Economics IIA Past Exam Paper Answers

NB: My tendency to make algebraic slips means you shouldn't place <u>great</u> faith in these, but I hope they are of use

<u>1999</u>

A1: $\begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 4 \end{pmatrix}$

A2: (1) $\begin{pmatrix} 3 & 0 & 6 \\ 0 & 5 & 5 \\ 1 & 2 & 4 \end{pmatrix}$ Dimension = 2

(2) No (3) Dimension = 1 $\begin{pmatrix} -2 \\ -1 \\ 1 \end{pmatrix}$

~~A2~~

A3: $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} + \left(\frac{1}{2}\right)^t \begin{pmatrix} 1 \\ -3/2 \\ 1/2 \end{pmatrix} + \frac{3}{2}\left(\frac{2}{3}\right)^t \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix}$

A4 (1) $c = 1/4$ (2) $\frac{1}{4}(2x-1)$ $\frac{1}{4}(3-2y)$

(3) $(x-y)/(3-2y)$ (4) No

A5: Answers given in question

A6 (1) $3/125$ (2) $5/24$

A7: Answer given in question

(2) $x_n = 1$ unstable

A8: (1) $x = \cancel{\ }+1$ stable;

A9: $u\left(\frac{3h}{2} + \frac{2k}{3} - \frac{3h^2}{32} + \frac{hk}{12} - \frac{k^2}{54}\right)$

B1 — B2 —

B3: (2) $\hat{\lambda} = \sum x_i/n$ (4) $\hat{\beta}$ is consistent

B4: (3) $pe^t + 1 - p$ (4) mean $= p$; var $= p(1-p)/n$

$u = kx_1^{3/4}x_2^{1/4}$

B5: (3) $e = u^{1/4}p_1^{3/4}p_2^{1/4}$ $C_1 = \frac{3}{4}\left(\frac{up_2}{p_1}\right)^{1/4}$ $C_2 = \frac{1}{4}\left(\frac{up_1^3}{p_2^3}\right)^{1/4}$ $D_1 = \frac{3M}{4p_1}$ $D_2 = \frac{M}{4p_2}$

B6 (2) $x_n = (1+h)^n - (1-h)^n$ (3) $x_n \to e^t - e^{-t}$ $y_n = e^t + e^{-t}$ ie steady state

$y_n = (1+h)^n + (1-h)^n$ eq^s tend to $x_{n+1} = x_n$, $y_{n+1} = y_n$

<u>2000</u>

A1: (a) $a = 9$ $b \neq 1$ (b) impossible (c) ($a = 4$ $b = 1$) or $a \neq 9$

A2: ~~[illegible]~~ Both two dimensional

A3 —

A4: —

A5: (a)(i) μ, ~~[illegible]~~ (ii) μ, $1/2$ (iii) 0, 2 (iv) μ, $1/n$

(b) Normal in all cases; (i) unbiased, efficiency = 1

(ii) " , " $1/2$

(iii) biased ; " 2

(iv) unbiased, " $1/n$

(c) Only (iv)

2000 cont

A6: (a) MLE is $\sum x_i/n$, $L = \frac{e^{-n\theta}\theta^{\sum x_i}}{\prod_i (x_i!)}$ (b) $\lambda = e^{n(\bar{x}-\theta_0)}\left(\frac{\theta_0}{\bar{x}}\right)^{n\bar{x}}$

A7: $k=1$

A8: (a) Yes (b) Constant (c) $G(K+g, L+h) = G(K,L) + g\frac{\partial G}{\partial K} + h\frac{\partial G}{\partial L}$

A9: $C(p,u) = \begin{pmatrix} \frac{u}{A}\left(\left(\frac{1-a}{a}\right)\frac{p_x}{p_y}\right)^{a-1} \\ \frac{u}{A}\left(\left(\frac{1-a}{a}\right)\frac{p_x}{p_y}\right)^{a} \end{pmatrix}$ $e(p,u) = \frac{u p_x}{aA}\left(\frac{1-a}{a}\frac{p_x}{p_y}\right)^{a-1}$

B1 (c) $a>1$ (d) Yes; if $ab=1$ No

B2 (b) $\frac{1}{1-\lambda}$ (c)(ii) No

B3 (a) $\hat{\mu} = \frac{1}{n}\sum x_i$ $\hat{\sigma}^2 = \frac{1}{n}\sum(x_i-\mu)^2$ (b) $\begin{pmatrix} n/\sigma^2 & 0 \\ 0 & n/2\sigma^4 \end{pmatrix}$

B4 –

B5: (a) $(-3, 3\alpha-3)$ and $(1, -(\alpha+3))$ (c) $\alpha < -5$ for stability, but $\alpha \le -9$ for stability and λ_1, λ_2 real (d) Yes??

B6: (b) $C(p,u) = \begin{pmatrix} 1+u(p_2/p_1)^{1/2} \\ 1+u(p_1/p_2)^{1/2} \end{pmatrix}$ (c) $e(p,u) = 2u(p_1p_2)^{1/2} + p_1 + p_2$

(d) $c=1$ (e) $0 \le c \le 1$

2001

A1: (b) $\underline{w}$ does not belong to span$\{\underline{u}, \underline{v}\}$

A2 (c) eg $\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$

A3 $\begin{pmatrix} 3 & 2 & 0 \\ 2 & 2 & 2 \\ 0 & 2 & 1 \end{pmatrix}$ N.B.: See correction to question noted in examiners' report. Q should end in $4x_2x_3$ not $4x_1x_3$. But neither form is positive definite.

A4 $(2,0)$ min; $(-2,0)$ max; $(1,3)$ and $(1,-3)$ saddles

A5 (a) $y = -\frac{7}{6}e^{-t} + \frac{2}{3}e^{2t} + \frac{1}{2}e^{-3t}$ (b) $x_n = \frac{5}{12}(3)^n - \frac{7}{24}(-3)^n - \frac{1}{8}$

A6 $(0,0)$ unstable; $(0,6)$ stable; $(4,0)$ stable; $(1,3)$ unstable

A7 (b) $\frac{1}{t^2}(e^t + e^{-t} - 2)$ (c) mean $= 0$; var $= 1/6$

A8 $\hat{\mu} = \sum x_i/n$

A9 All unbiased except the last; only the last two are consistent

B1 Dimensions of row & column spaces $= 3$; not productive

B2 (a) $w_t = 8(0.8)^t + 12(1.2)^t$ $r_t = 2(0.8)^t + 9(1.2)^t$ (b) 4 : 3

B3 (b) Degree $= (1-n)/n$ (iii) Not concave for (eg) $\alpha = 2$, $\beta = 1/3$ as F_{KK} can be made >0 by suitable choice of K

B4 (a) constant $= B/A$ (b) $\alpha\beta$

B5 –

B6 –

2002

A1 $t > 0$ for positive definite $t = 0$ for positive semidefinite

A2 $P = (ad + bc)/(d+b)$ $Q = (a-c)/(d+b)$; if d & b are strictly +ve, there is always one unique soln ?!

A3 Yes eg $\begin{pmatrix} 1 & 1 \\ -1 & -1 \end{pmatrix}$

A4 –

A5 ~~[illegible]~~ –

A6 –

A7. (a) $y = 3e^{-6t} - e^{t} - 2e^{-2t}$ (b) $X_n = \frac{1}{48}\left(5^n - 3(-5)^n + 2\right)$

A8 $(0,0)$, saddle $(0, 1/3)$ saddle $(2/3, 0)$ saddle $(2/9, 1/9)$ Min..

A9 $(2,1)$ stable ; $(-2,-1)$ unstable

B1 ~~[illegible]~~

B2 (b) $\underline{X}_t = \begin{pmatrix} -2 \\ 2 \end{pmatrix}(0.5)^t + \begin{pmatrix} 12 \\ 18 \end{pmatrix}$ (c) 2:3

B3 $MSE = c^2 k\theta^2 + \theta^2(c-1)^2$; $c = \frac{1}{k+1}$

B4 (b) $\hat{\theta} = \sum x_i/n$ (c) Yes : $\text{var}(\hat{\theta}) = \theta^2/n$

B5 (c) $$f' = \frac{\frac{\partial D}{\partial m} - \frac{\partial S}{\partial m}}{\frac{\partial S}{\partial p^*} - \frac{\partial D}{\partial p^*}}$$

B6 (c) $$x_1^* = \frac{1}{w_1}\left(\beta_1 + \gamma_{11}\ln w_1 + \gamma_{12}\ln w_2\right) \exp\left(\alpha \ln q + \{\beta_1 \ln w_1 + \beta_2 \ln w_2\} + 1/2\{\gamma_{11}(\ln w_1)^2 + \gamma_{22}(\ln w_2)^2 + 2\gamma_{12}\ln w_1 \ln w_2\right)$$

(d) $c = q^{\alpha} w_1^{\beta_1} w_2^{\beta_2}$

2003

A1 (b) $t \neq -8$ (c) $t = -8$ $s \neq -16$ (d) $t = -8$ $s = -16$

A2 inverse is $\begin{pmatrix} 4 & -1 & -3 \\ 3 & -1 & 0 \\ 3 & -1 & 1 \end{pmatrix}$

A3 —

A4 (a) $x = 1$ min ; $x = -1$ max (b) $(0,0)$ saddle ; $\pm(1,1)$ minimum

A5 $y = \frac{1}{2h}\left(e^{(1+h)t} - e^{(1-h)t}\right)$; as $h \to 0$ $y \to te^{t}$

A6 —

2003 cont

A7 (a) $(1-p)^{n-1}p$ (b) –

A8 (a) $N(0, 1+\rho^2)$ (b) don't reject (?)

A9 haven't done it yet

B1 (c) 80% subscribe to Mail

B2 –

B3 (a) (1, 1) stable (1, −1) unstable (2, ±2) unstable
(b) (0, 0) stable (c) (0, 0) unstable

B4 (a) $\frac{dQ}{Q} = c$ (b) – (c) Maybe I've missed something but what's to stop $x_1 \to \infty$ and hence $\pi \to \infty$? [Question is wrong] D_2 increases as p_2 increases!!]

B5 (a) $a = -\frac{1}{\lambda}\ln 0.95$ $b = -\frac{1}{\lambda}\ln 0.05$ (b) 0.33x to 19.5x (c) –

B6 (a) (±1, ±1) have prob $\frac{1}{4}\theta^2$; (0, ±1) and (±1, 0) have prob $\frac{\theta}{2}(1-\theta)$; (0, 0) has prob $(1-\theta)^2$
(b) 0 with prob $(1-2\theta+3\theta^2/2)$; 1 with prob $2\theta-2\theta^2$; 4 with prob $\theta^2/2$

2004

A1 (a) $(b-1)h \neq ma$ (b) $Y = \{-h(I^0+G) - a(M^s - M^0)\}/\{(b-1)h - ma\}$ etc
(c) decreases by $M\Delta G/[(b-1)h - ma]$

A2 (a) $24c(c+1)$ (b) zero

A3 (a) $c = \pm 5$ (b) 3 if $c \neq \pm 5$, otherwise 2 (c) no

A4 yes: 2

A5 $2c/9$

A6 $x = te^t$ $y = e^t$ if $c = 1$; otherwise $x = (e^{ct} - e^t)/(c-1)$ $y = e^{ct}$

A7 –

A8 –

A9 (a) – (b) $\theta^{\alpha-1+\Sigma x_i}(1-\theta)^{\beta-1+n-\Sigma x_i}$ Beta distⁿ

B1 (a) – (b) – (c) $\dim(\text{Im}(F)) = 2$; basis of Ker (F) is $\begin{pmatrix}1\\-2\\1\\0\end{pmatrix}, \begin{pmatrix}-7\\3\\0\\1\end{pmatrix}$

B2 (a) $\begin{pmatrix}0.8 & 0.4\\0.2 & 0.6\end{pmatrix}$ (b) $\lambda = 1: \begin{pmatrix}2\\1\end{pmatrix}$; $\lambda = 0.4: \begin{pmatrix}1\\-1\end{pmatrix}$ (c) £20

B3 (a) – (b) n even: $x = \pm y$; n odd: $x = y$ (c) $ac \geq b^2$ and $a + c < 0$ (i.e. $a<0$, $c<0$)
(d) $n = 1, a > 0, c > 0$ (e) $b < 0$ and $b < a < -b$ (via 3rd order conditions) and $a > 0$

B4 (a) Quasi concave (b) Neither (c) Yes, degree = product of homogeneities

B5 (a) – (b) $e^{-n\theta}\theta^{\Sigma x_i}/\Pi x_i!$ (c) – (d) it is fully efficient ($\text{var} = \theta/n$)

B6 (a) – (b) MSE = $1/n$ (c) $\theta^2\left(\frac{1}{12n} + \frac{1}{4}\right)$ biased and not consistent
$\theta^2/3n$ unbiased and consistent
(d) MSE $= [n\theta(1-\theta) - \theta^2(n-k)^2]/k^2$ and decreases as k increases from $n-1$ to n ~~to~~ to $n+1$