ECONOMICS TRIPOS PART I

Friday 16 June 2006 9 – 12

Paper 3

QUANTITATIVE METHODS IN ECONOMICS

*This exam comprises **four** sections. Sections A and B are on **Mathematics**; Sections C and D are on **Statistics**. You should do the appropriate number of questions from each section. The number of questions to be attempted is given at the beginning of each section.*

*Answers from the **Mathematics** and the **Statistics** Sections must be tied up in separate bundles, with the letter of the Section written on **each** cover sheet.*

This written exam carries 80% of the marks for Paper 3. Section A carries 24% of the marks, Section B carries 16% of the marks, Section C carries 24% of the marks and Section D carries 16% of the marks.

STATIONERY REQUIREMENTS	SPECIAL REQUIREMENTS
2 x 20 Page booklets	*List of statistical formulae*
Metric graph paper	*New Cambridge Elementary Statistical Tables*
Rough Work Pad	*Approved calculators allowed*
Tags	

You may not start to read the questions printed in the subsequent pages of this question paper until instructed to do so by the invigilator

SECTION A – MATHEMATICS

Answer **four** questions

1

$$A = \begin{pmatrix} a & 1 & 1 \\ 1 & a & 1 \\ 1 & 1 & a \end{pmatrix}$$

(a) For what value(s) of a is the matrix A singular?

(b) Find the adjoint matrix of A.

(c) Hence, or otherwise, find the inverse of A.

2 Suppose a firm produces an output Y according to the following production function:

$$Y = (K^{1/2}+L^{1/2})^2,$$

where K is capital and L is labour. Suppose too that the firm wishes to minimise its costs given some production target Y^*. If the wage rate is w and the rental rate is r:

(a) Use the Lagrangian method to determine the optimality conditions of the firm.

(b) Determine the firm's demand functions in terms of Y^*, r and w.

3 Consider the problem of minimizing the function $f(x, y) = x^4 + 4y^4$ subject to the constraint $2x + y = 5$.

(a) Find the critical point using the method of Lagrange.

(b) Using the substitution method, verify that the critical point you have found is indeed the constrained minimum of the function.

4 Solve the following integrals

(a) $\int \left(\frac{a}{x^3} + \frac{b}{x^2} + \frac{c}{x} \right) dx$.

(b) $\int_0^{\frac{\pi}{4}} \sin 2x \cos 2x dx$.

(c) $\int x^2 e^x dx$.

5 A profit maximizing monopolist faces an inverse demand schedule $p = 36 - 2q$, where p is price and q is demand. The total cost of producing q units is $q^3 - 6q^2 + 20q + 50$ if $q > 0$, and zero if $q = 0$.

(a) Find the profit maximizing quantity and price.

(b) What happens if the fixed cost increases to 70?

6 (a) Find the derivatives dy/dx of:

(i) $y = \ln\{ x^2/ (e^{2x} + 1)\}$.

(ii) $y = (x + 4)^{1/5} (x - 5)^{4/5}$.

(b) Find the partial derivatives of:

$f(x,y) = (x^3 - 6xy + y) (x - 2y)^2$.

[TURN OVER FOR SECTION B

SECTION B - MATHEMATICS

Answer **one** question

1 An individual has the following utility function for consumption of goods x and y:

$$U(x,y) = \tfrac{3}{4} \ln x + \tfrac{1}{4} \ln y.$$

The prices of goods x and y are p_x and p_y respectively. Suppose the individual initially owns 5 units of good x and 10 units of good y. Income is generated by selling these endowments, and there is no other source of income.

(a) Write down the individual's budget constraint and solve the utility maximization problem using the method of Lagrange.

(b) Calculate how much of the two goods the individual consumes when facing the following prices:

(i) $p_x = 6$ and $p_y = 2$.
(ii) $p_x = 9$ and $p_y = 0.5$.

(c) What are the individual's net purchases of each good in each of the two cases in (b)? (i.e. the difference between consumption and the initial endowment). Find the range of relative prices p_x / p_y for which the individual makes positive net purchases of good y.

(d) Derive the maximized level of utility as a function of prices p_x and p_y.

(e) Find the range of relative prices p_x / p_y for which the maximum utility is decreasing in the price of good y. Explain your answer intuitively.

2 Consider a closed economy described by the following equations:

$$C = C_0 + cY^d,$$
$$Y^d = Y - T,$$
$$I = I_0 - br,$$
$$G = \overline{G},$$
$$T = \overline{T},$$
$$M^s = \overline{m},$$
$$M^d = M_0 + kY - hr,$$

where C is consumption, Y^d is disposable income, Y is aggregate output, T is lump-sum taxes, I is investment, r the interest rate, G is government spending, M^s is the money supply, and M^d is money demand. All parameters are positive and c is less than one. C_0, I_0 and M_0 are constants; $\overline{G}, \overline{T}$ and $\overline{m}$ are policy variables.

(a) Derive an expression for the IS curve.

(b) Derive an expression for the LM curve.

(c) Using matrix algebra or otherwise, obtain the equilibrium level of output Y and interest rate r. Find an expression for the fiscal policy multiplier (i.e. the effect of changing G on Y).

(d) Now suppose that the original expression for money demand is replaced by:

$$M^d = kY + af(r),$$

where $f(r) = (1/(\sqrt{r}))$ and a is a positive constant. Calculate a first-order (i.e. linear) Taylor expansion of the function $f(r)$ around $r = r_0$. Hence write down a linear approximation of the money demand equation.

(e) What is the fiscal policy multiplier when the linear approximation of money demand is used? How is the size of the multiplier affected by the initial interest rate r_0? Explain the intuition behind your answer.

[TURN OVER FOR SECTION C

SECTION C – STATISTICS

Answer **four** questions

1 (a) Consider the following three models

$y = a - b/x$ (A)
$y = a + bx$ (B)
$\ln(y) = a + b \ln(x)$ (C)

In each of these models determine the elasticity of y with respect to x when $x = 1$.

(b) The following model is estimated for country X

$$\ln \hat{M}_t = \hat{\alpha} + \underset{(0.2)}{1.7} \ln Y_t + \underset{(0.6)}{0.5} \ln P_t \qquad R^2 = 0.9.$$

( ) denotes standard errors, $\hat{\alpha}$ is a constant, M_t is the volume of imports at time t, Y_t is total final expenditure in constant prices at time t, and P_t is the ratio of domestic to import prices, also at time t.

Interpret these results. How might the government of X pursue a sustainable strategy of demand led growth?

2 A company submits tenders for three contracts and estimates the probabilities of success in each case to be 0.1, 0.2 and 0.6, all independent events.

(a) Determine the probabilities of winning

(i) All three contracts.
(ii) At least one contract.
(iii) Precisely two contracts.

(b) A pack of fifty-two cards contains four aces, four twos, four threes, …four tens, four jacks, four queens and four kings. If a set of five cards is drawn (without replacing any cards) what is the probability that at least two cards have the same value (e.g. are both fours or both kings, etc).

3 Consider the following 'real wage resistance' model

$$\Delta \ln\{W_t(1 - \tau_t)\} = a + b\,\Delta \ln P_t + \varepsilon_t$$

where W_t is nominal wages and salaries per head of the working population at time t, τ_t is the average tax rate on wages and salaries, P_t is the consumer price index, ε_t is an error term and Δ is the first difference operator, so that $\Delta X_t = X_t - X_{t-1}$.

(a) Interpret the model if the null hypothesis is that $b = 1$.

(b) When a regression analysis on the model is carried out the following results are obtained:

	A	***b***
Estimate	*0.18*	*0.92*
Standard error of the estimate	*0.006*	*0.60*

(i) Use the results to test the "real wage resistance hypothesis" that processes of wage bargaining seek to maintain a real growth in average post tax real wages and salaries.
(ii) Suppose instead you had tested the null hypothesis that $b = 0$. What would you have concluded?
(iii) Discuss your findings.

4 The number of hours that a type of light bulb lasts is normally distributed with a mean of 560 hours and a standard deviation 50 hours.

(a) In a random sample of 150 bulbs how many can be expected to last:

(i) more than 640 hours.
(ii) between 540 and 600 hours.

(b) What is the bulb life below which only 2.5% of bulbs will fail?

(c) What should the mean life be if only 2.5% of bulbs have a life below 500 hours?

(d) Explain the notion of a confidence interval.

(e) In a random sample of 120 bulbs the number failing to meet some quality criterion is 12. Place a 95% confidence interval on the proportion failing to meet specification.

5 (a) If x and y are *independent* random variables, and given that

$\text{Var}(x) = \text{E}[X - \text{E}(X)]^2$ show that

(i) $\text{Var}(ax + by) = a^2\text{Var}(x) + b^2\text{Var}(y)$.
(ii) Using your result to (i) show that: $\text{Var}(x - y) = \text{Var}(x + y)$.

(b) If x and y are *independent* random variables, and $z = 3x + 2y$, and $\text{E}(x) = 1$ and $\text{E}(y) = 3$, and $\text{Var}(x) = \text{Var}(y) = 4$ then determine:

(i) E(z)
(ii) Var(z)

6 A company produces ready-made porridge, which it sells only in countries that have cold climates. The company wishes to expand its sales to warmer countries, but is not sure if there will be enough demand. A sample of 100 people is carried out in a warmer country where each person is asked if they are likely to buy the porridge regularly if it were to become available. Of those sampled, 45% reported that they would buy porridge regularly. To assess the importance of these results, the company used data from a sample taken in a cold country before the porridge was successfully introduced. In this earlier trial, 70% of a sample of 98 reported that they would buy the porridge regularly if it were to become available. Test the hypothesis that the proportion reporting that they would buy the porridge regularly is the same.

It is decided that the porridge will be put on sale in the warmer areas as long as the percentage of those who buy it regularly is no more than 20% lower than in the colder countries. On the evidence of the data collected so far, is it likely that the firm will market the porridge in warmer regions. What problems might there be in making this last inference?

SECTION D – STATISTICS

Answer **one** question

1 Focusing on a number of countries over a 16-year period a researcher wishes to examine Verdoorn's law, the hypothesis that, for manufacturing sectors, the rate of productivity growth is influenced by the rate of output growth. Wishing to undertake a cross-section analysis, and noticing that, for each country, the data on output and productivity each reveal two cyclical peaks the researcher records these (two) peak values of the variables, as well as the number of years between the (two) successive peaks of the cycles. The data obtained are reported in the table below.

Country	*Number of years between cyclical peaks*	*Index number of the level of manufacturing output at the second peak with the value at the first peak set to* 100	*Index number of the level of manufacturing productivity at the second peak with the value at the first peak set to* 100
A	10	197.4	147.7
B	8	138.3	123.4
C	15	174.2	125.2
D	12	146.8	123.4
E	14	149.0	120.9
F	10	133.0	117.4
G	12	139.1	122.8
H	9	125.8	121.5
I	10	125.9	124.6
J	10	118.5	106.7

(a) Explain why the researcher records the data shown rather than the first and last levels achieved over the 16-year period. How else might he or she have proceeded?

(b) Determine (to 3 decimal places) the average growth rate for both output and productivity for each country for the period between peaks.

(c) Assess Verdoorn's Law using the data available to you.

(d) How would you respond to the criticism that your results are not reliable because countries *A* and *J* are non-European whilst all the others are European.

2 Rates of change of the Consumer Price Deflator for Canada and the UK between 1989 and 2002

Year	*Canada*	*UK*
1989	2.9	4.9
1990	2.5	4.8
1991	2.7	6.0
1992	2.4	1.5
1993	2.5	1.8
1994	2.0	0.2
1995	2.6	2.3
1996	2.4	2.3
1997	1.8	1.6
1998	1.6	1.5
1999	1.3	1.7
2000	0.7	1.9
2001	1.2	2.3
2002	1.3	2.2

(a) Using the data given in the table above, test the hypothesis that the rates of inflation of Canada and the UK are uncorrelated.

(b) Given your answer to (a), test the hypothesis that the mean value of inflation was the same for Canada and the UK for the period 1989 to 2002.

(c) A researcher wants to know if there is a tendency for the rate of inflation in each country to fall over the period. To test this she runs regressions of the rate of inflation against time for each country. What does the researcher conclude? What might the problems be of testing for a fall in this way?

(d) Test the hypothesis that the rate of inflation is more variable for the UK before 1997, i.e., it is more variable in the period between 1989 and 1996 than in the period between 1997 and 2002.

(e) Discuss the methods used and, where appropriate, qualify your results.

END OF PAPER