Formulae for Part IA Natural Sciences/Computer Science Mathematics Course

At school, you probably had formulae sheets for the non-trivial formulae you needed to know. Unfortunately, at Cambridge you do not, and hence you will have to memorise them. This document lists the subset of the formulae that you should already know from school, and the sooner you begin work on memorising them, the better.

This document is based on "Mathematical Formulae: A Handbook", compiled for the Physics Department Teaching Committee by F. G. Gallagher and J. R. Shakeshaft, which covers most/all of the formulae you will need for the examination. It is on sale at the Cavendish Stores at the Cavendish Laboratory. It covers other formulae as well, but obviously you are not expected to know them if you have not been taught them at either school or during the course.)

Ian Rudy
Robinson College, University of Cambridge
All corrections/suggestions to: iar1 (with the usual @cam.ac.uk)
Document last updated: 5 May 2009
This document is available at:
http://www.robinson.cam.ac.uk/iar1/teaching/nst1a_maths_formulae.pdf

1. Series

1.1 Geometric Progression

$$S_n = a + ar + ar^2 + ar^3 + \ldots + +ar^{n-1} = a\frac{1-r^n}{1-r}$$

$$S_\infty = \frac{a}{1-r} \text{ for } |r| < 1$$

1.2 Binomial Expansion

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \frac{n(n-1)(n-2)}{3!}x^3 + \ldots$$

or, occasionally, the more general formula:

$$(a+b)^n = a^n + na^{n-1}b + \frac{n(n-1)}{2!}a^{n-2}b^2 + \frac{n(n-1)(n-2)}{3!}a^{n-3}b^3 + \ldots + b^n$$

1.3 Maclaurin Series

$$f(x) = f(0) + xf'(0) + \frac{x^2 f''(0)}{2!} + \frac{x^3 f'''(0)}{3!} + \ldots$$

1.4 Taylor Series

$$f(a+x) = f(a) + xf'(a) + \frac{x^2 f''(a)}{2!} + \frac{x^3 f'''(a)}{3!} + \ldots \text{, or, equivalently:}$$

$$f(x) = f(a) + (x-a)f'(a) + \frac{(x-a)^2 f''(a)}{2!} + \frac{(x-a)^3 f'''(a)}{3!} + \ldots$$

1.5 Power Series

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots + \frac{x^n}{n!} + \ldots$$

$$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \ldots + (-1)^{n+1}\frac{x^n}{n} + \ldots$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \ldots$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \ldots$$

2 Complex Numbers

2.1 De Moivre's Theorem

$$(\cos\theta + i\sin\theta)^n = \cos n\theta + i\sin n\theta$$

3. Trigonometric Formulae

3.1 General Formulae

$$\cos^2 A + \sin^2 A = 1 \qquad \sec^2 A = \tan^2 A + 1 \qquad \mathrm{cosec}^2 A = \cot^2 A + 1$$

$$\sin 2A = 2\sin A\cos A \qquad \cos(2A) = \cos^2(A) - \sin^2(A) = 2\cos^2(A) - 1 = 1 - 2\sin^2(A)$$

$$\sin(A \pm B) = \sin A\cos B \pm \cos A\sin B$$

$$\cos(A \pm B) = \cos A\cos B \mp \sin A\sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A\tan B}$$

$$\cos A\cos B = [\cos(A+B) + \cos(A-B)]/2$$

$$\sin A\sin B = [\cos(A-B) - \cos(A+B)]/2$$

$$\sin A\cos B = [\sin(A+B) + \sin(A-B)]/2$$

The following can be derived from the above, so either you can remember the derivation, or the formulae that you obtain:

$$\sin A + \sin B = 2\sin\frac{A+B}{2}\cos\frac{A-B}{2} \qquad \sin A - \sin B = 2\sin\frac{A-B}{2}\cos\frac{A+B}{2}$$

$$\cos A + \cos B = 2\cos\frac{A+B}{2}\cos\frac{A-B}{2} \qquad \cos A - \cos B = -2\sin\frac{A+B}{2}\sin\frac{A-B}{2}$$

3.2 Sine and Cosine Rule

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} \qquad a^2 = b^2 + c^2 - 2bc\cos A$$

4. Differentiation

Product rule: $\frac{d}{dx}(uv) = v\frac{du}{dx} + u\frac{dv}{dx}$

Quotient rule: $\frac{d}{dx}(u/v) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

Chain rule (function of a function): $\frac{d}{dx}(g(f(x))) = \frac{dg}{df}\frac{df}{dx}$

$\frac{d}{dx}(x^n) = nx^{n-1}$

$\frac{d}{dx}(e^{kx}) = ke^{kx}$

$\frac{d}{dx}(\ln x) = \frac{1}{x}$

$\frac{d}{dx}(e^{f(x)}) = e^{f(x)}\frac{df}{dx}$

$\frac{d}{dx}(\ln f(x)) = \frac{\frac{d}{dx}(f(x))}{f(x)}$

$\frac{d}{dx}(\sin x) = \cos x$

$\frac{d}{dx}(\cos x) = -\sin x$

$\frac{d}{dx}(\tan x) = \sec^2 x$

$\frac{d}{dx}(\sec x) = \sec x \tan x$

$\frac{d}{dx}(\cot x) = -\text{cosec}^2 x$

$\frac{d}{dx}(\text{cosec}\, x) = -\text{cosec}\, x \cot x$

4. Integration
Just the reverse of the differentiation formulae above!

5. Miscellaneous Formulae

5.1 Quadratic Forumula

$ax^2 + bx + c = 0$ has roots $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

5.2 Logarithms

$\log_a(xy) = \log_a(x) + \log_a(y)$

$\log_a(x/y) = \log_a(x) - \log_a(y)$

$\log_a(x^k) = k\log_a(x)$

5.3 Areas and Volumes

Surface area of a sphere: $4\pi r^2$

Volume of a sphere: $\frac{4}{3}\pi r^3$