2021

A1. -
2. (a) *all* $a>1$, *no* $a>1$; (b) $2(1-2^{-50})$.
3. -
4. (a) a triangle with sides $y=x$ (exclusive), $y=-x$ (exclusive), $x=1$ (inclusive), it is convex but not compact; (b) 1/3 at (2/3,1/3).
B5. (a) $x^*=1/(p+p^{1/(1-\alpha)})=p^{-1/(1-\alpha)}/(p^{-\alpha/(1-\alpha)}+1)$, $y^*=p^{1/(1-\alpha)}/(p+p^{1/(1-\alpha)})=1/(p^{-\alpha/(1-\alpha)}+1)$; (c) $\lambda^*=\alpha(p^{-\alpha/(1-\alpha)}+1)^{1-\alpha}$; (d) $\frac{-\alpha p^{-1/(1-\alpha)}}{(p^{-\alpha/(1-\alpha)}+1)^{\alpha}}$, tends to zero as α tends to zero.
6. (b) approximately 76.3; (c) 0.046; (d) approximately 66.
C7. (a) $1-(1-p)^k$; (b) $\frac{N}{k}(k+1-k(1-p)^k)$; (d) expected number of test falls: in the extreme case where one infection in a household means all infected, the new expectation is $\frac{N}{k}(p(k+1)+1-p)=\frac{N}{k}(k+1-k(1-p))$, ie approximately Np(k-1) less than in (b).
8. -.
9. 0.97.
10. (a) $\tau_1=0.54$, $\tau_2=0.96$; (b)(ii) If H_0 : no covid and H_1 : covid then $1-\tau_2$ and $1-\tau_1$ respectively; (c)(i) 13; (ii) 0.49.
D11. (e) Ratio $P(R_2|R_1)/P(R_2)$ is approximately 1.27.
12. (a) $\beta_2=Cov(Y_i,X_i)/Var(X_i)$; (c)(ii) zero.

2022

A1. (a) All x ; (b) All x, with $x<0:\ln 2, x\geq 0:(\ln 2)2^x$

2. (b) 2/3.

3. (b) $\max f=-1 \text{ at} \pm(1,1)$.

4. (a) Rotation about (1,1,1) by 120 degrees; (b) the identity matrix $\boldsymbol{I}$.

B5. (b) $x^*=\sqrt{2a}, y^*=1/2+\sqrt{a/2}$; (d) C^* decreases by 0.01 too.

6. (a) 6.93 years; (b) $K=100e^{0.1(e^{-t}-1)}$; (c) $K=100-10t+11t^2/2$, 2.18; (d) $K(t)\sim 90.97$, and in case it helps: $K=10t-100+200e^{-0.1t}$.

C7. (a) size 0.0579; power 0.942; (b) 119.70 ± 12.77 .

8. -.

9. (a) $2\sigma^2$; (b) for $\hat{g}:\text{bias}=0,\text{MSE}=\sigma^2/200$, for $\tilde{g}:\text{bias}=0,\text{MSE}=\sigma^2/3311$, so clearly prefer $\tilde{g}$.

10. (b) $\ln W_i=\alpha_f+(\alpha_m-\alpha_f)D+\beta_f Ed_i+(\beta_m-\beta_f)DEd_i+u_i$ where D = 0 for female, 1 for male.

D11. (a) $\text{MSE}=p(1-p)/N$; (b)(i) 9/14; (ii) 5/14; (iii) 1/3; (iv) 4/5, then 3/7 and 4/7 so management will recommend not going; (d) The denominator is common to both expressions, so I ignored that and then the ratio of the probabilities (drink : no drink) is (5 : 4), so management will recommend going.

12. (a)(i) $\hat{\beta}_2/s.e.\hat{\beta}_2=2.5$, reject H_0 at 5% but not at 1% or 0.1%; (ii) $\hat{\beta}_2/s.e.\hat{\beta}_2=4.58$, reject H_0 at all levels; $\hat{\beta}_2/s.e.\hat{\beta}_2=0.83$, do not reject H_0 at any level; (b) flips for (i) at 1% otherwise no changes; (c) $\hat{\beta}\pm(t_{crit})(s.e.\hat{\beta})$, 0.31 to 0.79.

2023

A1. (b) $(0,\infty)$; (c) $x\in(0,1): g(x)=\ln x; x\in(1,\infty): g(x)=x-1;$ range is $(-\infty,\infty)$.

2. -

3. (a) $2^9 A$; (b) Projection of any point on to the 45 degree line in direction of a and then a doubling of the magnitude.

4. (a) $q=(1-1/2\alpha)+p/2\alpha$; (b) As α increases, optimal profit decreases.

B5. (a) $u_A=3 \text{ at } x_A=10, y_A=5$; (b) $x_A=10, y_A=5, x_B=y_B=10$;

(d) $x_A=20/3, y_A=5/3, x_B=y_B=40/3$.

6. (a) r/w , apparently; (b) $f(K,L)=\frac{2}{3}(K+L)=\sqrt{(K^2-KL+L^2)}$.

C7. (c) zero.

8. Yes, as $Z=-3.87$.

9. (a)(i) 3/7; (ii) 0.063; (b) 0.038.

10. (b) The latter.

D11. (a) $bias=\beta\left(\sum w_i x_i-1\right)$, $var=\sum w_i^2\sigma^2$.

12. (a)(i) $E(\hat{\beta}_2)=\beta_2+\beta_1\frac{\sum x_i}{\sum x_I^2}$; (b)(i) $E(\tilde{\beta}_2)=\beta_2$.

2024

A1. (a) $t=\ln(1+P_e)+P_e$; (b) $\frac{dP_e}{dt}=\frac{1+P_e}{2+P_e}$, $2/3$.

2. (a) $a\ln a-a+1$; (b) $\frac{df}{dx}=(\ln x)^x(\ln(\ln x)+1/\ln x)$.

3. (a) Local minimum at $(1,1)$; (b) No.

4. (a) Max of 1 at $(\pm1,0)$; (b) The set is a 45 degree clockwise rotation of the first quadrant, it is open and convex.

B5. (a) $L=2500$, $M=80{,}000$, $\Pi=40{,}000$; (b) Profit falls by about 400; (c) $T=20+\sqrt{405}$; (d) 1st order Taylor is $40T$, upper bound on absolute error is 1.6.

6. (b) $x(t)=101e^t-1$; (c) stabilises at $x=1$, reaches 15 when $t=3.97$; (d) $x=100e^{\sin t}$, so minimum is $100e^{-1}$, maximum is $100e$.

C7. (a) 0.418; (b) 0.289.

8. pdf is $\frac{2}{\pi(1+x^2)}$ for $x\geq0$, zero otherwise.

9. (a) $Z=10.01$, so reject null hypothesis of equality.

10. (a)(ii) An interval of $\pm0.21\, around\, \beta^*$; (c)(i) $t=-1.8$, critical t for 2-sided test is -2.101 so do not reject null hypothesis, but critical t for 1-sided test is -1.734 so reject null hypothesis.

D11. (a)(i) 3/15; (ii) 2/3; (iii) 3/4; (b)(i) 1/12; (iii) 0.1715; (c) $\text{prob}\leq2/3$.

12. -.

2025

A1. (a) No; (b) Neither.

2. (a) zero; (b) $(0,1,0,\ldots,0)^T$.

3. (a) stationary points when $2x-y=\pi/2+n\pi, n$ integer ; (b) even *n*: a line of maxima; odd *n*: a line of minima.

4. (a) $a=1/2, b=1/4, P_1(1)=3/4$; (b) $|\frac{1}{2}\frac{e-e^2}{(1+e)^3}|\approx 0.045$.

B5. (a) $x=2, y=7, \lambda=6$; (c) Utility changes by approximately -0.09; (d) $x=0, y=9$.

6. (a) $U(1)=2-3e^{-1}\approx 0.9$; (c) $f^{(n)}(0)=(-1)^{n+1}(n-1)=(-1)^n(1-n)$, so n^{th} term of Taylor series is $(-1)^n(1-n)y^n/n!$; (d) upper bound on the error is $5^3/3\approx 41.7$.

C7. $P(a)>P(b)$.

8. $w_1=w_2=1/3$.

9. Critical $\bar{x}$ is 0.397, size is approximately 20%.

10. (d) Statement is true.

D11. (a) $E(X)=0, var(X)=2p$; (b) $E(|X|)=2p, E(I(X>0))=p$; (c) $k_1=1/2, k_2=1$;
(d) $\text{MSE}(\hat{p})=2p(1-2p)/4N, \text{MSE}(\tilde{p})=p(1-p)/N$; (e) Prefer $\hat{p}$ (difference of MSEs is $p/2N$)

12. (a) 0.6, 0.7, 0.2, 0.3; $|Z|=0.566$, do not reject H_0 at 5% significance; (c) 0.5, 0.4;
(d)(i) $P(R_i=1)=\alpha+\beta_1 D_i+\beta_2 M_i+\beta_3 D_i M_i+\epsilon_i$.; (ii) $H_0:\beta_2+\beta_3=0$