ECONOMICS QUALIFYING EXAMINATION IN ELEMENTARY MATHEMATICS

Friday 29 October 2000 9 to 12

*This exam comprises **two** sections. Each carries 50% of the total marks for the paper. You should attempt **all** questions from Section A and **two** questions from Section B.*

You are reminded that only the approved calculators may be used.

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Section A

1. *(a)* Find the derivatives dy/dx of:

$$y = x\ln(x+1)$$

$$y = \frac{(3+4x)}{e^x}$$

(b) Given that

$$\frac{(3+2xy^2)}{x^2} = 1$$

evaluate dy/dx.

2. *(a)* Suppose that a firm has a production function

$$q(K,L) = A(K^\beta + L^\beta)^{1/\beta} \quad \beta > 0, \quad A > 0$$

Show that the marginal product of labour, $\partial q/\partial L$, is positive if K and L are positive.

(b) How does an increase in the quantity of capital supplied affect the marginal product of labour?

3. Let

$$A = \begin{bmatrix} 1 & 2 \\ -1 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} x \\ y \end{bmatrix} \text{ and } R = \begin{bmatrix} 25 \\ 5 \end{bmatrix}$$

Find:

(a) AB
(b) A^{-1}

Hence solve the system of equations $AB = R$ for y and x.

4. Minimise

$$y = x_1^2 + 2x_2$$

subject to the constraint: $x_1 + x_2 = m$

(a) by substitution of the constraint

(b) by Lagrange's Method

5. *(a)* What are the maximum and minimum values of the function

$$f(x) = \ln x - x^2$$

for $x \geq 1/2$

(b) Sketch the graph of this function.

6. A firm has to produce an amount $\overline{q}$ and has production technology

$$q = k^{1/3} + l^{1/3}$$

where k = capital input and l = labour input. If capital costs r per unit and labour w per unit find the minimum cost of producing $\overline{q}$.

7. Given the production function $Q = AK^{\alpha}L^{1-\alpha}$ show that in a competitive economy revenues are just sufficient to pay capital and labour their marginal products

8. Evaluate the following integrals:

$$\int_{-1}^{1} \left(x - 3x^3\right) dx$$
$$\int_{0}^{2} xe^{-x} dx$$
$$\int_{1}^{5} 2x\sqrt{(x^2+1)} dx$$

9. Demand and supply in a particular market are given by

$$\begin{aligned} q_t^d &= 1400 - .5p_t \\ q_t^s &= 0.7p_{t-1} - 200 \end{aligned}$$

where q_t^d, q_t^s and p_t denote the quantity demanded, quantity supplied and the price in time t, respectively.

(a) Find the equilibrium price, p^*, such that if $p_{t-1} = p^*$, then $p_t = p^*$.

(b) Is p^* a stable equilibrium?

(c) What is the equilibrium quantity traded.

Section B

10. A consumer spends all his income, M, on two goods, books and cars. The price of the two goods are p_b and p_c. The consumer's preferences are given by the utility function, $u(b, c)$.

    (a) Write down and solve the consumer's problem.

    (b) Define what is meant by the income elasticity of demand for a good.

    (c) If the utility function has the form

    $$u(b, c) = b^{0.25} c^{0.5}$$

    are the two goods normal or inferior?

    (d) How do demands change if preferences are given by

    $$U(b, c) = \ln(u(b, c))$$

    Explain.

11. A firm produces output, Q, according to the production function

    $$Q = 3L^{\alpha} K^{1-\alpha}$$

    where L is labour and K is capital. Production uses a fixed cost, F, and the firm has to pay w to labour and r to capital. The firm is a price-taker in all markets. The price of output is p.

    (a) Taking output as given, find the marginal product of each factor and the amount of this factor which is used.

    (b) Suppose a consumer demands $\bar{Q} = 600$ units and that $w = 4$, $r = 8$, $F = 200$ and $\alpha = 1/2$. What is the minimum price that the consumer has to offer in order to receive $\bar{Q}$? How much labour and capital are used?

    (c) Suppose the firm can vary output but is constrained to use exactly 100 units of labour. Using Lagrange's method find the firm's demand for capital. What interpretation can you give to the Lagrange multiplier for labour?

12. Consider the following ISLM model of aggregate demand in an open economy

$$Y = C + I + G + X - M$$

$$C = \alpha Y$$

$$I = \beta - \gamma r$$

$$X = \overline{X}$$

$$M = \delta + eY$$

$$G = \overline{G}$$

$$m^d = hY - kr$$

$$m^s = \overline{m}$$

where C is consumption, I investment, Y denotes national income, G government spending, X exports, M imports, e the exchange rate, r the interest rate, m^s money supply and m^d money demand. The government controls G, and $\overline{m}$.

(a) Derive expressions for the IS and LM curves keeping the exchange rate fixed.

(b) Solve for equilibrium values of Y and r.

(c) What is the effect of an increase in e on equilibrium value of Y

(d) By how much would a government have to change the money supply to offset the effects on Y

13. Suppose demand and supply of a commodity are given by

$$q_s(p) = 16p - 10$$

$$q_d(p) = 20 - 4p$$

(a) Calculate the equilibrium price, p^*, and quantity, q^*.

(b) Suppose the government imposes a tax of t per unit on suppliers. Calculate the new equilibrium price, p_t^*. Compare $(p^* - p_t^*)$ with t and comment on the incidence of the tax.

(c) Suppose the tax is on consumers rather than on suppliers. How does this change p_t^*? Comment.

14. An industry consists of two firms who behave as Stackelberg oligopolists. The leader produces units of output at a constant marginal cost of £1; the follower produces at a constant marginal cost of £2. Total demand for industry output is given by

$$Q = 56 - 4p$$

(a) Derive the equilibrium outputs for the leader and follower.

(b) Find the equilibrium price.

15. A firm has total cost function $c = q^2 + \bar{c}$ where c denotes cost, $\bar{c}$ fixed cost and q the quantity produced. There are two countries in which output can be sold. In country A, demand is given by

$$q_A = 40 - 4p_A$$

In country B, demand is given by

$$q_B = 23 - 2p_B$$

where p_A and p_B are the prices in country A and B respectively.

(a) If the firm can supply both countries from a single plant and can charge a monoply price in the local market, how much will it sell in each country assuming that transport costs are zero. What will be the value of total profits?

(b) If consumers are able to purchase the good from either country at a common price, what will this price be?

(c) Would a move to free trade be Pareto improving?