MANAGEMENT STUDIES TRIPOS
DIPLOMA IN MANAGEMENT STUDIES

Wednesday 3 May 2006 9.00 – 12.00

Paper M2

QUANTITATIVE METHODS AND OPERATIONS MANAGEMENT

*Answer **four** questions, **two** from Section **A** and **two** from Section **B**.*

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*Answers to Sections **A** and **B** must appear in two separate booklets.*

*All **eight** questions carry the same number of marks.*

N.B. THE FINAL TWO PAGES OF THIS EXAMINATION PAPER CONSIST OF SPECIAL DATA SHEETS

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

SECTION A

1 Chemco is a small petrol production company located in the city of Aberdeen, Scotland. Chemco owns two petrol production lines that can be used to produce two types of petrol: unleaded and premium. Running production line I for an hour costs £6 and requires 2 barrels of crude oil A and 3 barrels of crude oil B. The output from running production line I for an hour is 2.5 barrels of unleaded petrol and 1.2 barrels of premium petrol. Running production line II for an hour costs £5 and requires 1 barrel of crude oil A and 3 barrels of crude oil B. The output from running production line II for an hour is 2 barrels of unleaded petrol and 0.5 barrels of premium petrol. Each week, Chemco can purchase up to 200 barrels of crude oil A at a cost of £2.5 per barrel, and up to 300 barrels of crude oil B at a cost of £3.6 per barrel. Both types of petrol can be sold at the following per-barrel prices: Unleaded, £9.6; premium, £11.8. Assume that each week, at most 48 hours of time are available for each of the two production lines.

(a) Chemco wishes to maximize its weekly profit, i.e., revenues less costs. Help the operations manager of Chemco to formulate a linear programming model for this production-planning problem. You are <u>not</u> required to solve your linear program.

(b) Define and explain sensitivity analysis with respect to the price of premium petrol and availability of an additional amount of crude oil A in your linear programming model in part (a).

(c) You are now told the additional condition that the number of hours for running production line II should be greater than or equal to that for running production line I, if the number of hours for running production line I is less than or equal to 20. How would your model, of part (a), change given this condition? You are <u>not</u> required to solve your model.

2 The construction company SkyLine plans to borrow £4,000,000 at the beginning of year 2007 from BlueMoon Bank to be paid in a lump sum at the end of three years. Interest on the loan will be calculated at the end of each year. BlueMoon charges its customers with three possible yearly-based interest rates: simple interest (interest does not generate further interest, and the interest rate remains unchanged during the loan period), compound interest (interest generates further interest, and the interest rate remains unchanged during the loan period), and simple-variable interest (interest does not generate further interest, but interest rates may change year by year). The simple interest rate and the compound interest rate are fixed at 9% and 8% per annum respectively until the year 2010. The variable interest rate varies depending on the national economy. An economic analysis from SkyLine shows that the variable interest rate of BlueMoon is either 8% with a probability of 0.4 or 10% with a probability of 0.6 in year 2008; and the variable interest rate of BlueMoon is either 7% with a probability of 0.7 or 9% with a probability of 0.3 in year 2009. After intensive negotiations, BlueMoon and SkyLine reached three options. In option (I), SkyLine can borrow the money using a simple interest rate for the whole loan period. In option (II), SkyLine can borrow the money using a compound interest rate for the whole loan period. In option (III), SkyLine can borrow the money using simple interest rates in year 2007, and simple-variable interest rates for year 2008 and year 2009.

(a) Calculate the total interest that SkyLine needs to pay at the end of three years if SkyLine elects to use option (I) to borrow money from BlueMoon.

(b) Calculate the total interest that SkyLine needs to pay at the end of three years if SkyLine elects to use option (II) to borrow money from BlueMoon.

(c) Construct a decision tree to help SkyLine decide which of the three options should be used to borrow money from BlueMoon based on the criterion of expected monetary value.

(d) Define and calculate the expected value of perfect information for SkyLine.

TURN OVER

3 The weights of the bus passengers in the city of Birmingham have a distribution with a mean of 75 kilograms and a standard deviation of 15 kilograms. The capacity for all buses on route 204 in Birmingham is 36. In March 2006, 80 passengers who use bus route 204 were randomly selected to answer the following two questions: (1) How many times did you use bus route 204 last month? (2) Are you happy with the service provided on bus route 204? The survey result shows that the average number of times that a passenger uses bus route 204 is 12 with a standard deviation of 4.4. The survey also shows that 64 passengers are happy with the service provided on bus route 204, and 16 passengers are not happy with the service.

(a) If a route-204 bus in Birmingham is full, what is the chance that the average weight of all passengers in the bus is between 72 and 79 kilograms?

(b) The 2005 report of public transportation for the city of Birmingham shows that 85% of route-204 passengers were happy with the service provided on route 204. Do you think the service on route 204 has deteriorated in March 2006 compared with year 2005? Show your calculations supporting your conclusion.

(c) Estimate the average number of journeys made by all passengers of route 204. Show your calculations supporting your conclusion. Explain your result intuitively to the mayor of the city of Birmingham who has little knowledge of statistics.

(d) Assume that the city of Birmingham's target for the monthly average number of times that a passenger uses bus route 204 is 13.2. What is the largest significance level at which this target is not significantly different from the corresponding value calculated from the survey in March 2006?

4 (a) Dr. Anna Smith, a specialist on lead poisoning, says that the highest prevalence of lead poisoning occurs in children who live in large industrial cities and whose families have low incomes. The research shows that extra amounts of lead in the blood can harm the brains of young children and consequently affect their overall IQ when they grow older. Dr. Smith is conducting a continuing study of 70 children, in an African country, who have hazardous amounts of lead in their blood. She collected data and presented the regression analysis on the relationship between the extra amounts of lead above the danger level in their blood when children were about two-years old and their overall IQ when they were about ten-years old as shown below.

SUMMARY OUTPUT

Regression Statistics	
Multiple R	0.7778707
R Square	0.6050829
Adjusted R Sq.	0.5992753
Standard Error	16.474758
Observations	70

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	1	28278.47042	28278.47	104.188	2.35E-15
Residual	68	18456.40101	271.4177		
Total	69	46734.87143			

	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*
Intercept	91.871254	4.110413778	22.35085	4.78E-33	83.66906	100.0735
Extra lead	-2.43436	0.238493192	-10.20725	2.35E-15	-2.910266	-1.958455

What is the value for the correlation coefficient between the extra amounts of lead in children's blood at two and their overall IQ at ten? Explain the meaning of the correlation coefficient in this situation.

(i) If a child's blood lead level was 20 units above the danger level, what would you predict his overall IQ to be at a given confidence level?

(ii) Give the *p*-value for the slope in this example and discuss its meaning.

(iii) Do you think the value for the slope obtained in the regression analysis is significantly different from zero at the 8% significance level?

(iv) Discuss the underlying assumptions of the regression analysis of Dr. Anna Smith.

TURN OVER

(b) A mathematics teacher in a high school in Cambridgeshire thinks that the frequency of absences from her lessons affects students' A-level mathematics examination marks. To check her hypothesis, she collected data for a sample of ten students as listed below.

	Jim	Ryan	Mark	Steve	James	Lucy	Lily	John	Linda	Andy
Maths mark	73	64	81	36	50	22	80	64	44	54
No.of absences	4	6	0	12	8	15	2	3	9	5

(i) Compute the regression equation based on the sample data.

(ii) The student Sally (who is not included in the above sample) was not absent for any maths lesson, and her maths mark is 45. Do you think Sally's low maths mark is due to statistical variations in the sample? Discuss how you arrived at your conclusion.

SECTION B

5. You and a clever friend from your college called Gene—an American from Chicago––are contemplating entering the high-end pizza market together. You feel that you can well compete with Pizza Express since your staff would consist entirely of you and Gene. Your business strategy is to provide 100% fresh, high quality pizza on a timely basis.

Gene would take orders over the telephone, which you estimate would take about one minute each. Orders would specify the type of pizza, fillings and toppings. Once the order is received, you would measure and mix the ingredients for the crust in about five minutes and then you would spoon the dough onto a pizza tray, which would take you another minute. Meanwhile, Gene would collect the order-specific filling and toppings (3 minutes) and clean and process them (2 minutes). You would then pour the filling onto the dough and spread the toppings on the top (2 minutes), load and set the oven-timer (1 minute). The actual baking would take about 8 minutes. During the next minute, Gene and you would unload the oven and place the pizza in a box.

(a) Draw a process flow chart identifying tasks, activity times, flows, and any potential storage.

(b) Calculate the flow time of the pizza process.

(c) Calculate the capacity of the pizza process.

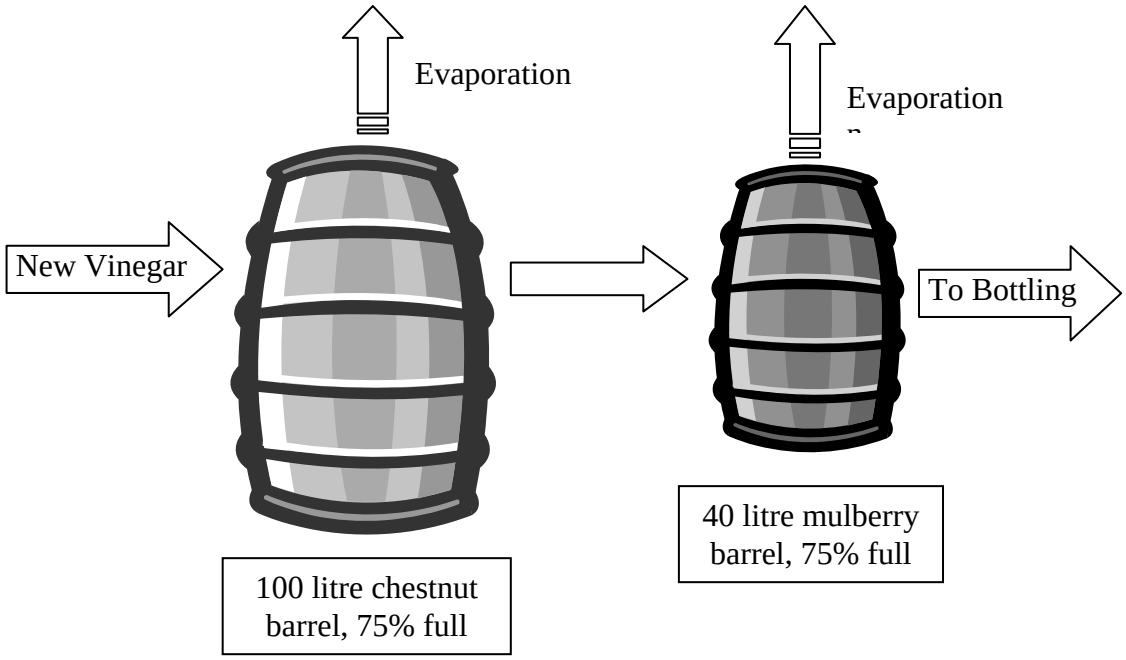
(d) Preferring higher revenues and equitable task assignments, you suggest cross-training your workers (i.e., both you and Gene can perform all 'human' activities). Recalculate the flow time of the pizza process.

(e) Still unable to meet all the demand (even with your cross-trained labour force), you want to increase flow time even further. What action or actions do you recommend?

TURN OVER

6 Balsamic vinegar is made from a combination of grapes. It spends time in a large (100 litre) chestnut barrel before moving into a smaller (40 litre) mulberry barrel, after which it is finally sold. The exact process is as follows:

- At the end of each year, half of whatever vinegar remains in the smaller barrel is drawn off into 250 ml (0.25 litre) bottles and sold.
- The smaller barrel is refilled from vinegar in the larger barrel. In order to allow breathing space for the necessary acetic oxidation, the smaller barrel is only filled to 75% of its capacity.
- The larger barrel is then refilled (again, only to 75% of its capacity) with new vinegar.
- Over the course of the year, 10% of the vinegar in each barrel will evaporate through the wood. For convenience, you should assume that this evaporation occurs all at once at the very end of the year (just before vinegar is removed from the smaller barrel).



(a) Calculate the number of 250 ml bottles of balsamic vinegar that are produced each year.

(b) Calculate the volume of new vinegar that is added to the larger 100 litre chestnut barrel each year.

(c) What is the average age of the balsamic vinegar when it is bottled (upon leaving the smaller mulberry barrel)?

(d) Give the formula for the EOQ with Production, making sure to define all the parameters [in 10 words or less, in each case], as well as the meaning of Q* [in 10 words or less].

(e) If the EOQ with Production formula was relevant to answering any of parts (a), (b), or (c), above, specify which part(s) it pertains to and *briefl*y explain why. If, however, it did not apply to any of (a), (b), or (c), *briefly* state why.

TURN OVER

7 Nottingham Corporation produces two different products on three machines. Product A begins with raw material, which is processed by machine 1, enters buffer b2, and is then converted by machine 2 into the finished product. Product B begins with a different raw material, which is processed by machine 1, enters buffer b3, and is then converted by machine 3 into the finished product.

Products A and B have different processing needs on machine 1, but machine 1 can produce either product at a rate of 50 units per hour. It takes 15 minutes to change machine 1 over from product A to product B, or from B to A. Machines 2 and 3 run at rates of 10 units per hour and 30 units per hour, respectively; if they are ever starved for input they fall idle and wait.

The production system is operated in cycles: machine 1 makes product A for a period of X minutes, then it is changed over and makes product B for a period of Y minutes, then it is changed over and held idle for a period of Z minutes (it may be that Z=0), and then the cycle repeats. Thus the total cycle is of length T=X+Y+Z+30 minutes.

In answering parts (a), (b), and (c) base your calculations on the situation that obtains after the system has been operating for a while.

(a) First suppose that the cycle length is set at T = 900 minutes (15 hours) and that production runs of product A on machine 1 are of length X=120 minutes (2 hours). What is the long-run average inventory level in buffer b2?

(b) What is the utilisation rate for machine 2?

(c) Assuming that management wants to keep both machine 2 and machine 3 busy all the time (that is, to produce both products at the maximum possible rate), what is the shortest possible cycle length T?

(d) *Briefly* give two different definitions of set-up time.

8 Parts (a), (b) and (c) of this problem concern a printed circuit board (PCB) assembly line. The most expensive pieces of equipment used in the line are automated chip placement (ACP) machines, which place and attach chips at precisely specified locations on circuit boards. An ACP machine executes an average of 22.47 chip placements per minute when it is operating.

The PCB assembly line that you are to analyse contains three ACP machines, and it is responsible for producing three types of circuit boards—here they will be called A, B and C boards. The line produces 320 boards per day of each type, because it feeds into another assembly line making 320 personal computers per day, and there is one board of each type in a personal computer. A board of type A requires 15 chip placements, one board of type B requires 80 chip placements, and one board of type C requires 9 chip placements. The PCB assembly line has been configured so that two of the three ACP machines are dedicated to production of type B boards, while the third ACP machine works only on boards of types A and C. Computerized controls allow the third machine to switch between boards of type A and type C with virtually no set-up time or changeover time. The PCB assembly line operates 15 hours per day—two eight-hour shifts with a half-hour break in the middle of each shift.

(a) Given the current scheme for dedicating ACP machines to board types, and the current schedule of 15 operating hours per day, what is the maximum number of personal computers that could be made per day with the current stock of 3 ACP machines? Assume there is adequate capacity for all resources other than ACP machines.

(b) Suppose that the production rate is increased by 50% to 480 personal computers per day, still with 15 operating hours per day, and that ACP machines continue to be dedicated either to type B boards or else to boards of types A and C. If management wants a planned idleness rate of at least 10% for all equipment, how many ACP machines will be needed?

(c) What would be your answer in part (b) if all ACP machines were able to work on all three types of boards?

TURN OVER

(d) Two heuristics for assembly line balancing are called *Ranked Positional Weights* and *Longest Sequential Chain of Followers*. Each of these two heuristics provides a method for assigning a priority ranking to operations. One of these heuristics specifies that ties are to be broken arbitrarily. Specify the name of that heuristic, and provide a *brief* explanation of why ties are broken arbitrarily in that heuristic.

END OF PAPER

SPECIAL DATA SHEET 1
Formulae for Correlation and Regression Analysis

Consider data pairs (X_1, Y_1), (X_2, Y_2), …, (X_n, Y_n).
Let m_X and m_Y denote the respective means of the X and Y data.
Let s_X and s_Y denote the respective standard deviations of the X and Y data.
The formula for covariance is given by the sum

$$\text{cov}(X, Y) = \frac{\sum_{i=1}^{n} (X_i - m_X)(Y_i - m_Y)}{n}.$$

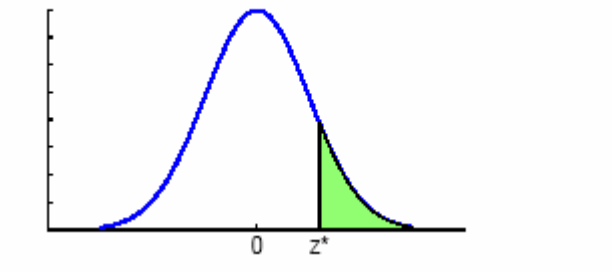
$$\text{correl}(X, Y) = r = \frac{\text{cov}(X, Y)}{s_X s_Y}.$$

The formula for the regression coefficient is
The formula for the line of best fit is

$$Y - m_Y = \frac{r s_Y}{s_X}(X - m_X).$$

SPECIAL DATA SHEET 2 – Standard Normal Distribution Table

(Areas under the standard normal curve beyond z*, i.e., shaded area)



z^*	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.4960	0.4920	0.4880	0.4840	0.4801	0.4761	0.4721	0.4681	0.4641
0.1	0.4602	0.4562	0.4522	0.4483	0.4443	0.4404	0.4364	0.4325	0.4286	0.4247
0.2	0.4207	0.4168	0.4129	0.4090	0.4052	0.4013	0.3974	0.3936	0.3897	0.3859
0.3	0.3821	0.3783	0.3745	0.3707	0.3669	0.3632	0.3594	0.3557	0.3520	0.3483
0.4	0.3446	0.3409	0.3372	0.3336	0.3300	0.3264	0.3228	0.3192	0.3156	0.3121
0.5	0.3085	0.3050	0.3015	0.2981	0.2946	0.2912	0.2877	0.2843	0.2810	0.2776
0.6	0.2743	0.2709	0.2676	0.2643	0.2611	0.2578	0.2546	0.2514	0.2483	0.2451
0.7	0.2420	0.2389	0.2358	0.2327	0.2296	0.2266	0.2236	0.2206	0.2177	0.2148
0.8	0.2119	0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1922	0.1894	0.1867
0.9	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685	0.1660	0.1635	0.1611
1.0	0.1587	0.1562	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423	0.1401	0.1379
1.1	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210	0.1190	0.1170
1.2	0.1151	0.1131	0.1112	0.1093	0.1075	0.1056	0.1038	0.1020	0.1003	0.0985
1.3	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869	0.0853	0.0838	0.0823
1.4	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0721	0.0708	0.0694	0.0681
1.5	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0594	0.0582	0.0571	0.0559
1.6	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455
1.7	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367
1.8	0.0359	0.0351	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294
1.9	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233
2.0	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183
2.1	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143
2.2	0.0139	0.0136	0.0132	0.0129	0.0125	0.0122	0.0119	0.0116	0.0113	0.0110
2.3	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084
2.4	0.0082	0.0080	0.0078	0.0075	0.0073	0.0071	0.0069	0.0068	0.0066	0.0064
2.5	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051	0.0049	0.0048
2.6	0.0047	0.0045	0.0044	0.0043	0.0041	0.0040	0.0039	0.0038	0.0037	0.0036
2.7	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027	0.0026
2.8	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021	0.0020	0.0019
2.9	0.0019	0.0018	0.0018	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014
3.0	0.0013	0.0013	0.0013	0.0012	0.0012	0.0011	0.0011	0.0011	0.0010	0.0010