

A1. x^2+x+1 . 2. $5\pi/4 \leq \theta \leq 3\pi/2$. 3. $\sin(1/x) - (1/x)\cos(1/x)$. 4. $0 < a < 1$.

5. (a) $t^2+2t-1=0$; (b) $\sqrt{2}-1$. 6. $-(x/3)(3-2x)^{3/2} - (1/15)(3-2x)^{5/2} + c$. 7. $2\pi/3$.

8. -. 9. $ar(1-r^{100})/(1-r)$, $r \neq 1$. 10. 1.42.

B11. (a)(i) $\operatorname{Re} = 2/5$, $\operatorname{Im} = 1/5$; (ii) $\operatorname{Re} = \cos(1)$, $\operatorname{Im} = \sin(1)$; (iii) $\operatorname{Re} = 0$, $\operatorname{Im} = \pi/2 + 2n\pi$;

(iv) $\operatorname{Re} = 0$, $\operatorname{Im} = 1$; (b) $\left(\frac{y-1}{a/2}\right)^2 + \left(\frac{x}{\sqrt{(a^2-4)}/2}\right)^2 = 1$ ellipse; (i) $\left(\frac{y-1}{2}\right)^2 + \left(\frac{x}{\sqrt{3}}\right)^2 = 1$;

(ii) $x=0, 0 \leq y \leq 2$.

12. (a)(i) $\sqrt{\pi}$; (ii) $3a^2\pi/2$ (b)(i) $\pi\sqrt{\pi}$ (ii) $\frac{2\pi}{3}(1-\cos\alpha)$.

13. (a)(i) $\sin x$; (ii) $y = \frac{-\cos^3 x}{3\sin x} + \frac{c}{\sin x}$; (b)(i) $y_c = e^{-\sqrt{3}x/2}(A \cos 3x/2 + B \sin 3x/2)$;

(ii) $y_{p,1} = (1/3)e^{-\sqrt{3}x}$; (ii) $y_{p,2} = x/3 - 1/3\sqrt{3}$;

(iv) $y = e^{-\sqrt{3}x/2} \left((2/3\sqrt{3} - 1/3)\cos(3x/2) + (2\pi/3)e^{\sqrt{3}\pi/2}\sin(3x/2) \right) + e^{-\sqrt{3}x}/3 + 2x/3 - 2/3\sqrt{3}$.

14. (a) $f = 1 + (7/2)\ln 3$ at $(0,0)$, $f = (7/2)\ln 3$ at $(-1,0)$.

15. (b)(i) $2 + 1/80 - 1/(50(128))$; (c)(i) $1 - x^2/3 + x^4/18$;

(ii) $(x-1) - (1/2)(x-1)^2 + (1/2)(x-1)^3 - (1/2)(x-1)^4 + \dots$.

16. (a)(i) Both $1/2$; (ii) $4/9$; (iii) $4/9$; (iv) $13/18$; (v) 3.5 ; (b)(i) $\binom{10}{n} p^n (1-p)^{10-n}$; (ii) $\sqrt{10p(1-p)}$.

17. (a)(i) $\ln(2-\cos x) - \ln(3-\cos x) + c$; (ii) $-\ln x/(3x^3) - 1/(9x^3) + c$;

(iii) $(x/2)\sqrt{1-x^2} + (\arcsin x)/2 + c$; (b) $J = c^4/4$.

18. (a)(i) $(0, \pm 1, 0), (0, \pm 1, 2), (\pm 1, \pm 1, 1)$; (ii) $(0, -1, 0), (0, -1, 2), (\pm 1, 1, 1)$; (b) $\lambda = \pm 3$;

(c)(i) $B = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$ or $B = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$, $C = \begin{pmatrix} -4 & -3 \\ -3 & 4 \end{pmatrix}$; (ii) $\lambda = 5$, $\mathbf{v} = (1/\sqrt{10}) \begin{pmatrix} 1 \\ -3 \end{pmatrix}$,

$\lambda = -5$, $\mathbf{v} = (1/\sqrt{10}) \begin{pmatrix} 3 \\ 1 \end{pmatrix}$.

19. (a) 3; (b) $(b(\cos\theta)/a, b(\sin\theta)/a, 1)r dr d\theta$; (c) $\pi b a^2$; (d) $4\pi a^3$; (e) $\pi b a^2$.

20. (a) 1, $2/3$; (c) $\phi = xy + yz + zx + c$; (e) three straight line segments, parallel to x, y, z axes respectively, works.

Paper I

A1. $6x^2 \sin x^3 \cos x^3 - 4 \sin x^3 \sin 2x + 6x^2 \cos x^3 \cos 2x - 4 \cos 2x \sin 2x$.

2. $(x-2)(x+1)$, $-1 < x < 2$. 3. $\theta = \pi/4, 3\pi/4$. 4. 4, 4/3. 5. Max $(1/2)e^{-1/2}$, Min 0.

6. $-\sqrt{2}/\sqrt{3}, 1/\sqrt{3}$. 7. Three. 8. $-1/4 e^{-x^4} + c$. 9. $2t/(1-t^2), (1+t^2)^2/(1-t^2)^2$.

10. $y = -9x/16 + 25/16$.

B11. (a)(i) Re=-15/4, Im=2; (ii) Re=161/16, Im=-15;

(b)(i) hyperbolae z : (1/2,2) to (2,1/2), z^* : (1/2,-2) to (2,-1/2); (ii) horizontal line (-15/4,2) to (0,2) to (15/4,2); (iii) parabola (161/16,-15) to (0,-8) to (-4,0) to (0,8) to (161/16,15); (c) $\text{Re} + 4 = (\text{Im}/4)^2$.

12. (a)(i) $f(x) = 1 - x(1 - e^{-1/x})$; (ii) $R/2r_0$ and $1 - r_0/R$; (b) $\pi a^2 d/2$.

13. (a)(i) $y = A \cos x + B \sin x$; (ii) $y = 1/2 + 1/6 \cos 2x$;

(iii) $y = 1/2 + 1/6 \cos 2x - 2/3 \cos x - 1/3 \sin x$; (b) $y = 0$ or $y = (ce^{-8x} - 2)^{-1/4}$.

14. (a)(i) $dF = Gm_2 dm_1/r^2 + Gm_1 dm_2/r^2 - 2Gm_1 m_2 dr/r^3$; (ii) 1%; (b) $-2\sqrt{2}c/9$; (c) r .

15. (c) $5/4 + 3/8(x-2) + (x-2)^2/16 - (x-2)^3/32$; (d) $x < \ln 2$? (am not sure about this).

16. (a) Mean = 10.5, Var = 35/4; (b)(i) 1/36; (ii) 5/9; (iii) 5/12; (iv) 5/108; (c) 1/10.

17. (a) $f(x)$; (b)(i) $F = \ln(-x)$; (ii) $G = \ln|x| + c$; (c)(i) $F = \ln|x + \sqrt{x^2 - 1}|$;

(d) $F = \ln|\sin^2 x + \ln x| - \ln(\ln \pi)$.

18. (a)(i) $a = \pm\sqrt{3}$; (ii) any b except 0 or 3; (b)(i) $\mathbf{B} = \begin{pmatrix} 1 & -2 \\ -2 & 1 \end{pmatrix}$; (ii) $\mathbf{B}^{-1} = -(1/3) \begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix}$;

(d)(i) $b=0, c=0, d=\pm 1$; (ii) $b=0, c=0, d=1$; (e) $(1/2) \begin{pmatrix} 2a & b+c \\ b+c & 6 \end{pmatrix} + (1/2) \begin{pmatrix} 0 & b-c \\ c-b & 0 \end{pmatrix}$.

19. (a)(i) $p > -1$; (ii) $p \geq 0$; (b) $1/(3x^3)$ error $O(1/x^5)$.

20. (a) 1; (b) $\int_{e^{\alpha x}}^{e^{\beta x}} t^2 \cos(xt^2) dt + \sin(xe^{2\beta x})\beta e^{\beta x} - \sin(xe^{2\alpha x})\alpha e^{\alpha x}$; equals zero when $\alpha = \beta$;

(c) $I = (1/2)\ln(\beta/\alpha)$.

Paper II

A1. (a) $\lambda = \cosh u + \sinh u : (1 \ 1)^T, \lambda = \cosh u - \sinh u : (1 \ -1)^T$. 2. $(-1 \ 1 \ 0)^T$.

3. $y = (1/3)e^{2x} + ce^{-x}$. 4. $\sqrt{(\pi/a)}$ and zero. 5. (a) $v = p/m, H = p^2/2m + V(x)$.

6. $(-xz \ -yz \ x^2 + y^2)^T, -2z$. 7. $c = 1 - e^{-1}, e^{-N}$. 8. $(-1)^{n-1} x^{2(n-1)}$. 9. $a = -1$. 10. π .

B11. (a) $\sqrt{13}$; (b) $1/\sqrt{2}$; (c)(i) $\sqrt{(\mathbf{a}_2 \cdot \mathbf{a}_2 - (\mathbf{a}_2 \cdot \hat{\mathbf{n}}_2)^2)} = |\mathbf{a}_2 - (\mathbf{a}_2 \cdot \hat{\mathbf{n}}_2)\hat{\mathbf{n}}_2|$; (ii) $\sqrt{2/3}$;

(d)(i) $d(t) = \sqrt{(t+1)^2 + 1/2}$; (ii) Asymptotes are $|t+1|$; (iii) $t_{min} = -1$, when $d(t) = 1/\sqrt{2}$.

12. (b) $f(0,1) = 1, f(0,-1) = -1$; (c) $f=0$ is $y=x^2$ a parabola, $f=1$ is $(0,1)$ a point, $f=-1$ is $y=-1 \pm x$; $(0,1)$ is a maximum, $(0,-1)$ is a saddle point.

13. (a) $(0 \ 0 \ 2a)^T$; (b) zero; (c) $a=0$; (d) $2a\pi$; (e) $2a\pi$; (f) $\phi = -e^{-r^2}/2$ when $a=0$.

14. (a) (ii) $1/2$; (iv) 2 ; (b) $1/4 - \pi/16$.

15. (a) $y = x - x/\ln x$; (b)(i) $d^2v/dt^2 - 4v = 2$; (ii) $v = Ae^{2t} + Be^{-2t} - 1/2$; (iii) $2Ae^{2t} - 2Be^{-2t}$

(c)(i) $y = 3x^2/4 - 3x/4 + c + de^{-2x}$; (ii) $y = 3x^2/4 - 3x/4 + 11/8 - (3/8)e^{-2x}$.

16. (a)(i) $(ay/(x^2+y^2), -ax/(x^2+y^2), 0)$; (ii) zero; (iii) $\psi = a \ln r, \nabla \wedge \mathbf{A} = (0, -a/r, 0)$; (b) $5/2$.

17. (b)(i) $\mathbf{L} = \begin{pmatrix} 1 & \lambda & 0 \\ \lambda & 0 & -1 \\ 0 & 4 & 1 \end{pmatrix}, \mathbf{r} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}, \mathbf{b} = \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}$; (ii) solutions if $\lambda \neq \pm 2$;

(iii) $x = (3\lambda - 4)/(\lambda^2 - 4), y = (\lambda - 3)/(\lambda^2 - 4), z = (\lambda^2 - 4\lambda + 8)/(\lambda^2 - 4)$; (c)(i) $\begin{pmatrix} a & -2c \\ c & a \end{pmatrix}$;

(ii) $\begin{pmatrix} 0 & \mp 2 \\ \pm 1 & 0 \end{pmatrix}$.

18. (a) $a_n = \frac{1}{L} \int_{-L}^L f(x) \cos \frac{n\pi x}{L} dx$; (b) $a_0 = L, a_n = 0$ if n is even, $a_n = -4L/n^2\pi^2$ if n is odd;

(e) $\pi^4/96 - 1$.

19. (a)(i) $\begin{pmatrix} 4 & 3 \\ 3 & 2 \end{pmatrix}$; (ii) Saddle point; (b) $\alpha = 3^{1/4}m, \beta = 3^{-1/4}m$.

20. (a) $\Phi = \frac{T_0}{2} - \frac{T_0}{2} e^{-4\pi^2 \kappa t/L^2} \cos \frac{2\pi x}{L}$; (b) $\Phi = \frac{-8T_0}{\pi} \sum_{m=1}^{\infty} \frac{e^{-(2m-1)^2 \pi^2 \kappa t/L^2}}{(2m-1)[(2m-1)^2 - 4]} \sin \frac{(2m-1)\pi x}{L}$.