UNIVERSITY OF CAMBRIDGE

ECT1
ECONOMICS TRIPOS PART I

Friday 12 June 2015 09:00-12:00

Paper 3

QUANTITATIVE METHODS IN ECONOMICS

There will be a 15 minute reading time prior to the beginning of the examination.

This paper is divided into four Sections:
Sections A and B are Mathematics; Sections C and D are Statistics.

You should do the appropriate number of questions from each Section.
The number of questions to be attempted is at the beginning of each Section.
Answer all parts of the questions.

Answers from the Mathematics and the Statistics Sections must be written in separate booklets with the letter of the Section written on each cover sheet.

Section A carries 30% of the marks
Section B carries 20% of the marks
Section C carries 30% of the marks
Section D carries 20% of the marks

Write legibly.

STATIONERY REQUIREMENTS
20 Page booklet x 2
Rough work pads
Tags

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAMINATION
Calculator - students are permitted to bring an approved calculator
New Cambridge Elementary Statistical Tables

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

SECTION A - Answer **ALL FOUR** questions from this Section.

1. (a) Calculate the n^{th}-order Taylor approximation for e^x about $x = 0$.

   (b) Use the Taylor series approximation formula of order 2 to approximate the function:

$$f(x) = \sqrt{4 + x}$$

   (c) Use your result in (b) to estimate both:

   i. $\sqrt{4.01}$

   ii. The maximum absolute value of the remainder given your answer to (c)i.

2. (a) The demand D_t and supply S_t for a good in period t are given respectively by:

$$\begin{aligned} D_t &= a - bp_t \\ S_t &= -c + dp_{t-1} \end{aligned}$$

   where p_t is the price in period t and a, b, c and d are positive constants.

   i. If market equilibrium determines actual prices at all times write down an equation linking p_t and p_{t-1}.

   ii. Determine the steady state p^* for the price process that you formulated in answer to (a)i.

   iii. If at $t = 0$ the price is p_0 determine a solution for p_t in terms of p_0 and p^*.

   iv. Hence or otherwise determine restrictions on a, b, c and d, if any, under which the system converges on the steady state.

   (b) Let

$$y(t) = ae^{-bt}$$

   where a and b are constants.

   i. Differentiate y with respect to t and express your result as a differential equation.

   ii. Drawing on your result in (b)i above or otherwise, solve the following. Suppose that for a small island economy with population at time t given by z_t, the maximum population supportable at all times is z_M, and that the rate of change of the population at time t is a constant, β, times the deviation of the population from this maximum. What is the current population if at time $t = 0$ it was z_0? Give your answer in terms of z_0, z_M and β.

3. Evaluate the following stating your answers, in each case, in terms of powers of e:

(a)

$$\int_2^3 \left[\left(2xe^{x^2-4} \right) + \frac{1}{x} - ln\,(1.5) \right] dx$$

(b)

$$\int_1^6 \frac{3}{2} e^{3(3x-2)^{1/2}} dx$$

4. Find and interpret the stationary points of the following functions:

(a) $f(x, y) = \frac{4}{3}x^3 + y^3 - 64x - 12y + 10$

(b) $f(x, y) = e^{\alpha}$ where $\alpha = -2y^2 - 6xy - 3x^3$

(TURN OVER)

SECTION B - Answer **ONE** question from this Section.

5. Consider the following function:

$$Q = K^{\alpha} L^{\beta}$$

Where Q is a firm's output level, K and L are respectively inputs of capital and labour, and α and β are positive constants.

(a) Determine the values of α and β for which the function is concave.

(b) Suppose now that $\alpha = \beta$, r is the unit price of capital and w is the wage rate.

i. What is the optimum level of long run production costs expressed in terms of output Q?

ii. Determine the marginal response of long run cost to a change in output

iii. How does the result determined in (b)ii compare to the value of the Lagrange multiplier at the optimum determined in (b)i? Explain the relationship.

iv. Suppose that capital is fixed in the short run at $\overline{K}$. Determine the short run cost curve in terms of $\overline{K}$ and Q.

v. Determine the relation between K and Q when the short run and long run marginal costs are equal, and comment on your answer.

6. The cost function facing a firm for a given product is given by

$$C(q) = \alpha^2 + q + q^2 \quad \alpha > 0$$

where q is the level of output produced. The demand for the product in question is given by

$$q^d = 52 - p$$

where p is the price.

(a) Compute the marginal cost curve and the average cost curve for the firm

(b) Suppose that, despite being the only firm in the market, this firm is competitive in that it takes the price as given in making all its decisions. If the firm is profit maximising determine how much the firm wants to produce at each price level, and hence determine the equilibrium levels of price and output.

(c) By how much would the optimal profit level at the equilibrium change if there is a very small change to α? Interpret your result.

(d) Suppose that after a period of time there are N firms in the market, each facing the above cost function, and all price takers. Determine the equilibrium values of p and q for each firm.

(e) Suppose that firms have entered the market until the point that each makes zero profits. Determine N if $\alpha = \frac{1}{2}$. Determine the equilibrium p and q.

(f) Check whether your solution values for p and q and N are consistent with the following equations,

$$\begin{aligned} p &= 1 + 2q \\ p &= \frac{1}{4q} + 1 + q \end{aligned}$$

and interpret your findings.

(g) Suppose now that the firm in part (b) had instead been a monopolist. What price would the firm have set and how much would it be able to sell? How do these results compare to those for the price-taking firm examined in (b)?

 (TURN OVER)

Section C - Answer **ALL FOUR** questions from this Section.

7. A driving test has both a theory section T and practical section P. Both must be passed at the same attempt in order to pass the driving test. The pass rate for the practical is 80% and for the theory 70%. Also 90% pass at least one or other section at each attempt at the driving test.

   (a) What is the probability of someone passing the driving test?

   (b) What is the probability that an individual passes the theory section given that they have passed the practical section?

   (c) The rules say that if someone fails the driving test at a particular attempt then they can resit the entire test. Treating successive attempts as independent, what is the probability an individual will take n attempts to pass for $n = 1, 2, 3, \ldots$?

   (d) Show that with enough resits an individual eventually passes the driving test.

8. A random variable X has an exponential distribution if its probability density function is given by

$$f(x) = \begin{cases} \lambda e^{-\lambda x} & x \geq 0 \\ 0 & x < 0 \end{cases}$$

   where $\lambda > 0$ is a parameter

   (a) Calculate $E(X)$, $E(X^2)$ and hence calculate $Var(X)$.

   (b) Show that the median of X is given by $\frac{ln(2)}{\lambda}$.

9. Random samples from two populations A and B with population means μ_A and μ_B and population variances σ_A^2 and σ_B^2 are collected. The following sample statistics are calculated

Sample	A	B
Sample size (n)	80	60
Sample mean ($\overline{x}$)	10.7	11.5
Sample variance (s^2)	6.4	4.3

   where $\overline{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$ is the sample mean and $s^2 = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \overline{x})^2$ the sample variance estimator.

   Stating clearly any assumptions you make

   (a) Obtain a 95% confidence interval for μ_A.

   (b) Test at the 5% significance level the hypothesis that $\mu_A = \mu_B$ against the alternative $\mu_A < \mu_B$.

10. Two economists get in an argument about how to best estimate the effect of education on wages. Both have the same data on wages and education, where *wage* is measured in pounds sterling per hour and *educ* is measured in years. They both estimate the regression function:

$$wage = \beta_0 + \beta_1 educ + u$$

One economist simply estimates the regression function, measuring *wage* in pounds sterling and *educ* in years. She estimates $\hat{\beta}_0 = 2$, $\hat{\beta}_1 = 3$. The other, being more precise, estimates the regression function using *wage* in pence (and thus multiplies *wage* by 100) and *educ* in days (and thus multiplies years of education by 365). He claims this will give more precise estimates and higher R^2.

(a) Calculate the second economist's estimates of β_0 and β_1.

(b) Which economist's regression will obtain a higher R^2?

Section D - Answer **ONE** question from this Section.

11. A hypothesis test H_0 concerns the value of a population parameter p. A test statistic is given that lies in some set Ω.

    (a) Explain carefully the acceptance region A and rejection region R for a hypothesis test. Why do we choose A and R to be mutually exclusive and exhaustive?

    (b) Explain carefully what is meant by the size and power of a test against some alternative hypothesis H_1.

    (c) To determine if a coin is fair I will toss it n times and reject the hypothesis that it is fair (and conclude it is biased against heads) if I see no heads in n tosses. How large must I choose n so that this test has size below 5%?

    (d) I decide to toss a coin 10 times and reject the hypothesis that it is fair if I observe 9 or 10 tails. What is the size of this test?

    (e) What is the power of the test described in (d) against the alternative $H_1 : Prob(head) = p$ as a function of p? Where will this power function be at a maximum? Explain why this is so.

12. You have data on 88 housing transactions. Each house is reported to have one of two styles: colonial or modern. There are 27 colonial and 61 modern houses in your dataset. Each of the houses in your dataset has at least one bedroom and no more than 7 bedrooms. There are 42 houses with 4 or more bedrooms and 46 houses with 3 or fewer bedrooms. Your dataset reports the following variables:

    - P, house price in 1000s of £
    - SQ, the square footage of the house
    - C, a dummy variable describing the style; 1= colonial, 0=modern
    - M, a dummy variable describing the style; 1=modern, 0=colonial
    - BD, a dummy referring to the number of bedrooms; $BD = 1$ for 3 or fewer bedrooms, $BD = 0$ for 4 or more bedrooms

    The following sample statistics are reported:

    - $E(P) = 293.55$, with standard deviation 102.71
    - $E(P|C = 1) = 272.37$, with standard deviation 111.69
    - $E(P|C = 0) = 302.92$, with standard deviation 97.98
    - $E(P|BD = 1) = 261.05$, with standard deviation 53.71

- $E(P|BD = 0) = 329.14$, with standard deviation 129.37

(a) Test, at the 95% confidence level, the hypothesis that there is no difference in price between colonial and modern houses.

(b) Test, at the 95% confidence level, the hypothesis that houses with four or more bedrooms, have a higher price than houses with 3 or fewer bedrooms.

(c) You estimate the least squares regression of house prices on square footage and obtain the following parameter estimates with standard errors reported in parentheses:

$$\hat{P} = \underset{(24.742)}{11.204} + \underset{(0.012)}{0.140}SQ \tag{1}$$

Interpret the coefficient on SQ and perform a hypothesis test, at the 5% significance level, that the coefficient is significantly different from zero.

(d) After estimating (1), you calculate the residuals of the regression and plot them in Figure 1.

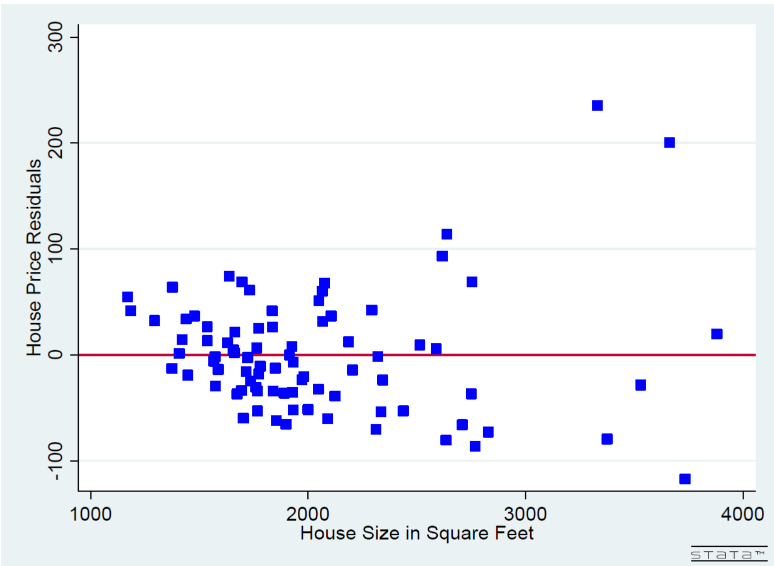

Figure 1 House price residuals from equation (1)

Which, if any, of the assumptions of the Gauss-Markov Theorem might be violated? Does the additional information provided in Figure 1 affect your answer to part (c) above? Explain.

(e) You next estimate the following two alternative specifications of the relationship between house prices and square footage.

$$\hat{ln(P)} = \underset{(0.076)}{4.824} + \underset{(0.000036)}{0.000401}SQ \quad (2)$$

$$\hat{ln(P)} = \underset{(0.641)}{-0.975} + \underset{(0.084)}{0.873}ln(SQ) \quad (3)$$

Interpret the coefficients on SQ in (2) and on $ln(SQ)$ in (3). (Assume that all assumptions of the Gauss-Markov Theorem are satisfied).

(f) Finally, you estimate the following:

$$\hat{ln(P)} = \underset{(0.079)}{4.773} + \underset{(0.000036)}{0.00039}SQ + \underset{(0.0450)}{0.0858}C \quad (4)$$

How do you interpret the coefficient on C? Suppose you estimate this equation again, but add an additional regressor, M (defined above). How will the inclusion of M affect your estimate of the coefficient on C? Justify your answer.

END OF PAPER