2016

A1. (a) $x-\frac{x^2}{2}+\frac{x^3}{3}-\cdots+(-1)^{n+1}\frac{x^n}{n}$; (b) $3.001\dot{6}$; (c) Based on the 2nd order term, 5x10^{-7}.

2. (a) 24/35; (b) $B=3, C=5$; (c) $\frac{3}{2}\ln(2x-3)+\frac{5}{2}\ln(2x+1)+c$.

3. (a) concave; (b)(i) convex; (ii) neither.

4. (a) $\begin{pmatrix}1-C' & -I' \\ M_Y^D & M_r^D\end{pmatrix}\begin{pmatrix}dY \\ dr\end{pmatrix}=\begin{pmatrix}dG \\ dM\end{pmatrix}$; (b) $dY=(M_r^D dG+I'dM)/(M_r^D(1-C')+I'M_Y^D)$.

B5. (a) $w(24-L)=S+pC, C=\frac{\beta}{p}(24w-S), L=(1-\beta)\left(24-\frac{S}{w}\right)$;
(b) $C^*=55, L=11, U=24.597\ dU=-0.1118dS$;
(c) $C=43, L=10.75, U=21.5$, tax take = 26.5; (d) $C=48.375, L=9.675, U=21.63$.
6. (a) $P_1=\frac{\alpha+k}{2}, P_2=\frac{\delta+k}{2}$; (b) $P_1=110, P_2=60, Q_1=7, Q_2=8, \Pi=610$;
(c) $Q=50-0.5P$ for $0<P\leq 80$, $Q=18-0.1P$ for $80\leq P\leq 180$, $Q=0$ for P≥ 180;
(d) $Q=7, P=110, \Pi=450$.

C7. (a) $\begin{pmatrix}P(X,Y) & X=1 & X=2 & X=3 \\ Y=1 & 1/12 & 1/4 & 1/6 \\ Y=2 & 1/4 & 1/12 & 1/6\end{pmatrix}$; (b) $\begin{pmatrix}P(X|Y) & X=1 & X=2 & X=3 \\ Y=1 & 1/6 & 1/2 & 1/3 \\ Y=2 & 1/2 & 1/6 & 1/3\end{pmatrix}$,
$\begin{pmatrix}P(Y|X) & X=1 & X=2 & X=3 \\ Y=1 & 1/4 & 3/4 & 1/2 \\ Y=2 & 3/4 & 1/4 & 1/2\end{pmatrix}$; (c) No; (d) 3/4.

8. -

9. (a) $E(Y_t)=t\mu, var(Y_t)=t^2+\sigma^2$; (b) $E(\bar{Y})=(T+1)\mu/2$,
$var(\bar{Y})=[(T+1)(2T+1)+6\sigma^2]/6T$; (c) Sample mean gives $var=1/T$.

10. (a) 10.

D11. (b) $Z=0.707$, do not reject H_0; (c) 0.76; (d) $\Phi\left(\frac{\sqrt{n}}{3}-1.645\right)$.

12. (a) $t=58$, clearly significant; (c) $AAAA=0.004278, CCCC=-0.1454. DDDD=-.01286, BBBB$ is likely to be 0.000; (d) $\hat{\delta}_0=\hat{\beta}_0+\hat{\beta}_1+\hat{\beta}_2$, $\hat{\delta}_1=-\hat{\beta}_1$, $\hat{\delta}_2=-\hat{\beta}_2$.

2017

A1. (a) $f(x)=6(x-1)+10(x-1)^2+10(x-1)^3+5(x-1)^4+(x-1)^5$; (b)(i) $\dot{Y}_t+\gamma(1-\beta)Y_t=\gamma(\alpha+I)$; (ii) $Y^*=(\alpha+I)/(1-\beta)$ if $\beta\neq 1$; (iii) $Y_t=Y^*+(3-Y^*)e^{(2-t)(1-\beta)\gamma}$; (iv) $\beta<1$.

2. (a) $g(x)>0$ if $x>3$ or $1<x<2$, $g(x)<0$ if $x<1$ or $2<x<3$; (b) $x^3-6x^2+11x-6$; (d) { $(x>3 \text{ or } 1<x<2)$ and $\lambda>0$ and $\alpha>0$ }, or { $(x<1 \text{ or } 2<x<3)$ and $\lambda<0$ and $\alpha<0$ }.

3. (c) 2 .

4. (a) $1+e-\dfrac{1}{1+e}-\dfrac{3}{2}$; (b) $128/15$.

B5. (a) $q_1=20$ $p_1=56$ $q_2=10$ $p_2=106$ $\pi=1700$ (b) $-1.12, -1.06$; (c) $p=66$ $q_1=16$ $q_2=14$ $\pi=1500$; (e) $p=126$ $q_1=0$ $q_2=8$ $\pi=660$; (f) -1.575 .

6. (a) $\dfrac{\alpha(x_2-b)}{\beta(x_1-a)}$ if $x_1\neq a$; (b) $x_1=\dfrac{\alpha M+\beta ap-\alpha bq}{p(\alpha+\beta)}$ $x_2=\dfrac{\beta M+\alpha bq-\beta ap}{q(\alpha+\beta)}$; (c) $\lambda=\dfrac{\alpha+\beta}{M-(bq+ap)}$.

C7. (a)

		Y		
		-1	0	$+1$
X	-1	$1/48$	$3/48$	$1/4$
	$+1$	$11/48$	$9/48$	$1/4$

, X and Y not independent;

(b)

Y/X	-1	0	$+1$
$prob$	$23/48$	$1/4$	$13/48$

,

$1/X$	-1	$+1$
$prob$	$1/3$	$2/3$

;

(c) $E(X)=1/3$, $E(Y)=1/4$, $E(Y/X)=-5/24$, $E(1/X)=1/3$; (d) $-7/24$.

8. (a) $1-e^{-1/5}$; (b) ≈ 0.672 .

9. (b) $E(X)=\dfrac{\theta}{2}$, $E(X^2)=\dfrac{\theta^2}{3}$, $\text{var}(X^2)=\dfrac{\theta^2}{12}$; (c) $\dfrac{X}{\theta}$ for $0\leq X\leq\theta$; $E(\bar{X})=\dfrac{\theta}{2}$, $\text{var}(\bar{X})=\dfrac{\theta^2}{12N}$; (e) eg $2\bar{X}$, $\text{var}=\dfrac{\theta^2}{3N}$.

10. -.

D11. (a) $E(X)=\beta$ $E(X^2)=2\beta^2$, $\text{var}(X)=\beta^2$; (b) $e^{-\frac{2}{\beta}}$; (c) approximately $N(\beta, \beta^2/100)$; (e) reject H_0 as $Z=3$; (f) ≈ 0.963 ; (g) $\ln(3.578)\approx 1.275$.

12. (a) 0.055974; (b) $VVVV=9.5713, YYYY=0.0445, ZZZZ=0.0674$; (c) zero; (d) 209.5; (e) 0.0961 (f) $lwage_i=\alpha+\epsilon_i$; (h) 0.055974; (i) 0.0961.

2018

A1. (a) $f(x)=-x-\frac{x^2}{2}-\frac{x^3}{3}-\ldots$; (b) $P(x)=-x-\frac{x^2}{2}-\frac{x^3}{3}-\ldots$; (c) -0.01005 .

2. (a) $\frac{1}{9}(3x-1)e^{3x}+c$; (b) $\frac{3}{8}$; (c) $\frac{5}{2}\ln|x+5|-\frac{1}{2}\ln|x+1|+c$

3. (a) $T=\begin{pmatrix}0 & -1 & 0\\ 1 & 0 & 0\\ 0 & 0 & -1\end{pmatrix}$; (b) $A^2=\begin{pmatrix}-1 & 0 & 0\\ 0 & -1 & 0\\ 0 & 0 & 1\end{pmatrix}$, $A^5=A$; (c) $A^{-1}=\begin{pmatrix}0 & 1 & 0\\ -1 & 0 & 0\\ 0 & 0 & -1\end{pmatrix}$.

4. (a) Max $pK^{\alpha}L^{\beta}-rK-wL$; (b) Min Cost = $rK+wL$ subject to $K^{\alpha}L^{\beta}=Q$;
(c) $K=1/4, L=4, \Pi=2$; (d) Indifferent, $0.04\ln 4$. If this answer to (d) puzzles you, consider the fact that the question says "improve" alpha, and since $K^*<1$, that means *decreasing* it.

B5. (a) $f(p)=100e^{(p^2-1)a/2}$, $g(p)=100e^{(p^2-1)b/2}$ (b) $ap_t^2-bp_{t-1}^2-a+b=0$, $p^*=1$;
(c) $p_t=\left(\left(\frac{b}{a}\right)^t(p_0^2-1)+1\right)^{1/2}$; (d) $|b|<|a|$, $p=1$.

6. (a) $\left(\frac{11}{6},\frac{1}{3}\right)$ is a minimum, no; (c) max at $(-11,-2)$, min at $\left(\frac{11}{6},\frac{1}{3}\right)$.

C7. (a) $f(x)=2x$, $f(y)=(1+y)/4$, $E(X)=\frac{2}{3}$ and $E(Y)=\frac{7}{6}$; (c) Yes.

8. (a) 0.5625 ; (b) 0.964 .

9. -.

10. (b) $Z=3.73$, reject null hypothesis; (c)(i) $\hat{W}=10.145+0.031E$, (ii) $\hat{W}=8.130+0.031E$;
(d) 2.015.

D11. (a) $f(x)F(x)$; (b) $F(x)=x$, $g(y)=2y$; (d)(i) $\frac{1}{2}$; (ii) 1.3% ; (e) Distribution of the sum is $N\left(\frac{90N}{N+1},\frac{90N}{(N+2)(N+1)^2}\right)$, so mean $\to 1$, var $\to 0$ as $n\to\infty$.

12. (a) $\hat{\alpha}^A=\bar{Y}^A$; If N_A and N_B are large, $\frac{(\hat{Y}_A-\hat{Y}_B)-(\mu_A-\mu_B)}{\sqrt{\left(\frac{\hat{\sigma}_A^2}{N_A}+\frac{\hat{\sigma}_B^2}{N_B}\right)}}$ is approximately $N(0,1)$, or if Y_A and Y_B are Normal and the variances are equal then we can do a t-test; (c) $\hat{\beta}=\bar{Y}_A-\bar{Y}_B$

2019

A1. (a) Rotation by 45 degrees clockwise and magnification by $\sqrt{2}$; (b) $\frac{1}{2^{608}}\begin{pmatrix}21\\-9\end{pmatrix}$.

2. -
3. 75.
4. The integral is the greater.

B5. (a) $x>S, y<T$; (d) $x=\frac{ATp_y+BSp_x-Am}{p_x(B-A)}, y=\frac{Bm-ATp_y-BSp_x}{p_y(B-A)}$, so neither of the goods is a Giffen good.

6. (a)(i) $r=1.00\%$; (ii) $m=£1245$; (iii) $D_t=1.01D_{t-1}-(1.005)^{t-1}m_0$;
(b) $D_t=(1+r)^t D_0-\frac{[(1+r)^t-(1+i)^t]m_0}{(r-i)}$, $m_0=1087$.

C7. (a) $x_1=0:\frac{1}{3}, x_1=1:\frac{2}{3}, x_2=0:\frac{1}{4}, x_2=1:\frac{3}{4}$, yes, they are independent;
(c)

	$x_2=0$	$x_2=1$
$x_1=0$	a	$\frac{1}{3}-a$
$x_1=1$	$\frac{1}{4}-a$	$\frac{5}{12}+a$

8. -.
9. 0.107.
10. (c) Intercept is $\alpha+E(\epsilon_i)$.

D11. (a) $E(x)=\sum_i x_i P(x_i), E(g(x))=\sum_i g(x_i)P(x_i)$; (d) $M(t)=pe^t+1-p$.

12. (c) Variance would be slightly higher.

2020

A1. (a) Yes; (b) Yes; (c) Min 0, Max $(0,1/2e)$.
2. (b) True; (c) False.
3. (b) $x=y=\pm\sqrt{(2019/2)}, z=0$.
4. 576047996/288000000.
B5. (a) Yes, by Extreme Value Theorem; (b) same as (a); (c) No; (d) Max u is -13 at (0.6) , Min u is -325 at $(12,-12)$; (e) Min u ~ -325.0025, true new u* will be slightly less negative.
6. (a) $p_{t+1}=p_t+\gamma(D_t-S_t)$; (b) $D=2(A+ap)$, $S=2(B+bp)$;
(c) $p_t=[1+2\gamma(a-b)]^t p_0+\left(1-[1+2\gamma(a-b)]^t\right)\dfrac{B-A}{a-b}$; (d) $|1+2\gamma(a-b)|<1$ and hence $\gamma<1/(b-a)$; (e) limit price is $p=\dfrac{B-A}{a-b}$.
C7. $^nC_k\left(\frac{1}{2}\right)^k\left(\frac{2}{3}\right)^n={}^nC_k\left(\frac{1}{3}\right)^n 2^{n-k}$.
8. 49/20.
9. (a) Bias = $-\theta/2$, MSE = $\theta^2/3$; Yes, MSE minimised by $k=3/2$.
10. (c) If you assume variances are equal and base your pooled variance on that, $t=-4.5$; if you use the large sample formula (which *doesn't* need the assumption of equal variances), $t=-4.41$; (d) Depends on what you get for (c), obviously, and is probably off the scale of tables, but if the LH tail area is p_0 then $p=1-p_0$, $p=p_0$, $p=2p_0$.
D11. (a) $C=8/5$; (b) $x_C=0.90$; (c) (question intended you to assume the $0.5x$ cost applies for $0\le x\le 0.9$) $0.1820+0.4747=0.6567$; (d) Z=1.71, reject hypothesis; (e) Depends what you assume: if you work with the *fraction* of overflows, then 0.856; if you work with the *number* of overflows and round the critical number up to 9, then 0.846; if you adjust the 9 to 8.5 due to something called the continuity correction (not taught in the course) then 0.879, and if you use a computer to find the true Binomial probability of overflows ≥ 9 then 0.883.
12. -