

Section A

1. (a) Find the derivatives dy/dx of:

$$y = xe^{-x}$$
$$y = \frac{(3 + 4x - x^2)}{\ln x}$$

(b) Find the partial derivative with respect to x_1, x_2 and x_3 of the function

$$y = f(x_1, x_2, x_3) = \frac{4x_1(3x_1 + x_2^2)}{x_3^2}$$

2. (a) Suppose that a firm has a production function

$$q(K, L) = AK^\beta L^{1-\beta} \quad 1 > \beta > 0, \quad A > 0$$

Show that the marginal product of labour, $\partial q/\partial L$, is positive if K and L are positive.

(b) How does an increase in the quantity of capital supplied affect the marginal product of labour?

(c) If $\beta = \frac{1}{2}$, and $A = 2$, write down an expression for output per worker in terms of capital per worker.

3. Let

$$A = \begin{bmatrix} 1 & -0.75 \\ -1 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} c \\ y \end{bmatrix} \quad \text{and} \quad R = \begin{bmatrix} 15 \\ 60 \end{bmatrix}$$

Find:

(a) AB

(b) A^{-1}

Hence solve the system of equations $AB = R$ for y and c .

4. Maximise

$$y = -x_1^2 - 4x_2 + 10$$

subject to the constraint: $x_1 + x_2 = m$

(a) by substitution of the constraint

(b) by Lagrange's Method

5. (a) What are the maximum and minimum values of the function

$$f(x) = -\frac{2}{3}x^3 + 2x^2 + 6x + 1$$

in the interval $[-2, 6]$.

- (b) Sketch the graph of this function in the interval $[-2, 6]$

6. A consumer's preferences are represented by the utility function

$$u = \ln(xy)$$

where u denotes utility and x and y are the quantities consumed. Given prices p_x and p_y and total income of m , find the demand functions for the two goods.

7. (a) Show that the function

$$z = f(x, y) = (x^2 + y^2)^{\frac{1}{2}} x^{\frac{3}{4}} y^{\frac{1}{4}}$$

is homogenous of degree 2.

- (b) Verify by calculation that

$$\frac{\partial z}{\partial x}x + \frac{\partial z}{\partial y}y = 2f(x, y)$$

as predicted by Euler's Theorem.

8. (a) Evaluate the following integrals:

$$\int_0^1 (x - 3x^2) dx$$
$$\int_0^2 e^{-2x} dx$$

- (b) Find the area under the curve $f(t) = 3t^3 - 3t^2 + 3t + 1$ between $t = -2$ and $t = 2$.

9. Demand and supply in a particular market are given by

$$\begin{aligned}q_t^d &= 600 - p_t \\q_t^s &= 2p_{t-1} - 1200\end{aligned}$$

where q_t^d , q_t^s and p_t denote the quantity demanded, quantity supplied and the price in time t , respectively.

- (a) Find the equilibrium price, p^* , such that if $p_{t-1} = p^*$, then $p_t = p^*$.
- (b) Find the explicit solution for p_t in terms of t given that $p_0 = 601$.
- (c) Is p^* a stable equilibrium?

Section B

10. A consumer has utility function for wine, w , and books, b :

$$u(w, b) = w^{\frac{1}{2}}b^{\frac{1}{2}}$$

Annual income is £1000. Wine is £2 per bottle, books are £20 each.

(a) Find the demands for wine and for books.

(b) The government decides alcohol is harmful and imposes a tax on wine of £2 per bottle. What effect does this have on the number of books bought?

(c) The government then embarks on an advertising campaign to change the preferences of the population. The campaign is successful and leads to a logarithmic transform of the utility function, such that utility is now given by

$$u(w, b) = \frac{1}{2} \ln w + \frac{1}{2} \ln b$$

An economics adviser to the government thinks the advertising campaign is a waste of money. Calculate the effect of the campaign on consumer demand. Explain.

11. A firm produces output, Q , according to the production function

$$Q = AL_s^\alpha L_u^{1-\alpha}$$

where L_s is skilled labour and L_u is unskilled labour, and A is constant. The cost of production is given by

$$C = w_s L_s + w_u L_u + F$$

where w_s and w_u are the wages of the skilled and unskilled, and F is the firm's fixed cost. The firm is a price-taker in all markets. The price of output is p .

(a) For each factor of production, find the marginal product and demand.

(b) Suppose a consumer demands $\bar{q} = 400$ units and that $w_u = 2$, $w_s = 8$, $A = 1$ and $F = 200$. What is the minimum price that the consumer has to offer in order to receive \bar{q} ? How much skilled and unskilled labour are used?

(c) Suppose the firm is constrained to use 100 units of skilled labour. Using Lagrange's method find the firm's demand for unskilled labour, the minimum cost of producing \bar{q} and the price that the consumer now has to offer.

(d) Additional skilled labour can be hired at a premium over w_s . What is the maximum premium that the firm would be willing to pay?

12. Aggregate demand is described as follows:

$$Y = C + I + G$$

$$C = \beta(Y - T)$$

$$I = \alpha - \gamma r$$

$$T = \bar{T} + tY$$

$$M^d = \delta(Y - T) - \varepsilon r$$

$$M^s = \bar{M}$$

where C is consumption, I investment, Y gross income, G government spending, r the interest rate, T total taxes, t the tax rate on income, \bar{T} a lump sum tax, M^s money supply and M^d money demand. Greek letters are parameters. The government controls G, \bar{T}, t , and \bar{M} .

(a) Derive expressions for the IS and LM curves.

(b) Solve for equilibrium values of Y and r .

(c) What happens to Y and r if the government increases \bar{T} ?

(d) By how much can the government increase G following the increase in \bar{T} , keeping the budget balanced? (i.e. such that $\frac{d(G - \bar{T} - tY)}{d\bar{T}} = 0$)

13. Suppose demand and supply of a commodity are given by

$$q_s(p) = 12p - 10$$

$$q_d(p) = 30 - 8p$$

- (a) Calculate the equilibrium price, p^* , and quantity, q^* .
- (b) Suppose the government imposes a tax of t per unit on suppliers. Calculate the new equilibrium price, p_t^* . Compare $(p^* - p_t^*)$ with t and comment on the incidence of the tax.
- (c) Suppose the tax is on consumers rather than on suppliers. How does this change p_t^* ? Comment.

14. An industry consists of two firms who behave as Stackelberg oligopolists. The leader produces units of output at a constant marginal cost of £3; the follower produces at a constant marginal cost of £4. Total demand for industry output is given by

$$Q = 104 - 4p$$

- (a) Derive the equilibrium outputs for the leader and follower.
(b) Find the equilibrium price.

15. A firm has total cost function $c = q^2 + \bar{c}$ where c denotes cost, \bar{c} fixed cost and q the quantity produced. There are two markets in which output can be sold. In market A , demand is given by

$$q_A = 20 - p_A$$

In market B , demand is given by

$$q_B = 32 - p_B$$

where p^A and p^B are the prices in market A and B respectively.

- (a) In which market would the firm prefer to be a monopolist?
- (b) If the firm can act as a monopoly supplier to both markets and the markets remain independent, how much will it sell in each market and what will be the value of total profits.
- (c) If consumers are able to purchase the good from either market, how much will the firm sell and at what price?
- (d) If there were free entry, would price fall to equal marginal cost?