2021

A1. -
2. (a) *all* $a>1$, *no* $a>1$; (b) $2(1-2^{-50})$.
3. -
4. (a) a triangle with sides $y=x$ (exclusive), $y=-x$ (exclusive), $x=1$ (inclusive), it is convex but not compact; (b) 1/3 at (2/3,1/3).
B5. (a) $x^*=1/(p+p^{1/(1-\alpha)})=p^{-1/(1-\alpha)}/(p^{-\alpha/(1-\alpha)}+1)$, $y^*=p^{1/(1-\alpha)}/(p+p^{1/(1-\alpha)})=1/(p^{-\alpha/(1-\alpha)}+1)$; (c) $\lambda^*=\alpha(p^{-\alpha/(1-\alpha)}+1)^{1-\alpha}$; (d) $\dfrac{-\alpha p^{-1/(1-\alpha)}}{(p^{-\alpha/(1-\alpha)}+1)^{\alpha}}$, tends to zero as α tends to zero.
6. (b) approximately 76.3; (c) 0.046; (d) approximately 66.
C7. (a) $1-(1-p)^k$; (b) $\dfrac{N}{k}(k+1-k(1-p)^k)$; (d) expected number of test falls: in the extreme case where one infection in a household means all infected, the new expectation is $\dfrac{N}{k}(p(k+1)+1-p)=\dfrac{N}{k}(k+1-k(1-p))$, ie approximately Np(k-1) less than in (b).
8. -.
9. 0.97.
10. (a) $\tau_1=0.54$, $\tau_2=0.96$; (b)(ii) If H_0 : no covid and H_1 : covid then $1-\tau_2$ and $1-\tau_1$ respectively; (c)(i) 13; (ii) 0.49.
D11. (e) Ratio $P(R_2|R_1)/P(R_2)$ is approximately 1.27.
12. (a) $\beta_2=Cov(Y_i,X_i)/Var(X_i)$; (c)(ii) zero.