ECONOMICS TRIPOS PART I

Friday 12 June 2009 9 to 12

Paper 3

QUANTITATIVE METHODS IN ECONOMICS

This exam comprises four sections. Sections A and B are on Mathematics; Sections C and D are on Statistics. You should do the appropriate number of questions from each section. The number of questions to be attempted is given at the beginning of each section.

Answers from the Mathematics and the Statistics Sections must be tied up in separate bundles, with the letter of the Section written on each cover sheet.

This written exam carries 80% of the marks for Paper 3. Section A carries 24% of the marks, Section B carries 16% of the marks, Section C carries 24% of the marks and Section D carries 16% of the marks.

STATIONERY REQUIREMENTS	SPECIAL REQUIREMENTS
2 x 20 Page booklet	*List of statistical formulae*
Metric Graph Paper	*New Cambridge Elementary Statistical Tables*
Rough Work pads	
Tags	*Approved calculators allowed*

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

SECTION A - MATHEMATICS
Answer **all** questions in this Section

A1 If the function $f(x,y)$ is a strictly concave function of x and y it possesses the following properties

(i) The Hessian matrix of second derivatives H satisfies

$$\begin{aligned} d'Hd &\leq 0 \ for\ all\ d \neq 0\ and \\ d'Hd &= 0\ for\ at\ most\ a\ finite\ set\ of\ points\ (x,y) \end{aligned}$$

(ii) For any λ satisfying $0 < \lambda < 1$

$$f(\lambda x^0 + (1-\lambda)x^1, \lambda y^0 + (1-\lambda)y^1) > \lambda f(x^0, y^0) + (1-\lambda)f(x^1, y^1)$$

(a) Show that the production function

$$q = f(x,y) = x^\alpha y^\beta$$

(with $\alpha > 0, \beta > 0$) is a strictly concave function of (x,y) provided $\alpha + \beta < 1$.

(b) Show that if $f(x,y)$ is a strictly concave function, and the point (x^0, y^0) is a critical point, then (x^0, y^0) is a global maximum.

A2 A seller on an Internet auction site is trying to sell an object. The exact quality of the object is unknown, but both you and the seller know that quality q varies between 0 and a known positive value a. The distribution of quality is described by the probability distribution function

$$f(q) = \frac{2}{a} - \frac{2}{a^2}q$$

(a) Verify that $f(q)$ satisfies the conditions

$$\begin{aligned} f(q) &\geq 0 \ for\ all\ q \leq a \\ \textstyle\int_0^a f(q)dq &= 1 \end{aligned}$$

and is therefore a valid probability distribution function.

(b) Suppose that you know that the maximum possible quality level, a, is equal to 9. If you are only concerned about the expected quality of the object you purchase, and are prepared to pay £10 per unit of expected quality, how much should your maximum bid be?

A3 A simple model of unemployment is as follows:

$$\begin{aligned} L &= U_t + E_t \\ R_t &= \frac{U_t}{L} \\ U_t &= (1-f)U_{t-1} + sE_{t-1} \end{aligned}$$

where L is the size of the labour force, U_t and E_t are the numbers of, respectively, unemployed and employed workers at time t, R_t is the unemployment rate at time t, f is the job finding rate and s is the separation (or job-loss) rate. The job-finding and separation rates, and the size of the labour force are all exogenous to the model and assumed constant.

(a) Derive the first-order difference equation that describes the evolution of the unemployment rate. What is the equilibrium unemployment rate if $s = 3\%$ and $f = 17\%$?

(b) Suppose the labour market is in equilibrium and a positive shock raises the job finding rate to 27%.What is the unemployment rate 3 periods later? In the absence of further shocks, how many periods pass before the unemployment rate has changed by half the difference between the initial unemployment rate (from part a)) and the new equilibrium.

A4 The population of a city t years from now is given by

$$f(t) = 40 - \frac{8-G}{t+2}$$

where G is a policy instrument (growth promoting public expenditures). Currently $G = 1$.

(a) Assuming there is no policy change, estimate the population in 5 years with a second-order Taylor series approximation around the current ($t = 0$) population. Comment on whether this a very good approximation (and why or why not).

(b) Use a linear (first-order Taylor series) approximation around the exact population in 5 years to calculate the decrease in time required to reach that population if public expenditures are doubled.

SECTION B - MATHEMATICS
Answer **one** question

B1 Consider the following simple macroeconomic model of a closed economy

$$\begin{aligned} Y &= C + I + G \\ C &= \bar{C} + c(1-t)Y \\ I &= \bar{I} + aY - br \\ G &= \bar{G} + \theta tY \\ M_d &= \alpha Y - \beta r \\ M_d &= M_s \end{aligned}$$

where Y, C, I and G respectively denote national income, consumption, investment and government expenditure. t denotes a proportionate tax rate on income, used to finance government expenditure. M_d, M_s and r denote respectively money demand, money supply and the interest rate. $\bar{C}, I, \bar{G}$ and the parameters c, a, b, α, β and θ are positive constants. The parameter θ summarises the government's fiscal rule relating discretionary government expenditure to tax revenue. Assume $a + c < 1$ and $0 < \theta < c$.

(a) Derive a system of two equations, one giving the relationship between r and Y consistent with goods market equlibrium (the IS curve), the other giving the relationship between r and Y consistent with money market equlibrium (the LM curve). Write this system of equations using matrix notation.

(b) Derive an expression for the value of Y which is consistent with a balanced budget.

(c) Derive the determinant associated with this system of equations.

(d) Hence, or otherwise, solve for the values of r and Y consistent with macroeconomic equilibrium. Evaluate the effect on the interest rate of an increase in the autonomous component of government expenditure $\bar{G}$. How does this vary with changes in the fiscal rule parameter θ? Explain the economic intuition behind your answer.

B2 A consumer consumes two commodities, x and y. The consumer has utility function $U(x, y) = (x - A)^2 y$ (where A is age, a shift parameter) and faces the budget constraint $p_x x + p_y y \leq m$ (where p_x and p_y are prices and m is money income.)

(a) Write down the Lagrangean expression for this problem and derive the first-order conditions which characterize a utility maximizing choice.

(b) Solve for the utility maximizing choices x^* and y^* (as functions of p_x, p_y, m and A).

(c) Solve for the Lagrange multiplier (λ) and give it an economic interpretation.

(d) Totally differentiate the first-order conditions and use Cramer's rule to determine the comparative static $\frac{\partial x^*}{\partial A}$.

SECTION C - STATISTICS

Answer ***all four*** questions

C1 (a) Various descriptive statistics of the following data set are calculated:

2 4 6 7 8 20 24 24 37 42 51 52 60 66 80

It is later discovered that an error was made, and that the 80 should really have Been entered as an 8. Which, if any, of the following measures need revising?

(i) Mean
(ii) Median
(iii) Mode
(iv) Variance
(v) Range

(b) (i) State Bayes' Theorem.

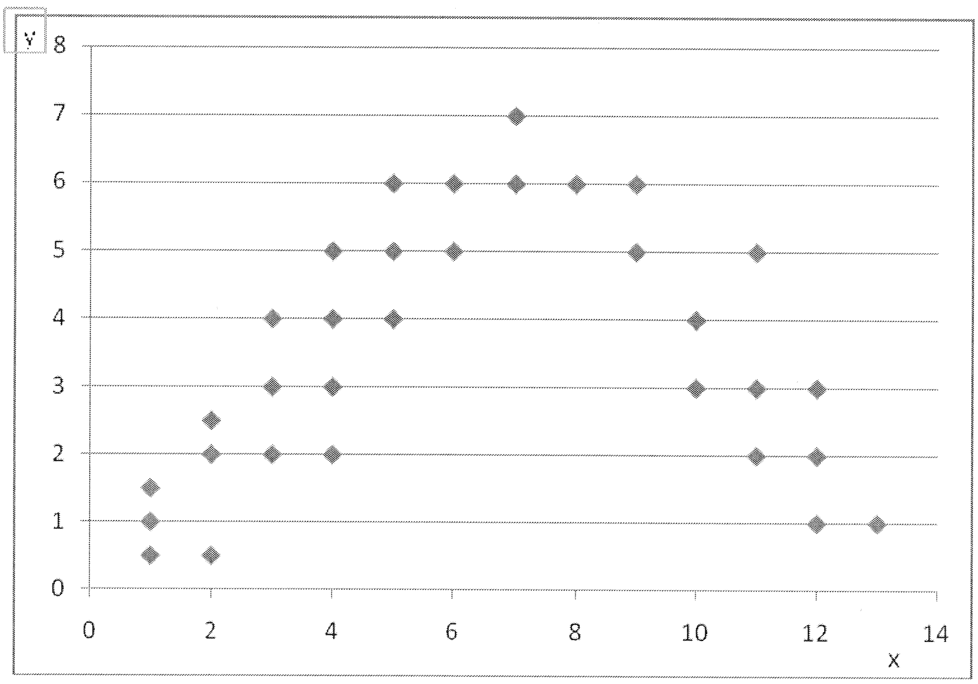
A statistics exam includes a question on Bayes' Theorem. Some Students have revised the topic recently, others have not. Of those who have revised 90% get it correct. Of those who have not revised it 10% get it right. Overall 70% get the question right.

(ii) What is the proportion of students that have revised the question recently?

(iii) Given that a specific student got the question correct, what is the probability that he or she revised?

C2

Plot 1



(a) Do you consider x and y in plot 1 above to be independent? Explain.

(b) Estimate (by guessing) an approximate value for the correlation coefficient for the shown data sample. Does your estimate suggest a limit to correlation analysis?

(c) The total data sample, call it A, is split into two subsamples with sub sample B including all observations where $x < 8$, and sample C including the remaining observations. You are asked to regress y on x. What functional forms might you try if the data to be used are the

(i) Total sample A
(ii) Sub sample B
(iii) Sub sample C.

(d) If in a sample of size 23 a correlation coefficient is calculated as .4, test the hypothesis that the population coefficient is 0.

(e) Determine a 95% confidence interval estimate of the population correlation coefficient.

C3 (a) A manufacturer of batteries wants to be reassured that fewer than 5% of the batteries produced are defective. When 300 are randomly selected from a large stock and each one tested, 10 are found to be defective. If a 1% significance level for a statistical test is used, is the manufacturer reassured?

(b) A researcher tests the (null) hypothesis that the proportions in two populations A and B that manifest a particular characteristic are the same. On the way to give a talk on the results the researcher discovers that the data have been left behind. So the researcher intends instead just to present the results. This researcher remembers that the sample proportion of observations drawn from population A manifesting the relevant chacteristic was 0.45; that the size of the sample drawn from A was precisely twice the size of that drawn from B; that the proportion of the sample drawn from population B that manifested the characteristic was 2/3 the value calculated for sample A; that a two tail test had been adopted employing a 5% significance level, and finally, given the noted ratios of sample sizes, and sample proportions, that the sample sizes had been the smallest possible at which the null hypothesis would have been rejected. What are the respective sample proportions and sample sizes?

C4 (a) Briefly discuss the logic of statistical hypothesis testing.

(b) A statistics lecturer generates a sample of observations using the following population model:

$$Y = a + bX + \text{random error term}$$

where the error term is normally distributed with constant variance.

The lecturer informs a set of students that in the sample the standard error of the estimate of the slope coefficient is 0.1 and that the slope coefficient itself, i.e. b, is either 0.5 or 0.7. The students are asked to test the hypothesis that b is 0.5 at the 5% level of significance.

(i) Explain carefully whether a 1 tail test or a 2 tail test should be employed.

(ii) Discuss the problems involved in reducing a type one error

(iii) Determine the type two error.

SECTION D - STATISTICS

Answer ***one*** question only

D1 (a) Briefly discuss the notion of independence as it figures in statistical analysis

(b) The **covariance** of 2 variables is a measure of how much two variables change together (the variance is a special case of the covariance when the two variables are identical). If the two variables are independent of each other the covariance between them is 0. The covariance between two real-valued random variables X and Y, with expected values $E(X) = \mu$ $E(Y) = \nu$ is defined as

$$\text{Cov}(X, Y) = E((X - \mu)(Y - \nu))$$

Where E is the expected value operator.

(i) If X and Y are independent of each other show that $V(aX + bY)$, i.e., the variance of $aX + bY$, is given by $a^2V(X) + b^2 V(Y)$.

(ii) Determine $V(X - Y)$ and $V(X + Y)$ when X and Y are independent of each other.

(c) A researcher wishes to compare the mean daily sales of two restaurants located in the same street in Cambridge. This researcher thus records total sales for each of 12 randomly selected days over a 3 month period. The data obtained are shown in the following table:

Daily Sales in the Two Restaurants (£s)

	Day	**Restaurant A X**	**Restaurant B Y**	**Differences in sales D**
1	Monday	1520	1360	160
2	Monday	1380	1270	110
3	Tuesday	1752	1656	96
4	Tuesday	1970	1880	90
5	Wednesday	2120	1960	160
6	Wednesday	2020	1840	180
7	Thursday	2800	2580	220
8	Thursday	2900	2640	260
9	Friday	4002	3700	302
10	Friday	3800	3460	340
11	Saturday	4300	4108	192
12	Saturday	4166	3960	206

The mean daily sales of Restaurant A are calculated as 2727.5 with variance 1183680.8 and the mean daily sales of Restaurant B are calculated as 2534.5 with variance 1066092.5

TURN OVER

(i) The researcher first ignores the data shown in the final column of the table, and tests the hypothesis that the population means are the same treating the figures on sales for restaurants A and B as two independent samples. How does the researcher proceed, and what does he or she conclude? Can you assess the validity of any assumptions made?

(ii) The researcher next focuses only on the 'differences in sales' figures, treating these daily differences D as a random sample of all daily differences, past and present. The researcher again tests the hypothesis that the mean differences are 0 now using only the final column data on sales differences. How does the researcher proceed, and what does the researcher conclude?

(iii) Compare and discuss your various findings.

D2 (a) Describe the method of ordinary least squares estimation, making clear the conditions under which its application is appropriate.

(b) Let $Y = a + bX + \textit{random error term}$ (A)

(i) Find the least-squares estimates of the parameters a and b using the following data: X = (2, 4, 6, 8) Y = (0, 1, 1, 2).

(ii) What are your predicted values of Y when X = 1, and 10?

(c) Let $X = c + d\,Y + \textit{random error term}$ (B)

(i) A researcher uses the estimates of the coefficients a and b of model (A) obtained in part (b) to infer estimates c and d of model (B). What values are obtained?

(ii) A second researcher does a least squares regression of X on Y directly, according to model (B), using the data given in (b) above , and so determines least squares estimates of the parameters c and d. How do these compare with the estimates obtained by the first researcher? Comment.

(d) The researcher discovers that the results achieved in a least square regression analysis are very sensitive to the inclusion of two outlying observations. How might he or she reasonably react?

(e) Interpret (theoretically and statistically) the following set of estimates

$$\widehat{\Delta \ln W_t} = \underset{(0.005)}{0.03} + \underset{(0.02)}{0.98}\,\Delta \ln L_t + \underset{(0.01)}{1.01}\,\Delta \ln P_t + \underset{(0.04)}{0.94}\,\Delta T_t + \underset{(4.7)}{9.6}\,(1/u_t)$$

$R^2 = 0.87$

Where W represents *Wages and Salaries*; L is *Employees in Employment*; P is the *Retail Price Index*; T the *tax rate on Wages and Salaries*; u the *unemployment rate*; the subscript t indicates time; ( ) denotes the standard error of the parameter estimate immediately above it; and R^2 is the coefficient of determination.

(f) Discuss briefly how you might visually detect positive and negative error autocorrelation.

END OF PAPER