2006

A1. (a) $a = -2$ or 1 (b) $A^{-1} = \frac{1}{(a+2)(a-1)^2}\begin{pmatrix} a^2-1 & 1-a & 1-a \\ 1-a & a^2-1 & 1-a \\ 1-a & 1-a & a^2-1 \end{pmatrix}$

A2. $K = \omega^2 Y^*/(r+\omega)^2$ $L = r^2 Y^*/(r+\omega)^2$

3. (a) $x = 2$ $y = 1$

4 (a) $-a/2x^2 - b/x + c\ln x$ (b) $1/4$ (c) $(x^2-2x+2)e^x$

5. (a) $q = 4$ $p = 28$ $\pi = 14$ (b) $q = 0$

6. (a)(i) $2/x - 2e^{2x}/(e^{2x}+1)$ (ii) $\frac{1}{5}(x+4)^{-4/5}(x-5)^{-4/5}(5x+11)$

(b) $\partial f/\partial x = (3x^2-6y)(x-2y)^2 + 2(x^3-6xy+y)(x-2y)$

$\partial f/\partial y = (-6x-1)(x-2y)^2 - 4(x^3-6xy+y)(x-2y)$

B1. (a) $x = \frac{15}{4} + \frac{15}{2}\frac{P_y}{P_x}$ $y = \frac{5}{2} + \frac{5P_x}{4P_y}$ (b)(i) $(6\tfrac{1}{4}, 6\tfrac{1}{4})$ (ii) $(4\tfrac{1}{6}, 25)$

(c)(i) $(1\tfrac{1}{4}, -3\tfrac{3}{4})$ (ii) $(-5/6, 15)$ $P_x/P_y > 6$ (e) $P_x/P_y > 6$

2 (c) $Y = [b(\bar{m}-M_0) + (C_0 - c\bar{T} + I_0 + \bar{G})h]/[h(1-c)+kb]$

$r = [k(C_0 - c\bar{T} + I_0 + \bar{G}) - (1-c)(\bar{m}-M_0)]/[h(1-c)+kb]$

Fiscal policy multiplier $= +h/[h(1-c)+kb]$

(d) $f(r) = \frac{3}{2}r_0^{-1/2} - \frac{1}{2}r r_0^{-3/2}$ so $M^d = kY + \frac{3a}{2}r_0^{-1/2} - \frac{a r r_0^{-3/2}}{2}$

Fiscal policy multiplier $= \dfrac{a r_0^{-3/2}/2}{a r_0^{-3/2}(1-c)/2 + kb}$ and $\downarrow$ as $r_0 \uparrow$.

C1. (a) A: $b/(a-b)$ B: $b/(a+b)$ C: b

2. (a)(i) 0.012 (ii) 0.712 (iii) 0.164 (b) 0.493

3. (a) $w_t(1-\tau_t)$ is proportional to p_t

(b)(i) H_0 $(b=1)$ not rejected vs H_1 $(b \neq 1)$ $z = -0.13$ crit $z = \pm 1.96$

H_0 $(a=0)$ rejected vs H_1 $(a>0)$ $z = 30$ crit $z = \pm 1.645$

So there is evidence for real wage resistance hypothesis

(ii) H_0 $(b=0)$ not rejected vs H_1 $(b\neq 0)$ $z = 1.5$

(iii) Std error in b is too big to conclude anything here - type 2 error too large. we need a bigger sample; (e) $0.10 \pm .05$

4. (a)(i) ~ 8.2 (ii) ~ 66.5 (b) 462 lbs (c) 598 lbs

5. (b)(i) 9 (ii) 52

6. H_0 (proportions equal) rejected as $z = 3.57$

H_0 $(\pi_1 - \pi_2 = 0.2)$ not rejected as $z = 0.735$

D1. (b) I'd use $\sqrt[n]{(i/100)} - 1$ where i = index n = no. of years

(c) $r = 0.85$ $t = 4.6$ clearly significant.

2. (a) $r = 0.5$ $t = 2.02$. Problem: they've not indicated the alternative hypothesis. If it is $\rho \neq 0$ then test is 2-tailed, and critical t is 2.179, so do not reject H_0 of no correl^n. If it is $\rho > 0$ then test is 1-tailed, and critical t is 1.782, so now we do reject H_0 of no correl^n! Suspect 2-tailed is correct and assumed so for (b)

(b) $t = 1.09$ critical $t = 2.056$ for 2-tailed for $\nu = 26$ do not reject H_0

(c) Canada $r = -0.90$ $t = -7.0$ UK $r = -0.55$ $t = -2.3$ reject H_0 ie there are falls (d) $F = 37.9$ critical $F = 4.876$ (1 tail)

2007

A1. –

2. (a) $x_n = \frac{a}{1-b} + b^n\left(\frac{1-b-a}{1-b}\right)$ (b) $b=-1$ unstable $|b|<1$ stable

see below: 3. (a) $q = a/2$ (b) $a/q - 1$ (c) $q = 2(a-10b)/(4+b) = 4$

4. (a) $\frac{1}{3}x^3\ln x - x^3/9 + c$ (b) $-\frac{2}{27}(3x+1)^{3/2} + \frac{2}{45}(3x+1)^{5/2} = \frac{2x}{9}(3x+1)^{3/2} - \frac{4}{135}(3x+1)^{5/2}$

(c) 7

5. (a) constant (b) $Y_L = A\alpha L^{-\gamma-1}(\alpha L^{-\gamma}+\beta K^{-\gamma})^{-\frac{1}{\gamma}-1}$ etc (c) $\left(\frac{\partial L}{\partial K}\right)_Y = -\frac{\beta}{\alpha}\left(\frac{K}{L}\right)^{-\gamma-1}$

6. (a) $\partial z/\partial x = \frac{1}{6}x^{-2/3}(x^{1/3}+y^{2/3})^{-1/2}$ $\partial z/\partial y = \frac{1}{3}y^{-1/3}(x^{1/3}+y^{2/3})^{-1/2}$

(b) $50x^{24} - 24x^{23}\ln(x^2+e^x) - x^{24}(2x+e^x)/(x^2+e^x)$

see below B1. (c) $Y = [\beta(\bar{C}+\bar{I}+\bar{G}) + bM_s]/[\beta(1-c-a+t(c-1))+\alpha b]$

$r = [\alpha(\bar{C}+\bar{I}+\bar{G}) - M_s(1-c-a+t(c-1))]/[\beta(1-c-a+t(c-1))+\alpha b]$

2 (a) $x^* = 3B/5p_x$ $y^* = 2B/5p_y$ (b) $x^*=5$ $y^*=4$ $\lambda^* = 1/60$

(c) $u^* = 1.267$, 1.283 increase of 0.0165, ie $\sim \lambda^*$.

C1. (a) – (b) £2 (c) 26, 36 (d) 0, 1.

2. $r = 0.073$, 95% CI is -0.52 to 0.62

3. (a) 0.1587 (b) 0.532

4. (a) Stdev = 6.4 (assuming >30 class is not a problem: I treated as = 33)

5. (a) Do not reject ($t = 0.94$, critical $t = 2.145$)

6. (a) 99%, no (b) $t = 0.5$ so do not reject H_0 ($\beta = 1$).

D1. (a) – (b) $y_i = \alpha + \beta x_i + \varepsilon_i$, $\varepsilon_i \sim N(0,\sigma^2)$, σ^2 indep. of x.

(c) $\hat{E} = 58.2 - 0.335S$ $R^2 = 0.88$ $t = 5.32$ so statistically significant (critical t is 2.776) $\hat{E}(40) = 44.8$ $\hat{E}(0) = 58.2$

I'd plot the graph if I were you (always)

2. (a) – (b) $\ln Y_t = \ln Z_t + S_t + R_t$ ie $\ln(Y_t/Z_t) = S_t + R_t$.

(c) $\hat{S}_1 = -0.099$ $\hat{S}_2 = -0.064$ $\hat{S}_3 = 0.005$ $\hat{S}_4 = 0.158$

(d) $\hat{\alpha} = 8.16$ $\hat{\beta} = 0.02$ $\gamma_1 = -0.257$ $\gamma_2 = -0.222$ $\gamma_3 = -0.153$

(e) as in (d), aside from the rounding.

Note re B1: The question as stated gives the determinant to be $-\beta(1-c-a+t(c-1)-\alpha b)$, which could be positive or negative. I suspect question should have said $a+c+t<1$, not $a+c<1$. Not sure what's right here.

Note re A3: you'll get $1 - a/q$ for (b) if you define price elasticity as $\frac{p}{q}\frac{dq}{dp}$ rather than $-\frac{p}{q}\frac{dq}{dp}$.

2008

A1. —

2. (a) $L^* = 160$ (b) $L^{**} = 120$

3. (a) $i'L = (2)$ $ii' = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$ (b) Q = zero matrix

4. $x^* = ma/p_x$ $y^* = m(1-a)/p_y$ $\lambda^* = \left(\frac{a}{p_x}\right)^a \left(\frac{1-a}{p_y}\right)^{1-a}$; $\frac{\lambda}{u} = \frac{1}{M}$

5. (a) $Y = 375$ $R = 5$ (b) $dY/dG = 15/8$

6. $y = x - 1$

B1. (a) $a - bp_t = -c + p_t^e$ (b) $bp_t + (\mu + \mu b - b)p_{t-1} = (a+c)\mu$
(c) $p^* = (a+c)/(1+b)$ (d) stable if $\mu < 2b/(1+b)$ (e) growth, oscillation.
(3578 to 3528)

2. (a) $Q_1 = 47$ $Q_2 = 37$ $P_1 = 53$ $P_2 = 43$ (b) 50 (c) — (d) $t = 47$
(e) (assuming 8 is a cost per unit you carry on plane (?!)) then
$Q_1 = 48$ $Q_2 = 36$ $P_1 = 52$ $P_2 = 44$

C1. Assuming individual bills are Normal, test stat $t = 2.36$; critical t is 1.833. So reject H_0 and conclude mean level of billing is excess of £17.10

2. —

3. (a) $(1+5)/36$ (b) $7/12$ (c) $125/3888$

4. —

5. (a) 4 (b) 1.21

6. (a) 0.33 to 0.47 (b) $z = 1.49$; critical $z = 1.96$, so do not reject null hypothesis of no difference

D1. (a) $N(0, \sigma^2)$ (b)(i) shouldn't happen (mean $\neq 0$) (ii) variance varies w/time
(c)(ii) 0.45 to 1.02 (iii) extrapolation gives 64.0, except relation is not linear, so it's a daft prediction (iv) both 56
(v) 1: (29.29, −9.29) 2: (32.98, −2.98) 8: (55.12, −0.12)
9: (58.81, −2.81) 10: (62.5, −6.5)

2. (a) College A: 56, 65, 56, 251, 15.8
College B: 61, 63, 61, 433, 6.6
(c) Assume individual marks are Normal
" variances equal (F-test gives 5.8, cf critical F of 5.82; so H_0 (variances equal) not rejected but it's very close)

test statistic = −0.77, critical t = 2.179
So do not reject H_0 (means equal).

(d) Uh? Normal approx does not hold for such small samples.
and (f) If we assume proportions equal then pooled estimate of $p = 4/14$ & then one can do Binomial calc^s (??)
(e) New mean for college A is 63.6. Unclear what is wanted beyond that

2009

A1. –

2. £30.

3. (a) $R_t = (1 - f - s)R^* + s$; 15%; (b) 11.715%; two periods.

4. (a) $f(t) = 36.5 + 1.7t - 0.875t^2$; (b) $t = 6 - G$ ie $t = 4$, so a decrease of 1.

B1. (a) $Y(1 - c(1 - t) - a - \theta t) + br = \bar{C} + \bar{I} + \bar{G}$ and $\alpha Y - \beta r = M_s$; (b) $Y = \frac{\bar{G}}{(1-\theta)t}$; (c) $-\beta(1 - c(1 - t) - a - \theta t) - \alpha b$; (d) $Y = \frac{\beta(\bar{C}+\bar{I}+\bar{G})+bM_s}{\beta(1-c(1-t)-a-\theta t)+\alpha b}$; $r = \frac{\alpha(\bar{C}+\bar{I}+\bar{G})-M_s(1-c(1-t)-a-\theta t)}{\beta(1-c(1-t)-a-\theta t)+\alpha b}$; r increases with $\bar{G}$, and the impact gets larger as θ increases.

2. (b) $x^* = \frac{1}{3p_x}(2m + Ap_x)$; $y^* = \frac{1}{3p_y}(m - Ap_x)$; (c) $\lambda = \frac{4}{9p_y}(\frac{m}{p_x} - A)^2$; (d) $\frac{\partial x^*}{\partial A} = \frac{p_y y^* - (x^* - A)p_x}{p_y y^* - 2(x^* - A)p_x} = \frac{1}{3}$.

C1. (a) all except (ii); (b) (ii) 0.75; (iii) 27/28.

2. (d) do not reject null hypothesis (sample t = 2; critical t = 2.080); (e) -0.01 to 0.70.

3. (a) No; (b) $n_A = 124$; $n_B = 62$; $p_A = 0.45$; $p_B = 0.3$.

4. (b) (iii) 0.36.

D1. (b) (ii) $V(X - Y) = V(X + Y) = V(X) + V(Y)$; (c) (i) No difference between restaurants (sample t = 0.45); sample F = 1.11 so variances could be equal; (c) (ii) There is a difference between restaurants (sample t=8.5).

2. (b) (i) a = -0.5; b = 0.3; (ii) -0.2; 2.5; (c) (i) c = 5/3; d = 10/3; (ii) c = 2; d = 3.

2010

A1. (a) $f(x) = x_0^{\frac{1}{2}} + \frac{(x-x_0)x_0^{-\frac{1}{2}}}{2} - \frac{(x-x_0)^2 x_0^{-\frac{3}{2}}}{8}$.

2. (a) $A^{-1} = \frac{1}{2+5\alpha}\begin{bmatrix} 2 & 5 \\ \alpha & -1 \end{bmatrix}$; singular if $\alpha = -\frac{2}{5}$; (b) $PBP' = \begin{bmatrix} \theta/2 & 0 \\ 0 & 3\theta/2 \end{bmatrix}$.

3. (b) $w = 1/\theta$.

4. (a) $800 + 0.15S$ if $S < 10000$, $1800 + 0.15S$ if $10000 < S < 15000$, $4300 + 0.15S$ if $S > 15000$; (b) $S = 10000, 15000$.

B5. (a) $\mathcal{L} = 10t_1^{1/2} + \frac{3}{2}t_2 - 30 - \lambda(t_1 + t_2 - 60)$; $t_1 = 11.1, t_2 = 48.9, g_1 = 86.6$, $g_2 = 66.7$.

6. (a) $\dot{Y} + \alpha(1 + \frac{lk}{h} - b)Y = \alpha(a + l\bar{M}/h + \bar{I} + \bar{G})$;

(b) $Y = \frac{a+\frac{l\bar{M}}{h}+\bar{I}+\bar{G}}{1+\frac{lk}{h}-b} + (Y_0 - \frac{a+\frac{l\bar{M}}{h}+\bar{I}+\bar{G}}{1+\frac{lk}{h}-b})e^{-\alpha(1+\frac{lk}{h}-b)t}$; (c) $\frac{a+\frac{l\bar{M}}{h}+\bar{I}+\bar{G}}{1+\frac{lk}{h}-b}$; (d) stable iff $1 + \frac{lk}{h} - b > 0$.

C7. (a)almost 0.5; (b) 0.58; (c) 0.896.

8. (a) (i) constant variance; (iv) horizontal line, with random noise above and below; (b) hypothesis $\delta = 0$ would not be rejected.

9. (a) (i) 1/3; (ii) 13/14; (b) (i) $\frac{X\beta_2}{\beta_1+\beta_2 X}$; β_2; $\frac{-\beta_2}{X(\beta_1+\beta_2 X)}$; $\frac{\beta_2}{\beta_1+\beta_2 X}$; (ii) 4.

10. (a) 96.04, so 97 (b) $Z = 24.6$, based on variance of 2400 (tricky to know if one should use 2400 or 2500, though conclusion would be the same); (c) $Z = 1.60$; critical one tailed Z is 1.645, so do not (quite) reject H_0.

D11. (a) TCs are 77465, 77810, ... , 81310, 80874; (b) (i)-1860.8, 944.8, 1668, -752, which add to zero; (iv) 329.6, 543.0; (v) $t = 2.55$, critical t is 2.306 so reject hypothesis of equality; (vi) $F = 2.71$, critical F is 9.605, so do not reject hypothesis of equality of variances.

12. (a) (ii) mean=14, var=304; (b) (i) 0.6; (ii) 0.5392; (iii) 2.