ECONOMICS TRIPOS Part I

Friday 13 June 2003 9 – 12

Paper 3

Quantitative Methods in Economics

*This exam comprises **four** sections. Sections A and B are on **Mathematics**; Sections C and D are on **Statistics**. You should do the appropriate number of questions from each section. The number of questions to be attempted is given at the beginning of each section.*

This written exam carries 80% of the marks for Paper 3. Section A carries 24% of the marks, Section B carries 16% of the marks, Section C carries 24% of the marks and Section D carries 16% of the marks.

You are permitted to use your own calculator where it has been stamped as approved by the University.

Cambridge Elementary Statistical Tables, graph paper, and a list of statistical formulae are provided.

You may not start to read the questions printed in the subsequent pages of this question paper until instructed to do so by the invigilator

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SECTION A – MATHEMATICS

Answer *four* questions

1. What are the extrema of:

$$f(x) = 2x^3 - 3x^2 - 12x + 12$$

over the interval $-3 \leq x \leq 3$

2. Euler's Theorem for the homogeneity of $f(x, y)$ is

$f(x, y)$ is homogeneous of degree r if and only if $xf_x + yf_y = rf(x, y)$

Use this to determine the homogeneity of the following functions

(a) $g(x, y) = x^3 + 2xy^2 - 3x^2y$

(b) $h(x, y) = \dfrac{3x^2}{2y} + \dfrac{y^2}{x}$

3. (a) Let $A = \begin{bmatrix} 2 & 3 & 3 \\ 3 & -1 & -2 \\ 5 & 1 & -1 \end{bmatrix}$, $B = \begin{bmatrix} 10 \\ 1 \\ 6 \end{bmatrix}$, $C = \begin{bmatrix} 3 & 6 & -3 \\ -7 & -17 & 13 \\ 8 & 13 & -11 \end{bmatrix}$

Find:

(*i*) AC
(*ii*) CA
(*iii*) CB

(b) Hence or otherwise solve for x, y and z

$$\begin{aligned} 2x + 3y + 3z &= 10 \\ 3x - y - 2z &= 1 \\ 5x + y - z &= 6 \end{aligned}$$

4. Find the critical points of the function

$$f(u, v) = 3u + 4v$$

subject to the constraint $g(u, v) = uv-3 = 0$ using the Lagrange multiplier method. Using the constraint transform $f(u, v)$ into a function of one variable and hence determine the nature of the critical points.

5. (a) If $f(x, y) = (x + 4)(4x + y)$ find f_x and f_y and evaluate these at the point (2, 1)

(b) Using $\frac{df}{dz} = f_x \frac{dx}{dz} + f_z$ or otherwise, find the total derivative of

$f(x, z) = 3x - z^2$ when $x = 2z^2 + z + 4$.

6. Let $R(Q)$ be the total revenue function and $C(Q)$ be the total cost function

$$R(Q) = 1000Q - 2Q^2$$

$$C(Q) = Q^3 - 62Q^2 + 1600Q + 1500$$

State the profit function $\Pi(Q)$ and find the profit-maximizing output Q.

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SECTION B - MATHEMATICS

Answer *one* question

1. An economy is described by the following equations:

$$C = 0.6(Y-T) + 320$$
$$I = -100r + 2680$$
$$T = 1650$$
$$G = 1650$$
$$M_S = 2960$$
$$M_D = 0.4Y - 500r + 3500$$

where Y is output, C is consumption, I investment, T taxes, G government spending, M_S money supply, M_D money demand and r the interest rate.

(a) Determine the equation of the *IS* curve

(b) Determine the equation of the *LM* curve

(c) Compute the equilibrium of this economy

(d) The size of the population of this economy is 25,200,000. Suppose the number of jobs offered (in millions) is $N = 0.003Y$. Compute the full employment level of output. Determine the value of G consistent with full employment.

2. A consumer has a utility function given by $U(x, y) = x^{1/2}y^{1/3}$, a budget B and faces prices P_x and P_y respectively.

(a) Use the Lagrange method to determine the optimal quantities of x and y for the consumer (*i.e.* his demand functions x and y)

(b) Compute how much the utility of the consumer changes when the price P_x increases by one unit

(c) What quantities will the consumer choose in the case $B = 25$, $P_x = 3$ and $P_y = 5$?

(d) Assume now the price of x increases and becomes $P_x = 5$. What are the quantities now chosen by the consumer?

SECTION C – STATISTICS

Answer *four* questions

1. Before embarking on a two-week tour of foreign companies the boss of a Cambridge firm asks Fred, a member of the staff, to water the plant in her office at the end of the first week. Fred is rather absent minded. There is a probability of 2/3 that he will forget to water the plant. The plant has in any case being showing signs of wilting and there is a probability of 1/2 that it will die, even if watered. If it is not watered the probability it will die is 3/4. If the boss returns to find that the plant is dead what is the probability that Fred forgot to water it?

2. A crisp firm has two plants A and B each possessing a machine for filling packets with crisps. The machine at plant A produces packets that have a mean weight of 170g and a standard deviation of 14g. The machine in plant B produces packets that have a mean weight of 175g with a standard deviation also of 14g. A sample of 49 crisps packets is known to have been produced wholly by one plant. But it is not known which of the two plants was involved. The mean weight of packets in this sample is 173.2 g.

   (a) Test the hypothesis, at the 5% level of significance, that the packets were produced by plant A
   (b) Determine the Type II error involved.

3. There is usually more than one way of organising and summarizing a large body of data". Briefly explain this statement giving examples. Does it matter?

4. Assume $X \sim N(\mu\ \sigma^2)$. Let $\overline{X}$ and s^2 denote respectively the mean and variance of a sample of size N taken from X.

   (a) Construct a 95% confidence interval for μ if:

   (i) N=64, $\overline{X}$ =400, σ^2 = 1600
   (ii) N=64, $\overline{X}$ =400, σ^2 is unknown and s^2 = 2500
   (iii) N=25, $\overline{X}$ =400, σ^2 is unknown and s^2 = 2500

   (b) Construct a 95% confidence interval for σ^2 if N= 81 and s^2 = 2500.

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5. (a) State the probabilistic results used to test the hypotheses concerning the population correlation coefficient ρ that:

   (i) $\rho = 0$
   (ii) $\rho = k$ where $k \neq 0$ and $-1 < k < 1$

   (b) If a correlation coefficient *r* of a sample of size 52 is calculated as 0.4 construct a 95% confidence interval for the population correlation coefficient ρ.

6. A researcher estimates the following relationship, using the method of ordinary least squares regression:

   $$y = \alpha + \beta x + \varepsilon \qquad \text{(i)}$$

   and reports the estimates $\hat{\alpha}$ and $\hat{\beta}$ of α and β respectively.

   When criticised for getting the dependent and independent variables mixed up the researcher reasons that the required estimates can be derived from those already obtained. Specifically it is reasoned that if the correct formulation is:

   $$x = \gamma + \delta y + \theta \qquad \text{(ii)}$$

   then the latter is but a reformulation of (i) as:

   $$x = -\alpha/\beta + (1/\beta)y - (1/\beta)\varepsilon \qquad \text{(iii)}$$

   so that the required estimates of γ and δ can be determined as $-\hat{\alpha}/\hat{\beta}$ and $1/\hat{\beta}$

   Critically comment upon the validity of the researcher's strategy.

7. The results of two surveys of secondary school economics students, questioned about their wish to further their studies at university, are reported below:

Aiming to study economics at university?	Year of survey:1975	Year of survey:1995
Yes	100	12
No	200	36
Don't know	0	0

   (a) Use these results to test the hypothesis that university enrolments in economics are declining.

   (b) Comment upon your findings and analysis.

SECTION D – STATISTICS

Answer *one* question

1.

x	y
10	10.14
8	9.14
13	9.74
9	9.77
11	10.26
14	9.10
6	7.13
4	4.10
12	10.13
7	8.26
5	5.74

(a) Using the data given in the table estimate the parameters of the following equation:

$$y = \alpha + \beta x + \varepsilon$$

(b) Interpret your results

(c) On the basis of your results what value of y would you predict for x = 16?

(d) Using your estimates of α and β, estimate the disturbance ε for each value of x, and plot your estimated values of ε against the corresponding value of x.

(e) What do the results of d) suggest about the validity of the results or predictions reported under a), b) and c)?

(f) Comment on your analysis and suggest how you might have improved your analysis.

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2.

UK Inward passenger movement by air: 1996 Quarter 1 to 2000 Quarter 2			
Quarter	**Passengers (1000s)**	**Quarter**	**Passengers (1000s)**
1996 Q1	10.3	1997 Q3	18.1
1996 Q2	13.6	1997 Q4	13.1
1996 Q3	16.6	1998 Q1	11.9
1996 Q4	12.1	1998 Q2	16.3
1997 Q1	11.0	1998 Q3	19.9
1997 Q2	14.9	1998 Q4	14.3
Source: Monthly Digest Of Statistics			

(a) You are asked to decompose the data in the table using the following model:

$$D = T + C + S + R$$

Where: D refers to the data series, T to the trend, C to the cyclical component, S to the seasonal factor, and R to the random residual

(i) Using the method of moving average estimate (T+C)
(ii)Using your results in (i) calculate the four seasonal factors S
(iii) Comment on the exercise

(b) A researcher uses quarterly data to see if food production P_t is related to food consumption a period earlier C_{t-1}. In order to capture the influence of seasonal factors the following dummy variables are constructed:

$Q_1 = (1, 0, 0, 0, 1, 0, 0, 0, 1, 0, 0, 0, 1, 0,\ldots\ldots)$
$Q_2 = (0, 1, 0, 0, 0, 1, 0, 0, 0, 1, 0, 0, 0, 1, 0,\ldots\ldots\ldots)$
$Q_3 = (0, 0, 1, 0, 0, 0, 1, 0, 0, 0, 1, 0, 0, 0, 1, 0,\ldots\ldots\ldots)$
$Q_4 = (0, 0, 0, 1, 0, 0, 0, 1, 0, 0, 0, 1, 0, 0, 0, 1, 0,\ldots\ldots\ldots)$

The estimated equation (with a constant suppressed) is:

$$P_t = \alpha_1 Q_{1t} + \alpha_2 Q_{2t} + \alpha_3 Q_{3t} + \alpha_4 Q_{4t} + \beta C_{t-1} + \varepsilon_t$$

And the following results are obtained (with standard errors shown in parentheses):

$$\hat{P}_t = \underset{(1.45)}{-17.05 Q_{1t}} \underset{(0.64)}{- 5.86 Q_{2t}} \underset{(0.88)}{- 4.13 Q_{3t}} + \underset{(2.6)}{11.32 Q_{4t}} + \underset{(0.45)}{1.43 C_{t-1}} \qquad R^2 = 0.94$$

After accidentally deleting these results the researcher again runs the regression using the same data. This time, though, the researcher includes a constant term α and drops the fourth quarter dummy variable Q_{4t}, and so estimates the following equation:

$$P_t = \alpha + \delta_1 Q_{1t} + \delta_2 Q_{2t} + \delta_3 Q_{3t} + \gamma C_{t-1} + \varepsilon_t$$

What estimates will be obtained for: R^2, γ, α, δ_1, δ_2 and δ_3

3. You wish to test whether two populations have the same mean, in conditions where you only possess small samples (fewer than 30 observation points) from each population.

(a) What assumptions must you make in order to proceed?
(b) What statistical results will you use?

Samples are taken of profit rates of small-scale electronic firms in two different regions of the country. The results are shown in the following table.

Region A	10, 11, 12, 13, 14, 15, 12, 10, 12, 11, 12
Region B	5, 5, 7, 5, 15, 7, 16, 13, 16, 6, 5

(c) For each regional sample determine:

(i) the sample size
(ii) the mean
(iii) the variance
(iv) the standard deviation
(v) the mode
(vi) the median
(vii) the range

(d) Test, at the 5% level of significance, the hypothesis that the population means are identical where the alternative hypothesis is that the population mean of region A is higher.

(e) If your sample statistics are as reported in c) excepting that that the sample size (identical for each sample) is found to be the minimum at which your null hypothesis is rejected, what is this sample size?

(f) Does the information provided in part c) throw any light on whether the assumptions listed in part a) are satisfied (do a formal test if you can).

(g) Interpret your findings

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4 The mean and standard deviation of the holding period returns on short, and long term UK government debt (per cent, per annum) computed over a given period of time, and also for two sub-periods are shown in the table below:

Holding Period Returns On UK Government Debt (per cent, per annum)

Period	Short Term		Long Term		Coefficient of correlation between long term and short term rates.
	Mean	Standard Deviation	Mean	Standard Deviation	
1981M1- 1990M12	11.22	16.59	11.84	39.17	0.847
1991M1- 2000M6	8.55	10.37	13.16	30.15	0.728
1981M1- 2000M6	9.92	13.94	12.48	35.00	0.800

(a) Test the hypothesis that mean long-term return is greater than the mean short-term return in each of the two sub-sample periods.
(b) Construct the 95% confidence interval of the expected (mean) gap between the long-term and the short-term returns over the full sample period 1980M1-2000M6
(c) Discuss the economic interpretation of your results.

END OF PAPER