch as Keynes almost dropping 'supply' in moving from the *Treatise on Money* (Keynes, 1930) to a final 'expenditure' version of the principle of effective demand. On the present reading, the principle of effective demand relates exclusively to 'supply', including the short-term expectations of entrepreneurs, but after Chapter 5 of *The General Theory*, Keynes assumes these expectations are determined by 'expenditure' and realised results.

If the fulfilment of the state of short-term expectation is a matter of secondary importance to Keynes, the point of effective demand cannot mean the position where short-term expectations are fulfilled, nor does Keynes's exposition in Chapter 3 assume that they are so fulfilled. This opens the question of the nature of the equilibrium process which determines effective demand (*GT,* p. 25), if this process does not take place over time and does not involve the convergence of expectations.

5. The Nature of the Equilibrium Represented by the Point of Effective Demand

In the previous section, the state of short-term expectation was defined as the set of equilibrium expected prices corresponding to the point of effective demand. Under Marshallian conditions of perfect competition and diminishing returns, these expected prices are sufficient to determine output and thus the value of effective demand, and correspond directly to Marshall's short-period normal prices. The question now is how these equilibrium prices are reached. Fundamental to the notion of competitive equilibrium is the independence of supply and demand, and in Keynes's exposition on p. 25 of *The General Theory* it is not obvious how entrepreneurs can face a demand curve which represents their own expectation of proceeds. The received interpretation solves this by taking the aggregate demand curve to mean the expenditure plans of consumers and investors, leading directly to the likelihood of disequilibrium through a mismatch between entrepreneur's expectations and aggregate expenditure plans, and the need for convergence over a number of days. However, Keynes is explicit that aggregate demand here refers to the price expectations of entrepreneurs, to which Asimakopulos (1982, pp. 19-22) can find no satisfactory answer. Nor does it help to interpret the equilibrium process as taking place wholly in the minds of entrepreneurs, individually or collectively, since in the competitive Marshallian system entrepreneurs form expectations of prices, not of demand curves; objective supply and demand determine expected prices. This suggests that Keynes's entrepreneurs are best understood as fulfilling two separate functions, as employers of labour on the one hand, and as dealers on the other.[15]

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[15] This suggestion develops Keynes's explanation of effective demand to Hawtrey (CW XIII pp. 596-632), notably 'The main point seems to me to affect the whole supply and demand theory and not my version of it in particular. I have the impression that you restrict the supply

Employers are specialised in managing the risks of production, and dealers, the risks of marketing finished goods; this division of enterprise is commonly observed in practice.[16] Production usually only takes place when a dealer or another employer places an order with an employer; not many goods or services are ordered by consumers directly from the manufacturer or provider. Production to order implies, under perfect competition, a set of forward markets (for all goods in production or producible today) for each day of Keynes's long period, extending to the horizon of short-term expectation determined by the period of production for industry as a whole. Whatever may be the expectations of individual dealers about future spot prices, the forward market establishes a price at which each dealer's demand is in equilibrium. If any speculation about future spot prices by employers is treated as a dealer activity, the forward prices become the shared expectations that determine aggregate employment and which permit definition of 'the' state of short-term expectation. On these definitions, divergent employer expectations are inconsistent with perfect competition; any divergences are thrown onto the dealers.[17]

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and demand method to market prices only, that is to say, they relate to the higgling of the market in respect of stocks which already exist. But that is not what Marshall or Pigou or most modern economists do. The demand which determines the decision as to how much plant [labour?] to employ must necessarily concern itself with expectations. And I am in this respect simply trying to put more precisely what is implicit in most contemporary economics.' (ibid, p. 602). Also, a few pages further on: 'The only thing that really matters is that the given state of expectation, whatever it is, does produce by its effect on the minds of entrepreneurs *and dealers* a determinate level of employment. But I should find it difficult to do without my schematism as a convenient method of quantifying the state of expectation' (ibid, p. 616, emphasis added).

[16] Under perfect competition, it is of no consequence if the employers are also dealers in their own or other employers' products; the analytical division corresponds merely to the two types of decision. It is also easier, but not essential, to think of the dealers as self-employed or as employing a fixed work-force.

[17] These forward markets extend only so far into the future as required to govern output over Keynes's long period; they do not represent a complete set of futures and insurance markets in the sense of Arrow & Hahn (1971). Keynes's 'correct foresight' (CW XIII, p. 603) or

The point of effective demand is a short-period equilibrium position, meaning that entrepreneurs adjust employment to maximise profit. Since Keynes's short period is his day, and the day is the quantum unit of time, this means that aggregate demand and supply are in static equilibrium at all times (every day); the equilibrium process of finding the point of effective demand described on p. 25 of *The General Theory* takes place on a single day. The equilibrium prices of the output resulting from today's employment are determined each day as the set of prices which clear the supply offers by employers and the demand bids by dealers in the forward markets for delivery of each type of good on the different days corresponding to the end of their production periods. It is this process, of continuous market clearing at a point in time rather than convergence over time, to which Keynes refers (*GT,* p. 25). Each day employment moves directly to the equilibrium position corresponding to the forward prices, although, as noted earlier, there may be a difference between the short-period and long-period employment because of the production time required to adjust the capital equipment. Thus if the effective demand for any given day corresponds to a vector of forward prices for each industry, there is also a matrix of such vectors, with one column for each day of the long period (reminiscent of Hicks's production plan). This matrix **Π** may be regarded as the numerical expression of the state of short-term expectation, and may itself change as a whole from day to day.

Today's matrix of forward prices is a set of market prices in the same sense as today's spot prices, so that this mental device allows us to convert short-term expectations into market prices. These forward prices correspond to Keynes's expected prices and Marshall's short-period normal prices. The 'expectations matrix' defines not only today's forward prices and effective demand, but also the effective demand for each day of the long period in a given state of expectation. Today's forward price vector determines today's employment, and the expectations matrix implicitly plots the convergence path of employment to its long-period position, so that both the short-period and long-period equilibrium positions are fully determined by a given state of expectation. Even if the expectations matrix changes every day, today's equilibrium

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'rational expectation' is limited to the short term and is contingent on the state of expectation: such foresight will be disappointed if the state of expectation changes.

employment is always on a convergence path towards a long-period position (see Appendix B).

Since production is undertaken only against forward orders, the employer's expectations are fulfilled by contract, and can be disappointed only if the dealer defaults upon delivery (this would represent a windfall loss for the employer, which does not affect present or future employment decisions). Whether the dealers' expectations are fulfilled is literally a question for another day, and depends on whether they have correctly judged future demand at the time of delivery in a given state of expectation, and whether the state of expectation changes in the interim. The state of short-term expectation, seen as a matrix of equilibrium wholesale forward market prices available to employers, defines levels of daily and long-period employment which are independent of whether dealers' expectations are fulfilled. If there is a change in the state of expectation, both employers and dealers may make windfall gains or losses on existing capital-goods, including work in progress under forward contracts.

This device captures Keynes's definition of the aggregate demand function in terms of the expectations of entrepreneurs, and also helps to clarify the distinction between the cases where a change in demand has been foreseen (or 'sufficient notice' has been given), and where it is more or less unforeseen. Sufficient notice here means that forward orders for any good are placed for delivery not earlier than the end of its period of production, defining a new state of expectation but without 'more disturbance to the price of consumption-goods than is consequential, in conditions of decreasing returns, on an increase in the quantity which is produced' (*GT,* p. 122). Unforeseen changes in demand affect in addition the prices of goods already in production, with consequent current or prospective windfall gains or losses for dealers on their forward contracts. These changes are unforeseen in the sense that the state of expectations has changed since yesterday, but given the change, the windfall gains or losses are now foreseen, being the difference between the forward prices for a given day in the new expectations matrix and in the old. If the forward contracts can be traded as futures contracts, prospective windfalls can be crystallised today. However these windfalls do not affect the employment decision, which remains based on today's new forward price vector (*GT,* p. 288).

The disappointment of dealer expectations plays no direct causal role in relation to the determination of equilibrium employment, but exerts its influence through changing the state of expectation. Keynes writes:

> the importance [of the difference between expected and actual income resulting to an entrepreneur from a particular decision, due to a mistake in the short-period expectation] … lies in the fact that this difference will be one of the relevant factors in determining subsequent effective demand. I began … by regarding this difference as important. But eventually I felt it to be of secondary importance, emphasis on it obscuring the real argument. For the theory of effective demand is substantially the same if we assume that short-period expectations are always fulfilled. ... For other economists, I find, lay the whole emphasis, and find the whole explanation in the differences between effective demand and income; and they are so convinced that this is the right course that they do not notice that in my treatment this is *not* so (CW XIV, p. 181)

In our terms, the dynamic passages of *The General Theory* relate to the convergence of daily employment on its long-period equilibrium position after a change in the state of expectation which then persists. Any windfalls, arising from the initial change being unforeseen, have no effect on employment. The theory of employment is the same if the change was fully anticipated and the state of expectation persists.

Keynes then writes:

> Entrepreneurs have to endeavour to forecast demand. They do not, as a rule, make wildly wrong forecasts of the equilibrium position. But, as the matter is very complex, they do not get it just right; and they endeavour to approximate to the true position by a method of trial and error. Contracting where they find that they are overshooting their market, expanding where the opposite occurs. It corresponds precisely to the higgling of the market by means of which buyers and sellers endeavour to discover the true equilibrium position of supply and demand. … The main point is to distinguish the forces determining the position of equilibrium from the technique of trial and error by means of which the entrepreneur discovers where the position is. …

> Let us suppose identity of *ex ante* and *ex post*, my theory remains. *Ex ante* decisions may be decided by trial and error or by judicious foresight, or (as in fact) by both. I should have distinguished more sharply between a theory based on *ex ante* effective demand, however arrived at, and a psychological chapter indicating how the business world reaches its *ex ante* decisions. It is only in this chapter that income, investment and saving, which are *ex post* concepts, come in. (CW XIV, pp. 182-183)

This passage, taken in isolation, certainly invites confusion between the point of effective demand and the 'true equilibrium position' where expectations are fulfilled, which is only dispelled in the final paragraph quoted. Here Keynes acknowledges that the equilibrium employment determined by the principle of effective demand may not be a 'true equilibrium' in the different sense of fulfilling expectations, but this admission does *not* appear in *The General Theory* itself. It is this distinction that is captured by the separate definition of D^e (the aggregate demand of *The General Theory*) and D (the 'true' aggregate demand) (Casarosa, 1981; Chick, 1983). Nevertheless it is a mistake to identify this 'higgling' with the equilibrium process of p. 25 of *The General Theory*, rather than with the revision of short-term expectation of p. 50 (Hansson, 1985, p. 333);[18] trial and error or higgling is an *alternative* to that process: for 'judicious foresight' read rational expectations (CW XIII p. 603). As Keynes makes clear, effective demand is an entirely *ex ante* concept relating solely to expectations, but capable of revision in line with *ex post* income. From his reference to the 'psychological' nature of the missing chapter on the formation of dealer expectations, we may infer a reluctance to model this process in formal terms (such as Hicks, 1939; Parinello, 1980; or Amadeo, 1989).

If the point of effective demand is to be characterised as a position of system equilibrium under competition, the questions of existence, uniqueness and stability arise. Keynes approaches these questions from a strict Marshallian rather than from either a Walrasian or 'neoclassical' perspective (where the latter term has the meaning of *GT,* p. 177): 'I am more Classical than the Swedes, for I am still discussing the conditions of short-period *equilibrium*' (CW XIV, p. 183; emphasis added). Marshall's system is *always* in a position of market-period equilibrium provided that output has a market value (*GT,* p. 64); questions of disequilibrium arise only in the sense of departures from the normal. The established approach to stability, beginning with Hicks (1939), presumes that unemployment reflects short-period disequilibrium,

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[18] See Keynes's response to Robertson: 'I do not remember attributing the disappointment of entrepreneurs "to a divergence between aggregate demand and aggregate supply price". I attribute their failure to produce *more* to this; but their disappointment, if any, I attribute (like you) to a divergence between aggregate demand price and income' (CW XIV, p. 89; cf. Robertson, 1936, p. 169).

and is concerned with the conditions for the system to be self-righting, while (as this paper has argued) *The General Theory* implies 'unemployment equilibrium'. By the definition of effective demand as a point of short-period equilibrium and the axiom of optimisation that the level of employment represents that equilibrium, the question of existence does not arise, since some level of employment (therefore, equilibrium employment) is observed. Keynes does not share the Walrasian concern to prove that this equilibrium position represents the optimal allocation of factor resources, since manifestly it does not, given involuntary unemployment. Keynes recognises the possibility of neutral equilibrium (being one of multiple adjacent positions of stable equilibrium) in his discussion of Say's Law (*GT,* pp. 26, 29), and the need to distinguish different distributions as well as levels of employment (*GT,* p. 286), but is content simply to assume that 'corresponding to a given level of aggregate effective demand there is a unique distribution of it between different industries' (*GT,* p. 282).

Given normal price elasticities of demand and supply in accordance with Marshallian axioms, a shift in the demand function for any particular industry will lead immediately to a new set of market-clearing equilibrium prices in the forward markets corresponding to today's employment. Under these conditions, there are no theoretical grounds for instability or indeterminacy in the point of effective demand. A second question is the stability of the state of expectation when expectations are not fulfilled. Keynes's remark 'that in practice the process of revision of short-term expectation is a gradual and continuous one' (*GT,* pp. 50-51) implies a belief, based on observation, that small daily changes are not associated with extravagant disequilibrium dynamics in either expectations or production, and that employment will track its long-period equilibrium value closely. Keynes does not spell out the theoretical basis of this belief, but again Marshallian axioms imply that a divergence between market and normal prices will lead to a stabilising adjustment in expectations (see *GT,* p. 124; CW XXIX, p. 90n; Hicks 1939, Ch. V).

Like Keynes's Chapter 3, this paper has made no use of the multiplier, which plays such an important role in the Keynesian interpretation of effective demand. By now the reader may perceive that in this analysis the multiplier is a market-period (*ex post*) relationship between the realised values of consumption and investment output, which may well influence the state of expectation, but is not directly itself a causal element of the principle of effective demand. The multiplier is best understood as an

equilibrium relation or logical condition which 'holds good, continuously, without time-lag, at all moments of time' (*GT*, p. 122). A marginal propensity to consume less than unity is simply the 'psychological law' that ensures a set of market prices exist so that a value can be placed on output (*GT*, p. 64), and is the mathematical equivalent at the macroeconomic level of the conditions for stable market-period price equilibrium in terms of Marshallian supply and demand elasticities (see Appendix C).

6. Concluding remarks

This paper's point of departure has been a reconsideration of Keynes's use of Marshall's equilibrium periods in *The General Theory*. Keynes's quantum day corresponds to both the market and short equilibrium periods; the day and the period of production are not the same, and the long and short term are not the same as the long and short period. Keynes does not admit long-term long-period equilibrium in the sense of Marshall; his long period extends only to the horizon of short-term expectation, at the end of the period of production. So, there are three units of calendar time grounded in production (the day, microeconomic production period and macroeconomic period of production); a correspondence between the market and short equilibrium periods and the day; and a correspondence between the long equilibrium period, the macroeconomic period of production and the horizon of short-term expectation.

In consequence, employment is always, for analytical purposes, in short-period equilibrium, and the dynamic passages of *The General Theory* describe a convergence of employment over time from short-period to long-period equilibrium, in a given state of short-term expectation, because of production time. Employment is determined solely by the state of expectation through effective demand at any time, and neither the convergence of expectations nor the multiplier plays any direct role. From this perspective, passages in *The General Theory* that have troubled (among others) Patinkin (1976), Asimakopulos (1982), Clower (1997) and Gnos (2004) come into focus; effective demand comes into its own as an equilibrium principle based on supply and demand in the markets for the heterogeneous output resulting from today's employment in each industry; and aggregate effective demand proves to be conceptually and numerically quite different from aggregate income, expected or realised.

If the objective of economics is to explain the world, economists need to accept that the theory of competitive equilibrium can only provide a useful model for a given state of expectation, which for most purposes confines its application to a point in time. Far from failing to understand the Classical theory of value, as has so often been suggested, Keynes took it to a new level of refinement and clarified what equilibrium can mean in a monetary economy. The Keynesian synthesis and its surviving remnants emasculate *The General Theory* by ignoring its micro-foundations: one implication of this paper is that the Keynesian cross and Swedish interpretation of the point of effective demand, on which we were all raised, should be altogether abandoned in favour of the Post Keynesian *Z* diagram, and even that should be interpreted only in static terms as describing the point of equilibrium. Keynes recognised the strict limits within which competitive equilibrium theory can remain rigorous, in the old sense of conformity with the facts. His achievement was to devise as new primary elements, alongside the endowment, technology and preferences of the Classical system, the independent variables of the propensity to consume, the marginal efficiency of capital, and the state of liquidity-preference that reflect the historical nature of time. These not only allowed him to make a logical connection between the observed level of aggregate employment and something as intractable, but inescapable, as the state of long-term expectation, but also provided a method for moving beyond the equilibrium model itself in order to concentrate attention on changes in its independent variables. There are, of course, significant limitations to the Marshallian system, which need to be overcome, yet *The General Theory* should be the starting point of any theory of the economic system as a whole based on supply and demand: after 70 years we have still scarcely begun to take its message seriously, as Arrow & Hahn also remarked in closing their book, 35 years ago.

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Appendix A

In order to compare Keynes's principle of effective demand with the Walrasian concept of general equilibrium, the discussion of Section 5 can be expressed in symbols, broadly following the nomenclature of Arrow & Hahn (1971). Keynes summarises the definition of effective demand (*GT,* pp. 25-29) as the solution to three equations:

$$D = f(N) = D_1(N) + D_2 = \chi(N) + D_2 \tag{A1}$$

$$Z = Z(N) = \phi(N) \tag{A2}$$

$$D = Z \tag{A3}$$

where D_1 reflects the propensity to consume χ and is a function of aggregate employment, and D_2 reflects the inducement to invest and is independent of D_1, and largely of *N*. It is convenient for the avoidance of doubt to write the solution to this system with asterisks as (*D**, *N**), although Keynes chooses not to do this, perhaps because as a strict Marshallian he treats *D* and *N* as always in equilibrium.

To make the relative prices and the supply and demand for the products of individual industries explicit, Keynes's system of equations can be expanded to:

$$D = D(\mathbf{n}, \bar{\mathbf{x}}) = (\mathbf{x} - \bar{\mathbf{x}})' \, \mathbf{p^d} \tag{A1a}$$

$$\mathbf{x} = \chi(\mathbf{n}, \bar{\mathbf{x}}, \mathbf{p^d}) + \Theta(\mathbf{p^d}, \Omega, r) \tag{A1b}$$

$$Z = Z(\mathbf{n}) = \mathbf{y}' \mathbf{p^s} \tag{A2a}$$

$$(\mathbf{y}, \mathbf{p^s}) = \Phi(\mathbf{n}, \bar{\mathbf{x}}) \tag{A2b}$$

$$D = Z \tag{3}$$

where column vectors are defined as follows

n   quantities of homogeneous labour employed today in each industry n_i

x   quantities bid by households today x_i for delivery of a dated good *i* at the end of its production period, where goods include both consumption- and capital-goods produced by an industry. Each good may have a different production period, so that output is heterogeneous both by industry and by date of delivery.

$\overline{\mathbf{x}}$ today's opening stock of producible capital-goods (Keynes abstracts from storage of consumption-goods by their definition). Capital-goods are held by households, and rented or sold to firms.

$\mathbf{y}$ quantities offered by firms today y_i for delivery of a dated good i at the end of its production period. Negative elements of $\mathbf{y}$ represent bids by firms for the output of other firms and for existing capital-goods (user cost).

$\mathbf{p^d}$ forward bid prices corresponding to $\mathbf{x}$ in terms of numeraire, say wage-units.

$\mathbf{p^s}$ forward offer prices corresponding to $\mathbf{y}$ in terms of numeraire.

Other variables and functions are as in *The General Theory*, except that χ and $\boldsymbol{\Phi}$ are now vector functions, and D_2 is replaced by $\boldsymbol{\Theta}$, which allows investment to vary with the price of capital-goods. $\boldsymbol{\Theta}$ takes as parameters a given state of long-term expectation Ω and a given structure of interest rates r (see CW XIII, p. 441). The functions $\chi, \boldsymbol{\Phi}, \boldsymbol{\Theta}$ and the parameters $\overline{\mathbf{x}}, \Omega, r$ are exogenous, while all of $\mathbf{n, x, y, p^d}$, $\mathbf{p^s}$ are the endogenous variables whose equilibrium values are represented by the solution to the system. Although no time subscripts are shown, this is an inter-temporal or dynamic system in the sense of Hicks (1939), since all price and quantity variables are dated; yet it remains static in the usual sense, since the solution represents the spot and forward values at a point in time of a 'temporary equilibrium' (Arrow & Hahn, 1971, pp. 33-40, 347).

The money-value variables D and Z are scalar (dot) products of the relevant vectors (A1a, A2a), and similarly $N = \mathbf{e'n}$ where $\mathbf{e'}$ is the unit row vector, the transpose of $\mathbf{e}$. The transformation illuminates Keynes's reasoning for working in aggregates only of money-value and labour, meaning that his system of scalar equations (A1), (A2) and (A3) captures the heterogeneity of output quite as correctly as does the vector system, since the aggregate functions $f(N)$ and $\phi(N)$ are defined if the vector system has a solution (at least they are defined for $N = N^*$; we get correspondences rather than functional relations away from the equilibrium point).

Equation (A1a) represents the money-value of consumption and investment demand, while equation (A2a) represents the aggregate supply price of output. Note that both D and Z are aggregates of values at different dates. Equation (A1b) shows the demand for the product of each industry varying with (the aggregate income arising from)

employment and the ownership of the capital equipment, and on all relative prices, thus allowing for complementarity. Equation (A2b) shows the price and quantity offers of the product of each industry varying with employment and the capital equipment.

Assuming that each level of effective demand is uniquely distributed (*GT*, p. 282, which incidentally implies that an equilibrium solution exists), there exists, as an exercise in comparative statics, a functional relation between the equilibrium values of the effective demand *D** in terms of money (or wage-units, without loss of generality) and of the level of aggregate employment *N**, corresponding to different rates of investment and propensities to consume, in our terms, different values of the parameters Ω and *r*, and different functional forms χ. As Keynes emphasises (*GT*, p. 286), the distribution of employment may differ at two levels of aggregate employment, or in the above terms, an equilibrium employment vector $\mathbf{n_1}^*$ need not be a scalar multiple of $\mathbf{n_2}^*$. This functional relation between *D** and *N** (the 'employment function') is the very backbone of *The General Theory*.

A change in any of the functions, but particularly in the parameters Ω and *r*, will lead to a change in the expectations matrix $\mathbf{\Pi}$. If this matrix is defined to include the expected prices for goods already in production, the windfall gains or losses can be identified as the differences between the price vectors representing the columns for the same day in both the original $\mathbf{\Pi}$ and the revised matrix $\mathbf{\Pi}'$.

The Walrasian model differs from Keynes's enhancement of Marshall's system in a number of ways. The structural difference, apart from the major question of the process of price adjustment, is that the Walrasian demand and supply vectors include the endowment of non-producible factor services, and the future goods producible with the capital-goods producible today. Implicit in the inclusion of factor services is the idea that households have as good access to product markets as do firms, but prefer to sell factor services rather than products. In Keynes's model as expressed above, $\mathbf{x}$ and $\mathbf{y}$ refer only to the *products of industries*, including the stock of producible capital-goods carried forward each day. In the Walrasian model there is no difference between factor services and any other goods, so that $\mathbf{x}$ becomes $\mathbf{\breve{x}}$, a vector including as negative elements the supply of non-producible factor services by households ($\mathbf{\bar{x}}$ defined accordingly) and $\mathbf{y}$ becomes $\mathbf{\breve{y}}$, a vector including as negative elements the demand for factor services by firms. The corresponding price vectors are

also denoted by cups. Given optimisation, the scalar products $\breve{\mathbf{x}}'\breve{\mathbf{p}}^{\mathbf{d}}$ and $\breve{\mathbf{y}}'\breve{\mathbf{p}}^{\mathbf{s}}$ are both zero, reflecting respectively the household budget constraint and the zero profit condition. Say's Law is implicit in the equation $\breve{\mathbf{x}}'\breve{\mathbf{p}}^{\mathbf{d}} = 0$. The Walrasian excess demand function is then defined as $\mathbf{z}(\breve{\mathbf{p}}) = \breve{\mathbf{x}} - \bar{\mathbf{x}} - \breve{\mathbf{y}}$, where $\breve{\mathbf{p}} = \breve{\mathbf{p}}^{\mathbf{s}} = \breve{\mathbf{p}}^{\mathbf{d}}$, and various mathematical theorems identify the conditions under which the equilibrium solution $\mathbf{z}(\breve{\mathbf{p}}) = \mathbf{0}$ exists and is unique and stable. Walras' Law requires also that the scalar product $\breve{\mathbf{p}}'\mathbf{z}(\breve{\mathbf{p}}) = 0$. An excess supply of any good (including labour) is compatible with equilibrium only if its price is zero and disposal is free, which tells us nothing helpful about involuntary unemployment, although it does accommodate Ricardo's no-rent land. In Keynes's system, by contrast, both the household budget constraint and the firm's supply decision depend on the amount of labour firms in aggregate decide to hire, as expressed in the employment vector **n,** as well as on the endowment $\bar{\mathbf{x}}$. The endowment of non-producible factors of production is not part of Keynes's system, because their owners do not make production and employment decisions, which are reserved by definition to entrepreneurs. The existence of a wage-dependent labour force is a sufficient condition for preferring Keynes's treatment over Walras'.

The second structural difference is that in the Walrasian system there is no division between short- and long-term expectation, corresponding to employment and investment decisions; there is only inter-temporal equilibrium in what amounts to a 'fixed and reliable' state of expectation (*GT,* p. 293), within stochastic limits. In formal terms this means $\breve{\mathbf{x}}$ and $\breve{\mathbf{y}}$ include not only the goods currently in production or producible today, but also the future goods producible in turn with the capital-goods in production or producible today. This amounts to making the state of expectation endogenous and money inessential, and postulating some nexus (*GT,* p. 21) that brings the propensity to consume $\boldsymbol{\chi}$ into harmony with the investment function $\boldsymbol{\Theta}$, such as a complete set of futures and insurance markets to fix the state of long-term expectation and remove the need for liquidity-preference (Arrow & Hahn, 1971, pp. 33-34). The absence of this nexus in practice is precisely the primary concern of *The General Theory*.

Appendix B

The relationship between Keynes's short- and long-period equilibrium employment can also be described using the notation of Appendix A. The short-period equilibrium employment vector on any given day can be expressed as $\mathbf{n}^*(t)$ and the long-period employment vector as $\mathbf{n}^{**}\big|\Omega_t, r_t$ such that $\mathbf{n}^*(t+\lambda) = \mathbf{n}^{**}\big|\Omega_t, r_t$, if $\Omega_{t+\lambda} = \Omega_t$ and $r_{t+\lambda} = r_t$. In this expression, Ω_t means the state of long-term expectation on day t, r_t represents the structure of interest rates on day t, and λ is the length of the long period measured in days. This condition holds the valuation of investment opportunities constant while allowing the cost of capital-goods to vary. Where the analysis is conducted in a constant state of expectation, express reference to Ω_t and r_t can be omitted. λ is finite if the minimum labour unit is a discrete quantum, so that the process of asymptotic convergence comes to an end.

Today's state of short-term expectation, conditional on yesterday's employment vector and today's state of long-term expectation and interest rates, may then be expressed as a matrix of expected prices $\mathbf{\Pi}_t\big|\mathbf{n}^*(t-1), \Omega_t, r_t = [\mathbf{p}(t), \mathbf{p}(t+1), ... \mathbf{p}(t+\lambda)]$, where $\mathbf{n}^*(t-1)$ is yesterday's employment and $\mathbf{p}(t)$ is the equilibrium price vector corresponding to the point of effective demand on day t. $\mathbf{p}(t)$ depends on the aggregate demand function (equation A1b) and the aggregate supply function (equation A2b), with $\mathbf{p}(t) = \mathbf{p_s}(t) = \mathbf{p_d}(t)$ in equilibrium. The propensity to consume χ, investment $\mathbf{\Theta}$ and aggregate supply $\mathbf{\Phi}$ functions are held constant during the long period. With a given state of long-term expectation, interest rates and initial employment vector, the short-term price expectations matrix will be the same for each day of the long period and can for convenience simply be written $\mathbf{\Pi}$, a numerical expression of the state of short-term expectation. The convergence path of employment over the long period from any given initial position to the position of long-period employment in a given state of expectation is then expressed by the dynamic process $\mathbf{n}^*(t\big|\mathbf{\Pi}) \to \mathbf{n}^{**}$.

This raises the question of the conditions under which the process is convergent and the long-period equilibrium is stable. Keynes's description of the equilibrium solution path on p. 49 of *The General Theory* can be expressed as the solution to three equations:

$$n_I^*(t) = \alpha \dot{n}_C^*(t) + n_I^{**} \tag{B1}$$

$$\dot{n}_C^*(t) = \beta\left(n_C^{**} - n_C^*(t)\right) \tag{B2}$$

$$n_C^*(t) = \gamma\left(n_C^*(t) + n_I^*(t)\right) \tag{B3}$$

$n_I^*(t)$ and $n_C^*(t)$ are the sums of the elements of the daily employment vector **n***(t) corresponding to the capital- and consumption-goods industries respectively (*i.e.* the levels of employment in each sector). These are all short-period equilibrium values, corresponding to the point of effective demand each day. n_I^{**} and n_C^{**} are the long-period equilibrium values determined by supply and demand in forward markets.

Equation (B1) then expresses the level of employment in the capital-goods industries $n_I^*(t)$ as the sum of an exogenously given long-period level n_I^{**} together with a dynamic term $\alpha \dot{n}_C^*(t)$, making it a function of the rate of change of employment in the consumption-goods industries. This represents the employment in the capital-goods industries needed to produce the additional capital equipment required by the consumption-goods industries to support their increased employment.

Equation (B2) is an expositional device: it merely states that, if there is a long-period equilibrium, the level of employment in the consumption-goods industries will tend towards it at a rate β proportional to the level of disequilibrium. We will consider later why this should be the case. Similarly, equation (B3) establishes a relation between the level of employment in the consumption-goods industries and total employment. The proportion γ is a variant of the employment multiplier.

Reducing the system of equations (dropping the time subscripts for convenience) gives an equation of motion for the consumption-goods industries, from which employment in the capital-goods industries and total employment follow directly:

$$\ddot{n}_C^* + \frac{1}{\alpha}\dot{n}_C^* + \frac{\beta}{\alpha\gamma}n_C^* = \frac{\beta}{\alpha\gamma}n_C^{**} \tag{B4}$$

This process is convergent since $\alpha, \beta, \gamma > 0$. The condition for the damped simple harmonic motion described by Keynes is $\gamma < 4\alpha\beta$. Since $\gamma < 1$, oscillation takes place unless the capital intensity of the new employment in consumption-goods production (α), and/or the rate of response to disequilibrium (β) are sufficiently low. Equation (B4) does not capture the whole of Keynes's argument, since it cannot

describe the cyclical effects of a change in the composition of a given level of total employment: to do this with mathematics would require a higher order system of partial differential equations, with one equation for each industry. Since not much can be said *a priori* about the value or behaviour of the parameters α and β, a technical exercise of this sort would add little.

The rate of response to disequilibrium (β) and the length of the long period depend on the supply and demand elasticities in each industry, taking into account the user cost of existing stocks of capital-goods. By assumption, the long-period employment vector is known, determined in the markets for the final day of the long period by the aggregate supply functions of employers and the aggregate demand functions of dealers. On a particular day, the demand for a consumption-good relative to its long-period value will reflect the level of employment expected by the dealers, the relative price on that day, and the consequent degree of substitution in favour of other products, or deferral of purchase until a later day. Conversely the supply price of a product will reflect not only the diminishing returns associated with increased output, but also the opportunity cost of using existing stocks now rather than later. If the new long-period employment represents an increase, the level of output in the early days of the long period will be constrained by a shortage of capital-goods and a limitation of demand by both the high price and the shortfall in the level of employment from its new long-period value. If today's market price is too low in relation to a later day within the long period, it may pay to hold the capital equipment in stock rather than use it in production, thus increasing the spot price. A process of inter-temporal arbitrage takes place, on both the demand and supply sides, between the forward markets for different days of the long period, until an equilibrium convergence path is arrived at. Whether the equilibrium convergence path involves overshoot oscillations depends mainly on the physical supply and demand elasticities.

Appendix C

A marginal propensity to consume less than unity can be shown to be the macroeconomic equivalent of the Marshallian conditions of market-period price equilibrium. Let the vectors **x, y, p^d, p^s** be defined as in Appendix A, with the important difference that they now refer to spot deliveries of goods today, and not to forward deliveries at the end of their production periods. For the most part today's deliveries will be based on finishing work-in-progress. We can then define:

Y aggregate value of income $\equiv \mathbf{y'p^s} = \left(\mathbf{x} - \bar{\mathbf{x}}\right)' \mathbf{p^d}$ in price equilibrium

C aggregate value of consumption $\equiv \mathbf{x'_c p^d_c} = \mathbf{y'_c p^s_c}$ in price equilibrium, where the subscript denotes that the vectors include only the consumption-goods industries (there are no stocks of consumption-goods by definition, but this does not affect the result)

I aggregate value of investment (exogenous for the present purpose).

The postulated functional relations are:

$$\mathbf{x} = \mathbf{x}(\mathbf{p^d}, Y) \tag{C1}$$

$$\mathbf{y} = \mathbf{y}(\mathbf{p^s}) \tag{C2}$$

$$\mathbf{p^d} = \mathbf{p^d}(Y) \tag{C3}$$

$$\mathbf{p^s} = \mathbf{p^s}(Y) \tag{C4}$$

and without relative prices, the aggregate functions are:

$$C = C(Y) \tag{C5}$$

$$Y \equiv C + I \qquad \text{(national income identity)} \tag{C6}$$

Note that the demand and supply functions (C1) and (C2) are those for the daily market period and not for the full production periods of effective demand set out in Appendix A. Some variation in supply is possible in the market period, through the depletion of stocks, the finishing of work-in-progress and the provision of services on demand, so that **y** need not be assumed constant. All variables are undated, i.e. simultaneous, so this is a static equilibrium problem. The traditional (point) multiplier can be obtained by differentiating the identity (6), giving

$$\frac{dY}{dI} = \frac{1}{1 - \frac{dC}{dY}} \tag{C7}$$

which for stability requires $\frac{dC}{dY} < 1$.

We can instead include the relative prices of consumer goods and write:

$$Y \equiv C + I \equiv \mathbf{x_c'}\mathbf{p_c^d} + I \equiv \mathbf{y_c'}\mathbf{p_c^s} + I \tag{C8}$$

Dropping the subscript, the multiplier can now be written in two forms corresponding to demand and supply:

$$\frac{dY}{dI} = \frac{1}{1 - \left[\left(\frac{\partial \mathbf{x}}{\partial \mathbf{p^d}} \frac{d\mathbf{p^d}}{dY} + \frac{\partial \mathbf{x}}{\partial Y} \right) \mathbf{p^d} + \mathbf{x'} \frac{d\mathbf{p^d}}{dY} \right]} \tag{C9}$$

$$\frac{dY}{dI} = \frac{1}{1 - \left[\left(\frac{d\mathbf{y}}{d\mathbf{p^s}} \frac{d\mathbf{p^s}}{dY} \right) \mathbf{p^s} + \mathbf{y'} \frac{d\mathbf{p^s}}{dY} \right]} \tag{C10}$$

Note the extra term in the demand version (C9), reflecting the direct influence of aggregate income on demand independent of its influence through the demand price. In the case of supply, the quantity is determined solely by the supply price, which is thus the only channel through which aggregate income can affect supply.

The condition of price equilibrium is that expressions (C9) and (C10) for the multiplier are equal. In each industry this means (abstracting from complementarity between industries):

$$\left(x + p^d \frac{\partial x}{\partial p^d} \right) \frac{dp^d}{dY} + p^d \frac{\partial x}{\partial Y} = \left(y + p^s \frac{dy}{dp^s} \right) \frac{dp^s}{dY} \tag{C11}$$

In equilibrium $x = y$, $p^s = p^d$ so we can write:

$$\left[\frac{dp^d}{dY} \frac{Y}{p^d} + \frac{\partial x}{\partial p^d} \frac{p^d}{x} \frac{dp^d}{dY} \frac{Y}{p^d} + \frac{\partial x}{\partial Y} \frac{Y}{x} \right] \frac{p^d x}{Y} = \left[\frac{dp^s}{dY} \frac{Y}{p^s} + \frac{dy}{dp^s} \frac{p^s}{y} \frac{dp^s}{dY} \frac{Y}{p^s} \right] \frac{p^s y}{Y} \tag{C12}$$

which can be restated as

$$\varepsilon_{p^d Y} \left(1 + \varepsilon_{xp^d} \right) + \varepsilon_{xY} = \varepsilon_{p^s Y} \left(1 + \varepsilon_{yp^s} \right) \tag{C13}$$

where ε_{xY} denotes the aggregate income elasticity of demand $\dfrac{\partial x}{\partial Y}\dfrac{Y}{x}$, ε_{p^sY} the

aggregate income elasticity of the supply price $\dfrac{dp^s}{dY}\dfrac{Y}{p^s}$ and the other elasticities are

written accordingly.

For normal goods, $\varepsilon_{xY} > 0$, so that

$$\frac{\left(1+\varepsilon_{xp^d}\right)}{\left(1+\varepsilon_{yp^s}\right)}\frac{\varepsilon_{p^sY}}{\varepsilon_{p^dY}} < \quad \text{and therefore } \varepsilon_{xp^d} < \varepsilon_{yp^s}, \text{ if } \varepsilon_{p^sY} \leq \varepsilon_{p^dY} \tag{C14}$$

Putting (C14) into words, the condition for aggregate money-income to be defined is equivalent to the standard Marshallian condition that in each industry the market-period price elasticity of demand is less than the market-period price elasticity of supply for normal goods, provided that the aggregate income elasticity of the supply price is less than or equal to the aggregate income elasticity of the demand price. The latter pair of 'aggregate' elasticities do not appear explicitly in Marshall.

In the market period, the price elasticity of demand is negative for normal goods, and the price elasticity of supply is either zero, or positive mainly through the depletion of stocks. *A priori* one would expect the second half of the condition to hold as an inequality for normal goods below full employment, and the strict equality to hold at full employment. The definition of a normal good is that the quantity consumed increases at least in proportion to total consumption. Although all supply prices will increase as aggregate income increases through diminishing returns in market-period production and increasing demand for a given quantity of capital-goods, an increase in the quantity consumed implies that the demand price increases with aggregate income by more than the supply price. Only at full employment, when no increase in the quantity produced is possible except by substitution of other goods, can one expect supply prices to increase *pari passu* with aggregate income and demand prices.