ECONOMICS TRIPOS PART I

Friday 11 June 2010 9:00-12:00

Paper 3

QUANTITATIVE METHODS IN ECONOMICS

This exam comprises four sections:
Sections A and B are Mathematics; Sections C and D are Statistics.

You should do the appropriate number of questions from each Section.
The number of questions to be attempted is given at the beginning of each Section. Answer all parts of the question.

Answers from the Mathematics and the Statistics Sections must be tied up in separate bundles with the letter of the Section written on each cover sheet.

This written exam carries 80% of the marks for Paper 3. Section A carries 24% of the marks; Section B carries 16% of the marks; Section C carries 24% of the marks and Section D carries 16% of the marks.

Write your ***number*** *not your name on the cover of each booklet.*

STATIONERY REQUIREMENTS	SPECIAL REQUIREMENTS
20 Page booklet x 2	*Approved calculators allowed*
Metric graph paper	*List of statistical formulae*
Rough work pads	*New Cambridge Elementary*
Tags	*Statistical Tables*

You may not start to read the questions
printed on the subsequent pages of this question paper
until instructed that you may do so by the Invigilator

SECTION A

Answer **ALL FOUR** questions.

1 Consider the function

$$y = f(x) = x^{1/2}, \quad x > 0$$

and considering only $f(x) > 0$.

(a) Find the second order Taylor series approximation around x_0 (ignore all terms higher than the second order).

(b) Show that if you use the approximation in (a) then using the differential $dy = f^{'}(x)dx$ to estimate the impact on y of a change in x of amount dx leads to an overestimate of the actual change in y.

2 Let A be the square matrix given by

$$A = \begin{bmatrix} 1 & 5 \\ \alpha & -2 \end{bmatrix}$$

(a) Compute the inverse matrix A^{-1}. For what, if any values, of the unknown parameter α is A a singular matrix?

(b) Let B and P be 2 x 2 square matrices defined by

$$B = \begin{bmatrix} \theta & -\frac{\theta}{2} \\ -\frac{\theta}{2} & \theta \end{bmatrix}$$

$$P = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix}$$

where θ is a positive constant. Show that the matrix $P'P$ is equal to the identity matrix I, and that the matrix PBP' has all off-diagonal elements set to zero.

3 You are an employer trying to hire a worker to undertake a given task. The wage which you are prepared to pay depends on the worker's quality, but unfortunately you cannot observe this. However both you and the worker know that quality q is positive, and that the distribution of quality is described by the probability distribution function

$$f(q) = \theta e^{-\theta q}$$

where $\theta > 0$.

(a) Verify that $f(q)$ satisfies the conditions

$$\begin{aligned} f(q) &\geq 0 \quad for\ all\ q \geq 0 \\ \int_0^\infty f(q)dq &= 1 \end{aligned}$$

and is therefore a valid probability distribution function.

(b) Suppose that you are willing to pay the worker a wage w which is equal to his expected quality. Derive an expression giving the value of w in terms of θ.

4 Suppose that a sales person earns a basic monthly salary of £800 plus a commission and possible bonuses based on her level of sales. Suppose that the commission rate is 15% and the possible bonuses are a lump-sum amount of £1000 if her monthly sales exceed £10,000 and a further lump-sum of £2500 if her monthly sales exceed £15,000.

(a) Find the function that relates sales to earnings.

(b) At which points is the function discontinuous?

(c) Discuss the incentives created by this pay scheme.

SECTION B

Answer **ONE** question.

5 A student is preparing for exams in two subjects. She estimates that the grades she will obtain in each subject, as a function of the amount of time spent working on them, are

$$\begin{aligned} g_1 &= 20 + 20\sqrt{t_1} \\ g_2 &= -80 + 3t_2 \end{aligned}$$

where g_i is the grade in subject i and t_i is the number of hours per week spent in studying for subject i, $i = 1, 2$. She wishes to maximize her grade average $\left(\overline{g} = \frac{g_1+g_2}{2}\right)$ and she cannot spend in total more than 60 hours studying in the week.

(a) Formulate the problem and write down the Lagrangian function.

(b) Solve for the optimal allocation and find individual subject grades at the optimum.

(c) Discuss the properties of the marginal contribution to the grade average from studying each subject.

(d) Interpret the Lagrangian multiplier.

6 Consider the following IS-LM model:

$$\begin{aligned} C &= a + bY - lR \quad \text{consumption demand;} \qquad a, b, l > 0 \\ I &= \overline{I} \quad \text{investment demand} \\ G &= \overline{G} \quad \text{government demand} \\ L &= kY - hR \quad \text{money demand;} \qquad k, h > 0 \\ M &= \overline{M} \quad \text{money supply} \end{aligned}$$

where Y is output and R is the interest rate.

Assume that the money market clears instantly. However output adjusts gradually in response to the demand-supply gap in the following way

$$\dot{Y} = \alpha(C + \overline{I} + \overline{G} - Y), \qquad \alpha > 0$$

Assuming that $Y(0) = Y_0$,

(a) Write down the differential equation for Y.

(b) Solve the equation in (a) given the initial condition.

(c) What is the steady state value of Y?

(d) Discuss the stability conditions of the system.

SECTION C

Answer **ALL FOUR** questions.

7 (a) It is known that each year 90% of all students who are made conditional offers to read Economics in Cambridge, satisfy the conditions set i.e. they obtain the grades necessary to claim their place. In a particular year, 150 students throughout the UK receive conditional offers to read Economics. In this case, what is the probability that between 80% and 90% meet the conditions?

(b) In a particular school, 10 students are made offers. What is the probability that 80% to 90% of these students satisfy the conditions set if the success rate (i.e. the probability of meeting their conditions) is the same for all schools?

(c) In due course it is discovered that the school in question actually has an above average success rate, and that in fact, 95% of candidates from this particular school make their offers. If this is the case, what is the probability of success for all the other UK students (assuming the figure of 90% in 7(a) remains correct)?

8 (a) Consider the following Population Regression Function:

$$C_i = \alpha + \beta Y_i + \varepsilon_i \tag{1}$$

where Ci is Household Consumers Expenditure for household i, Y_i is Household Disposable Income, and ε_i is a disturbance term.

(i) State the assumption usually made of the variances of ε_i.

(ii) Explain why this assumption in (i) might not hold for this case, i.e. in a simple regression of Consumption against Income for a cross section of individuals.

(iii) Show that the Variance of ε_i, $V(\varepsilon_i) = E(\varepsilon_i)^2$ if $E(\varepsilon_i) = 0$

(iv) Explain what you would expect to find if you were to draw a graph of $\hat{\varepsilon}_i^2$ against Y_i, if the assumption in (i) held?

(b) A researcher estimates (1) and uses the residuals obtained from this estimation ($\hat{\varepsilon}_i$) to run the following regression:

$$\hat{\varepsilon}_i^2 = \gamma + \delta Y_i + u_i$$

Explain why this regression can be used to assess the assumption made in part (i). If the assumption does in fact hold, what result would you expect to get from a test of the hypothesis that $\delta = 0$? Under what circumstances might the assumption in (i) not be testable in this way?

9 (a) An Estate Agent purchases 4 plots of land in area A and 6 plots of land in area B. All plots of land are equally desirable and sell for the same amount.

(i) What is the probability that the first two of the newly acquired plots sold by the estate agents are both from area B?

(ii) What is the probability that of the first four sold, at least one is from area A?

(b) Consider the following models:

$$Y_t = \beta_1 + \beta_2 X_t \tag{1}$$

$$\ln Y_t = \beta_1 + \beta_2 \ln X_t \tag{2}$$

$$Y_t = \beta_1 + \beta_2 \left(1/X_t\right) \tag{3}$$

$$Y_t = \beta_1 + \beta_2 \ln X_t \tag{4}$$

(i) What is the elasticity of Y with respect to X in each case?

(ii) Engel suggested that "the total expenditure that is devoted to food tends to increase in arithmetic progression as total expenditure increases in geometric [exponential] progression". If Y denotes expenditure on food and X denotes total expenditure, which of the above models would you use to test Engel's suggestion? Explain your answer.

10 In previous years, a computer services company carried out a survey of its customers to discover how much they were spending on computer services over the calendar year, finding that the average amount spent was £20,000 per firm with a standard deviation of £2,500.

(a) It is decided to carry out a sample survey to determine whether or not the average amount spent has changed if, assuming the population variance is unchanged, the 95 per cent confidence interval for the mean is to be no wider than £1000. What sample size would be just sufficient to satisfy this criterion?

(b) A sample survey of a size just sufficient to satisfy this criterion is carried out; a sample mean of £26,000 is observed - the sample standard deviation is £2,400. Is there any evidence to suggest that the firm's clients now spend more on computer services?

(c) Whilst carrying out the survey in (b) the firm also asks its customers if they are satisfied by the services they receive. 50% reply they are very happy with the services they receive, compared with 42% from the original complete survey. Is there evidence to suggest that the firm is now providing a significantly better service?

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SECTION D

Answer **ONE** question.

11 It is supposed that the data on the number of road casualties in Great Britain can be decomposed as follows:

$$Y_{it} = (TC)_{it} + S_{it} \qquad \text{where } S_{it} \sim N\left(\mu_i, \sigma_i^2\right) \qquad i = 1, ..4$$

where Y_{it} is the number of casualties in quarter i in year t, $(TC)_{it}$ is a combined trend and cyclical component, S_{it} is a (quarterly) seasonal factor. It is assumed that the seasonal factors for any specific quarter i are normally distributed over time, with a fixed population mean and variance.

(a) Using the data shown in Table 1, and employing the method of moving averages, estimate the TC component.

Table 1

Total Number of Road Casualties				
Year	**Quarter 1**	**Quarter 2**	**Quarter 3**	**Quarter 4**
1995	73428	72380	81417	83281
1996	72139	76423	81531	90209
1997	75344	81287	81551	89621
1998	74061	80602	81880	88669
1999	74119	77056		
Source: Annual Abstract of Statistics				

(b) The following table, for under 16 year olds only, shows NOT the total number of road casualties, but quarterly fluctuations once any trends and cycles have been removed from the data.

Table 2

Road casualties amongst under 16 year olds (with trend and cycles removed)				
Year	**Quarter 1**	**Quarter 2**	**Quarter 3**	**Quarter 4**
1995	-1855	848	2150	-1087
1996	-2080	1174	2300	-291
1997	-1595	1205	1594	-395
1998	-1812	1086	1176	-737
1999	-1962	411	1120	-1250

(i) Estimate (to one decimal place) the mean seasonal factor for each quarter.

(ii) Check that the estimated mean seasonal factors do not exhibit a (annual) trend.

(iii) Interpret your results. Do you think the data in Table 1 reveal the same quarterly pattern as those in Table 2? Comment.

(iv) Determine (to one decimal place) the standard deviations for the seasonal factors in quarter 2 and quarter 3.

(v) Using your calculations for (i) and for (iv), test the hypothesis that, for the under 16 year olds, the population mean seasonal factors for quarter 3 and for quarter 4 are the same.

(vi) Can you assess the validity of any assumptions you have made in (v)?

(vii) Comment on your analysis.

12 Two real-valued random variables X and Y have expected values $E(X) = \mu$, $E(Y) = v$, respectively where E is the expectations operator. The covariance between the two variables is defined as

$$Cov\,(X, Y) = E\,((X - \mu)\,(Y - v))$$

(a) (i) If X and Y are random variables then show that $V(aX - bY)$, i.e., the variance of $aX - bY$, is given by $a^2V(X) - 2abCov(X, Y) + b^2V(Y)$

(ii) Hence or otherwise find the variance of $3X - 2Y$ as well as its mean value where

$$E[X] = 6 \quad E[Y] = 2 \quad \text{Covariance } (XY) = -8$$

and Variance $(X) = 16 =$ Variance (Y)

(b) State Bayes' Theorem.

On any given day Fred works either entirely in office A or entirely in office B (in a different town) or he takes the day off (and spends it mostly at home in the countryside). The probability he spends any given day at A is 0.4, and it is 0.3 that he spends it at B. When he works in office A the probability he eats lunch is 0.8 and when he works in office B the probability he eats lunch is 0.6. He never bothers with lunch when he is not working. His behaviour on any particular day is independent of his behaviour on any other day.

(a) (i) He did not eat lunch yesterday. What is the probability he stayed at home?

(ii) He had lunch the previous two days. What is the probability he was in the same place on both days.

(iii) What is the expected value of the number of lunches over the next four days? Interpret your result.

END OF PAPER