2005

A1 (a) $c = 1$ or 4 (b) eg $\begin{pmatrix}1\\1\\-4\end{pmatrix}, \begin{pmatrix}1\\6\\1\end{pmatrix}$ and $\begin{pmatrix}1\\4\\-4\end{pmatrix}, \begin{pmatrix}1\\6\\4\end{pmatrix}$

A2 (a) $x_2 = 2$ (b) $4, 0$.

A3 —

A4. —

A5 (a) — (b) $x = y = 0$ (though by inspection – Lagrange is invalid here)

A6 (a) $y = e^{(1-1/t)/7}$ (b) $y = 5 - 3e^{-t}$ (c) $\frac{5\sqrt{6}-5}{2\sqrt{6}}\begin{pmatrix}1+\sqrt{6}\\-1\end{pmatrix}e^{(3+\sqrt{6})t}$
$- \frac{(15\sqrt{6}-5)}{2\sqrt{6}}\begin{pmatrix}1-\sqrt{6}\\-1\end{pmatrix}e^{(3-\sqrt{6})t}$

A7 (a) $E(X) = np$ $\text{var}(X) = np(1-p)$
(b) (i) $\binom{n}{x}p^x(1-p)^{n-x}$ (ii) $E(X|N) = Np$ $\text{var}(E(X|N)) = p^2 \text{var}\, N$
(c) — (b) — (c) No (shows $P(Y_1|Y_2) \neq P(Y_1)$)

A8 (a) mean $= \mu_1 + \mu_2$ var $= \sigma_1^2 + \sigma_2^2$ (d) $5/9$

A9 (a) 0.65 (b) 0.71825 (c) M_2 $r = 4$, $(0,1,0)^T$

B1 (c) $r = 1$, $(1,0,1)^T$; $r = 3$, $(1,0,-1)$;
$P = \begin{pmatrix}1/\sqrt{2} & 1/\sqrt{2} & 0\\ 0 & 0 & 1\\ 1/\sqrt{2} & -1/\sqrt{2} & 0\end{pmatrix}$ if one normalises the eigenvectors

B2 (a) — (b) $K\begin{pmatrix}1\\2\end{pmatrix}$ K = scalar, Null(A) + Row(A) is $\mathbb{R}^2$
(c) —

B3 —

B4 (a) $V = 2^{2/3}M/3p_1^{1/3}p_2^{2/3}$ $E = 3p_1^{1/3}p_2^{2/3}u/2^{2/3}$

B5 (a) Yes (b) Unbiased (c) $\hat{p} \pm 1.96\sqrt{\frac{\hat{p}(1-\hat{p})}{5000}}$

B6 (a) $e^{\theta(e^t-1)}$, $e^{70(e^t-1)}$ (b) — (c) She can refute ($p = 0.0267$)

2006

A1. (a) cn^{-1}, $n^{-1} + n^{-2}$ (b) $c < 1 + 1/n$

A2. —

A3. Call $E(X_i) = p$, then Np, $p^2\text{var}(N)$, $p^2\text{var}(N) + p(1-p)E(N)$

A4. (a) $a = 1$ $b = -1$ (b) —

A5. —
$(0,5,0)^T e^t + (-1,-5,-1)^T e^{2t}$

A6. (a) $\underline{x} = (-1,-5,1)^T +$ (c) $5/7$ will be unemployed

A7. (a) $1, 0.3, (5,2)^T, (1,-1)^T$ (b) $\begin{pmatrix}1 & 0\\ 0 & 0.3\end{pmatrix}$

A8. (c) $m = 2$ $n = 2$ $p = 3$ $q = 3$
A_1 is symmetric; A_4 is symmetric, A_2 and A_3 are transposes
(c) eg $\underline{v}_3 = 7\underline{v}_1 + \frac{8}{5}\underline{v}_2$

A9. (a) $c \neq 30.6$ (b) $(0.216, 0.021, 0.112)^T$

B10. (a) $-n\log(\sqrt{2\pi}\sigma^2) - \sum_{i=1}^{n}(y_i - \beta x_i)^2/2\sigma^2$ (b) $\hat{\beta} = \sum x_i y_i / \sum x_i^2$
(c) $I(\beta) = \sum x_i^2/\sigma^2$; $\hat{\beta}$ is $N(0,1)$ if it is standardised? (d) No – var is $\sigma^2/\sum x_i^2$.

B11. (a) $\binom{n}{y}p^y(1-p)^{n-y}$? (b) Beta(11, 168) (c) mean $= \alpha/(\alpha+\beta) = 11/179$
$\mu_2' = 132/32220 = 0.0041$
(d) 0.026 to 0.096 var $= 3.2 \times 10^{-4}$

2006 (cont)

B12. optimum is at $x=-2/7$ $y=5/14$, conditions are sufficient since objective f^n is concave and constraints are convex (linear, in this case)

B13 –

B14. (a) a_{ij} = units of good i required to make one unit of good j
(b) $(858, 1216, 777)^T$ millions (c) $p^T = w\{a_0^T (I-A)^{-1}$ or $p = (I-A)^{-1T} w a_0$
(d) $(3.26, 2.13, 4.61)$

B15. (a) – (b) – (c) Null: just the origin Col: the 3 columns of A
(d) ? B is any point in $\mathbb{R}^4$ other than Col(A), so the 3 columns of A plus (eg) $\begin{pmatrix}0\\0\\0\\1\end{pmatrix}$ will do, as $\begin{pmatrix}0\\0\\0\\1\end{pmatrix}$ is not in Col(A).

2007

A1.(a) $Y = [h(C_0 - c_1 t_0 + I^0 + G) + (M_S - M^0)(c_2 + a)] / [h(1 - c_1 + c_1 t - a_0) + m(c_2 + a)]$
$r = [m(C_0 - c_1 t_0 + I^0 + G) - (M_S - M^0)(1 - c_1 + c_1 t - a_0)] / [h(1 - c_1 + c_1 t - a_0) + m(c_2 + a)]$
(b) –

A2. (a) either none or many (b) no sol^{n}s if $s = -3/2$ $t = -6$ (c) $\begin{pmatrix}x\\y\\z\end{pmatrix} = \begin{pmatrix}0\\-7/5\\13/15\end{pmatrix} + x\begin{pmatrix}1\\0\\-8/3\end{pmatrix}$

A3. (a) – (b) expansionary = 2/5 recession = 3/5

A4. –

A5. (a) No (try approaching along $y = \sqrt{x}$) (b) Yes, both are zero. They are not continuous there though

A6. (a) $y = 7 + e^{(1-t^3)/3}$ (b) $\underline{x} = \begin{pmatrix}1\\1\end{pmatrix} e^{6t}$

A7. (a) cdf is $1 - e^{-\lambda t}$ (b) I doubt re-entry would be memoryless like (a).

A8. (a) $MGF_X = (pe^t + q)^n$ $MGF_A = p(1-(qe^t)^{n+1})/1-qe^t$
(b) Binomial with $2n$ and the same p and q as (a); (c) $\dfrac{p^2(1-(qe^t)^{n+1})^2}{(1-qe^t)^2}$

A9. (a) $-\sigma^2/n$, zero, both asymptotically unbiased; (b) $2(n-1)\sigma^4/n^2$
(c) $MSE(S^2) = \left(\frac{2n-1}{n^2}\right)\sigma^4$; $MSE(\tilde{S}^2) = \frac{2\sigma^4}{n-1}$ so we'd prefer s^2 at $n=20$.

B10 (b) $A^{-1} = \begin{pmatrix}-1 & -1 & 2\\ -1/2 & 1/4 & 1/2\\ 1/2 & 1/4 & -1/2\end{pmatrix}$

B11. (a) – (b)(i) first 2 columns of A (ii) 2, 3 (iii) $\begin{pmatrix}-3\\3\\1\\0\\0\end{pmatrix}, \begin{pmatrix}-2\\5\\0\\1\\0\end{pmatrix}, \begin{pmatrix}-1\\1\\0\\0\\1\end{pmatrix}$

B12. (b) Yes, in that $-$(objective f^n) is concave and constraints are convex
(d) $x = 28/37$ $y = 86/37$ (second constraint is not binding, Lagrange multiplier for first constraint $= 15/37$)

B13. –

B14. (a) Joint mass f^n is $\theta^{\Sigma x_i}(1-\theta)^{n-\Sigma x_i}$; log-likelihood is just log of this
(b) (I assume it means unbiased estimator: all except $\hat{\theta}_4$ (c) $n/(\theta(1-\theta))$
Cramér-Rao lower bound $= \theta(1-\theta)/n$; $\hat{\theta}_3$ and $\hat{\theta}_5$ fully efficient
(d) $\hat{\theta}_3$ is consistent

B15. (a) $\sum_i -\ln\sqrt{2\pi}\sigma - \frac{1}{2\sigma^2}\sum_i (Y_i - \alpha_0 - \beta_0 X_i)^2$ (b) – (c) $\hat{\beta}_0 = \sum x_i Y_i / \sum x_i^2$
(d) –

2008

A1. (a) Both false (b) $r=4$

A2 (a) Not unless V_3 is zero (b) —

A3. —

A4. (c) Continuous if $n>0$ Differentiable if $n>2$

A5 (a) $k = \int_0^T (p-c)e^{-rt}\,dt$ so $p = c + rk/(1-e^{-rT})$

(b) $V_t = \int_t^T (p-c)e^{-r(t'-t)}\,dt'$ so $V_t = \dfrac{k(1-e^{-r(T-t)})}{1-e^{-rT}}$

(c) $-dV_t/dt = 0.058k$ whereas by conventional accounting, $-\dfrac{dV_t}{dt} = 0.1k$.

A6. (a) $\phi(k) = sf(k) - \delta k - kg(f(k))$. If anyone knows why $f'(k)>0$ at $k=k_1$, I'd like to know....

A7. —

A8. (Denote mean number of events in time t by μ)

(a) prob dist^n of G is $\binom{n}{r}\left(\frac{\mu}{r}\right)^r\left(1-\frac{\mu}{n}\right)^{n-r}$ $r=0,1,2\ldots$ (b) $\dfrac{\mu^r e^{-\mu}}{r!}$

A9. (a) Type I error $= \alpha$ (we choose?). If z-value corresponding to an area α in RH tail of $N(0,1)$ ~~of~~ is z_c, then power is just the area to the right of z_c in $N(2,1)$.

B10. (a) (i) $J \geq 5$; at least 5 columns of V linearly independent

(ii) $J > \text{Rank}(V)$

(b) (i) $\begin{pmatrix}2\\1\\0\\-1\end{pmatrix} + \alpha\begin{pmatrix}1\\-1\\2\\0\end{pmatrix} + \beta\begin{pmatrix}0\\0\\-4\\1\end{pmatrix} + \gamma\begin{pmatrix}2\\3\\-2\\3\end{pmatrix}$ ie (col space of V) + ω

$z_4 = 0$ $z_3 = -25/3$ $z_2 = 20/3$ $z_1 = -14/3$ $\Rightarrow$ income $-58/3$. a riskless loss...? ¿

(ii) all portfolios can be duplicated as $\text{Rank}(V) = 3 < 4$.

B11. (a) $\begin{pmatrix}y\\s\\c\end{pmatrix}_t = \begin{pmatrix}5&2&4\\-3&6&2\\3&-3&1\end{pmatrix}\begin{pmatrix}y\\s\\c\end{pmatrix}_{t-1}$ $r^3 - 12r^2 + 41r - 42$

(b) $r_1 = 2$ (not 0.2) $r_2 = 3$ $r_3 = 7$

(c) $\underline{v}_1 = \begin{pmatrix}2\\3\\-3\end{pmatrix}$ $\underline{v}_2 = \begin{pmatrix}-2\\-4\\3\end{pmatrix}$ $\underline{v}_3 = \begin{pmatrix}-1\\1\\-1\end{pmatrix}$

(d) $\begin{pmatrix}y\\s\\c\end{pmatrix}_t = (0.2y_0 - s_0 - 1.2c_0)(2)^t(2\ 3\ -3)^T + (-0.6y_0 - 0.4c_0)(7)^t(-1\ 1\ -1)^T + (-s_0 - c_0)(3)^t(-2\ -4\ 3)^T$

$$= \begin{pmatrix} 0.4(2)^t + 0.6(7)^t & -2(2)^t + 2(3)^t & -2.4(2)^t + 2(3)^t + 0.4(7)^t \\ 0.6(2)^t - 0.6(7)^t & -3(2)^t + 4(3)^t & -3.6(2)^t + 4(3)^t - 0.4(7)^t \\ -0.6(2)^t + 0.6(7)^t & 3(2)^t - 3(3)^t & 3.6(2)^t - 3(3)^t + 0.4(7)^t \end{pmatrix}\begin{pmatrix}y_0\\s_0\\c_0\end{pmatrix}$$

B12. (a) when $c = c_c$, $v = 20$ km/hr, $q = 857$ (I don't know why it's not 857(3))

(b) $\underset{\frac{d}{dq}(qc)}{MSC} = 82.5$ $\underset{c}{PC} = 45$ (c) Assuming toll varies as the traffic varies, $v = 31.6$ $q = 525.7$, toll $= q\frac{dc}{dq} = 9.2$p/km cyclists $= 1600 - 525.7 = 1074.3$

(d) 28.9p/km

B13. (c) $C = y\alpha^{-\alpha}(1-\alpha)^{\alpha-1}\omega^{1-\alpha}r^{\alpha}$

B14. (a) Prob $= 10p(1-p)^9$.

(b)

p:	0.01	0.02	0.03	0.06	0.1	0.2
posterior:	0.116	0.297	0.232	0.262	0.079	0.014

(normalised)

B15. (a) $(\sum x_i)\log\lambda - n\lambda - \sum_i \ln x_i!$ (b) $\hat\lambda = \sum x_i/n$ var $= \lambda/n$ (I don't think Cramer-Rao can be used here, as the hint implies)

(c) $E(\tilde\lambda) = \lambda$ var$(\tilde\lambda) = \lambda/n - 1/3n$

(d) $\tilde\lambda$ is more efficient; both are unbiased, so prefer $\tilde\lambda$.

2009

A1. –

2. Yes.

3. Yes.

4. –

5. –

6. (a) Binomial $\binom{n}{y} p^y (1-p)^{n-y}$? (b) Beta distribution with $\alpha = 18;\ \beta = 103$.

7. –

8. (b) $y = 2\ln\frac{2p}{\sqrt{w_1 w_2}}$.

9. (a) $P_{t+1} = P_t(1 - a(h-z)) - a(g-c)$; $P_t = \frac{c-g}{h-z} + \left(P_0 - \frac{c-g}{h-z}\right)(1 - a(h-z))^t$; (b) system is stable if $z \le h < z + 2/a$.

10. $g''\big(f(x)\big) = -\frac{f''(x)}{(f'(x))^3}$.

B11. (a) (i) $Y(1-b) + r(c+a) = I^0 + G$; $mY - hr = M^s - M^0$; (ii) Y = 300; r = 50.

12. (b) (i) $r = 1\ \underline{v} = \frac{1}{\sqrt{2}}\binom{1}{-1}$; $r = 3\ \underline{v} = \frac{1}{\sqrt{2}}\binom{1}{1}$.

13. –

14. (a) $L = \frac{1}{\left(\sqrt{2\pi\sigma_\varepsilon^2}\right)^n} e^{-\sum_i (y_i - x_i\beta)^2/2\sigma_\varepsilon^2}$; (b) $\hat{\beta} = (X^T X)^{-1} X^T \underline{y}$; (d) $var\big(\hat{\beta}\big) = \sigma_\varepsilon^2 (X^T X)^{-1}$ on the leading diagonal, and zero off it.

15. (a) Marshallian demands = $(\frac{p_2}{p_1}, \frac{y - p_2}{p_2})$; Hicksian demands = $(\frac{p_2}{p_1}, u - \ln\left(\frac{p_2}{p_1}\right))$; expenditure function = $p_2(u - \ln\left(\frac{p_2}{p_1}\right) + 1)$; indirect utility = $\ln\left(\frac{p_2}{p_1}\right) + \frac{\mathrm{y} - \mathrm{p}_2}{\mathrm{p}_2}$; (c) -1; (d) $\left(1, \frac{y}{y - p_2}\right)$ (possibly with a minus sign on the front, depending on how you define elasticity of demand).

16. (a) $\Pi_\mathrm{V} = (\frac{\mathrm{a-c}}{2})^2$; (b) $\Pi_\mathrm{V} = (\mathrm{a-m})^2/8$; will make and sell product if $\sqrt{2}(a-c) > a - m$; yes; (c) requires $c > m$, which is true.

2010

A1. (c) all 2nd order polynomials.

2. (a) $x_i = \frac{b}{1+na}$.

3. –.

4. (b) zero.

5. -2.

6. (a) $\frac{1-r}{Q_0}$; (b) "its time" is $\frac{1}{Q_0}$.

7. –

8. (a) $\binom{n}{m}\alpha^m(1-\alpha)^{n-m}$; (b) $1-(1-\alpha)^n$.

9. –

B10. (a) $\alpha \neq 3 \ or -2, \beta =$ anything; (b) Same pivots, but not necessarily same null space.

11. (b) (ii) by Cramer: $\ln(P_1) = \frac{\begin{vmatrix} m_1-k_1-b_1y & a_{12} \\ m_2-k_2-b_2y & a_{22}-n_2 \end{vmatrix}}{\begin{vmatrix} a_{11}-n_1 & a_{12} \\ a_{21} & a_{22}-n_2 \end{vmatrix}}$, $\ln(P_2) = \frac{\begin{vmatrix} a_{11}-n_1 & m_1-k_1-b_1y \\ a_{21} & m_2-k_2-b_2y \end{vmatrix}}{\begin{vmatrix} a_{11}-n_1 & a_{12} \\ a_{21} & a_{22}-n_2 \end{vmatrix}}$;

(iii) $\ln(P_1) = \frac{\begin{vmatrix} m_1-k_1-b_1y-a_{11}\ln(1+\frac{t}{100}) & a_{12} \\ m_2-k_2-b_2y-a_{21}\ln(1+\frac{t}{100}) & a_{22}-n_2 \end{vmatrix}}{\begin{vmatrix} a_{11}-n_1 & a_{12} \\ a_{21} & a_{22}-n_2 \end{vmatrix}}$, $\ln(P_2) = \frac{\begin{vmatrix} a_{11}-n_1 & m_1-k_1-b_1y-a_{11}\ln(1+\frac{t}{100}) \\ a_{21} & m_2-k_2-b_2y-a_{21}\ln(1+\frac{t}{100}) \end{vmatrix}}{\begin{vmatrix} a_{11}-n_1 & a_{12} \\ a_{21} & a_{22}-n_2 \end{vmatrix}}$;

(iv) I'm unconvinced one can say without knowing the a_{ij}.

12. (a) $U_t = \frac{\alpha(1-\beta^t)}{1-\beta} + U_0\beta^t$, converges iff $\beta < 1$; (b) $U_t = \frac{\alpha(1-\beta^t)}{1-\beta} + U_0\beta^t + \sum_{i=0}^{t-1}\beta^i e_{t-i}$.

13. (a) $p = \frac{1}{2n}\sum_{i=1}^n a_i$, $S = \frac{1}{2}(\sum_{i=1}^n a_i^2 - \frac{3}{4n}(\sum_{i=1}^n a_i)^2) + \sum_{i=1}^n m_i$;
(b) $p = \frac{a_i}{2}$, $S = \frac{1}{8}\sum_{i=1}^n a_i^2 + \sum_{i=1}^n m_i$; difference is $\frac{3n}{8} var(a_i)$.

14. (a) (i) $\begin{matrix} 97902 & 5 & 97907 \\ 1998 & 95 & 2093 \end{matrix}$; (ii) 95/1998; (iii) 95.46% (iv) "posterior odds" now 47.5.

15. (a) $\frac{1}{\theta}$; (b) $\theta_{ML} = \frac{r}{(\sum_{i=1}^r X_i)+8(n-r)}$, where r=number of individuals with $X < 8$ and X_i is the data value for any individual with $X < 8$; estimated mean unemployment rate is $1/\theta_{ML}$;
(ii) assuming this means asymptotic variance, $\frac{\theta^2}{r}$.