UNIVERSITY OF CAMBRIDGE

ECT1
ECONOMICS TRIPOS PART I

Friday 15 June 2018 9:00-12:00

Paper 3

QUANTITATIVE METHODS IN ECONOMICS

Answer **ALL FOUR** questions from Section A, **ONE**: question from Section B, **ALL FOUR** questions from Section C and **ONE** question from Section D.

Answers from Sections A and B (Mathematics) and from Sections C and D (Statistics) must be written in separate booklets with the letter of the Sections written on each cover sheet.

Sections A and C each carry 30% of the marks. Section B and D each carry 20% of the marks.

Each question within each section will carry equal weight.

Write you **candidate number** (not your name) on the cover of each booklet.

Candidates are asked to note that there may be a reduction in marks for scripts with illegible handwriting.

If you identify an error in this paper, please alert the **Invigilator**, who will notify the **Examiner**. A **general** announcement will be made if the error is validated.

STATIONERY REQUIREMENTS
20 Page booklet x 2
Rough work pads
Tags

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAMINATION
Calculator - students are permitted to bring an approved calculator
New Cambridge Elementary Statistical Tables

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

SECTION A – Answer **ALL FOUR** questions from this Section.

1. Let $f(x) = \ln(1 - x)$.

   (a) Provide a Taylor series expansion for $f(x)$ around the point $x = 0$.

   (b) Provide a polynomial function $P(x)$ which is at most 0.0001 away from f for all $x \in (0, 0.1)$.

   (c) Hence find a number that estimates $\ln(0.99)$ that is correct to at least 4 decimal places.

2. Evaluate the following integrals (making sure you show your work):

   (a) $\displaystyle\int xe^{3x}\,dx$

   (b) $\displaystyle\int_{e}^{e^2} \frac{dx}{x(\ln x)^3}$

   (c) $\displaystyle\int \frac{2x}{x^2 + 6x + 5}\,dx$

3. Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ be a linear transformation such that

$$\begin{array}{rcl} (1,0,0) & \mapsto & (0,1,0) \\ (1,1,0) & \mapsto & (-1,1,0) \\ (1,1,1) & \mapsto & (-1,1,-1) \end{array}$$

   (a) Provide a matrix representation for T according to the standard unit basis.

   (b) Denoting this matrix by A, compute the matrices A^2 and A^5. (*Hint: Feel free to use the geometric interpretation of T.*)

   (c) Determine whether T is singular or not. If it is non-singular, express T^{-1} in matrix representation. If it is singular, explain why.

4. A firm uses two inputs K and L to produce an output according to its technology

$$f(K, L) = K^{\alpha} L^{\beta}$$

where $0 < \alpha, \beta < 1/2$ are parameters that capture the efficiency of the firm's technology.

(a) Let w and r be the prices of L and K respectively, and p the price of the output. Write the firm's profit maximisation problem.

(b) Which problem do you need solve to obtain the cost function?

(c) If $\alpha = 0.25$, $\beta = 0.25$, $w = 0.25$, $r = 4$, and $p = 4$, compute the numerical value of the firm's profit.

(d) Now suppose the firm has access to two costly innovations A and B. Innovation A improves α by 0.01, whereas innovation B improves β by 0.01. Which innovation does the firm prefer? And in approximate terms, what is the most the firm would be firm willing to pay for its preferred innovation?

SECTION B – Answer **ONE** question from this Section.

5. Consider a market for an ordinary good where the price in each period is determined by the standard market clearing condition: supply equals demand. Let the demand curve for this good be $f(p)$ and the supply curve be $g(p)$.

(a) You are told that at every price p, the price elasticity of market demand for this good is ap^2 and the price elasticity of market supply is bp^2 where a and b are given constants. If both the market demand and the market supply at $p = 1$ are 100, derive the demand curve f and the supply curve g.

(b) Suppose demand D_t and supply S_t in period t are determined according to

$$D_t = f(p_t) \qquad \text{and} \qquad S_t = g(p_{t-1})$$

where p_t is the market price in period t. Obtain a difference equation for p_t, and evaluate all steady states if there are any.

(c) Obtain a formula which expresses the price in period t as a function of t and p_0.

(d) Determine the conditions under which p_t converges, and whether there is a stable long run equilibrium. Comment on the nature of the convergence path.

6. Let $f : \mathbb{R}^2 \to \mathbb{R}$ be given by

$$f(x, y) = 2x^2 + 5y^2 - 4xy - 6x + 4y + 13$$

(a) Identify and classify all stationary points of f. Does f admit a global maximum?

(b) Now suppose the domain of f is restricted to $D \subset \mathbb{R}^2$ described by

$$(x - y)^2 + \frac{3}{2}y^2 \leq 87$$

Explain why f must admit a maximum and a minimum on domain D.

(c) Identify where f takes its maximum and minimum values on domain D. Make sure you show your work in getting your answers.

SECTION C – Answer **ALL FOUR** questions from this Section.

7. Let X and Y be continuous random variables with joint probability density function

$$f(X,Y) = \frac{x + xy}{2} \quad 0 \leq x \leq 1,\ 0 \leq y \leq 2$$

   (a) Calculate the marginal probability density functions for X and for Y.

   (b) Calculate $E(X)$ and $E(Y)$.

   (c) Are X and Y independent? Justify your answer.

8. (a) In a particular Cambridge College, 18% of all students both play football and row, and 32% of all students play football. What is the probability that a student rows given that the student plays football?

   (b) We have two bags each containing black and white balls. One bag contains three times as many white balls as black balls. The other bag contains three times as many black balls as white balls. Suppose we choose one of these bags at random. From this bag we select five balls at random, replacing each ball after it has been selected. The result is that we find 4 white balls and one black. What is the probability that we were using the bag with mainly white balls?

9. A (null) hypothesis is to be tested by calculating a test statistic t that takes values in some sample space Ω

   (a) Explain what is meant by acceptance (A) and rejection (R) regions for the test. What properties should we require of these regions?

   (b) Explain what is meant by a Type I and Type II error and define the size and power of the test.

   (c) What problems might occur if we choose A such that the probability of a Type I error is zero?

   (d) If we carry out several independent tests of size α show that the probability we reject at least one of the nulls, even if they are all true, is $1 - (1 - \alpha)^n$, where n is the number of tests carried out. Hence show that if we make each individual test size $\frac{\alpha}{n}$ then the overall testing procedure will have overall size (ie the probability we reject at least one of the nulls, even if they are all true) approximately α.

10. To investigate the relationship between wages, experience, and union status we have a dataset containing 528 observations, each corresponding to a person, for whom we observe the hourly wage (W), union status (1 if the worker is in a union (U), 0 otherwise), and years of work experience (E).

(a) Consider the regression of wages on union status:

$$\widehat{W} = \underset{(0.245)}{8.659} + \underset{(0.573)}{2.140}U$$

Provide an interpretation for the coefficient on U.

(b) Test the null hypothesis that, on average, the wages of unionized and non-unionized workers are the same against the alternative that they are not.

(c) Next, consider the regression of wages on union status and experience:

$$\widehat{W} = \underset{(.394)}{8.130} + \underset{(0.577)}{2.015}U + \underset{(0.018)}{0.031}E$$

Based on this regression:

i. write an expression for the fitted least squares line for unionized workers.

ii. write an expression for the fitted least squares line for non-unionized workers.

(d) If two people have the same labor market experience, but one is unionized and the other is not, what is their predicted average wage difference?

(e) List the assumptions under which the least squares estimators are unbiased. Is the coefficient estimate on the union variable in part (a) unbiased? Explain why or why not.

SECTION D – Answer **ONE** question from this Section.

11. Random variables X_1 and X_2 are drawn independently from a distribution with probability density function $f(x)$ and cumulative distribution function $F(x)$.

(a) Calculate $P(X_1 = x, X_2 \leq x)$ in terms of f and F and hence show that

$$P(max\{X_1, X_2\} = x) = 2f(x)F(x)$$

Now suppose that $f(x)$ is the uniform distribution $U[0,1]$ i.e.

$$f(x) = 1 \quad 0 \leq x \leq 1$$

(b) Calculate $F(X)$ and hence write down the probability density function of the random variable $Y = max\{X_1, X_2\}$.

(c) Show that $E(Y) = \frac{2}{3}$ and $Var(Y) = \frac{1}{18}$.

(d) A game is played where each of 90 students draws independently two numbers from a $U[0,1]$ distribution and keeps the highest of their two draws. The resulting 90 values are summed and the entire group gets a free meal if the sum exceeds 60 and a free meal with wine if the sum exceeds 65. Stating clearly any assumptions you make:

i. What is the probability they get the meal?

ii. What is the probability they get the meal with wine?

(e) Explain how your calculations above would change if each student can draw N times from the $U[0,1]$ distribution and keep their largest draw.

12. A researcher has access to data for a sample of size N on a random variable Y. The sample contains data on two sub-populations, each containing N_A and N_B $(N = N_A + N_B)$ observations on, respectively, random variables Y_A and Y_B.

(a) Using the following linear regression equation

$$Y^A = \alpha^A + \varepsilon^A \qquad (1)$$

where α^A is an unknown parameter and ε^A is an error term. Find an expression for the ordinary least squares (OLS) estimator of α^A

(b) If the mean in population A is μ_A and the mean in population B is μ_B how would you test the hypothesis $H : \mu_A = \mu_B$? State any additional assumptions that you need to make to undertake your test.

(c) Consider now the following linear regression equation

$$Y = \lambda + \beta D + \varepsilon \qquad (2)$$

where λ and β are unknown parameters and ε is an error term. D is a dummy variable that is equal to 1 for membership of subpopulation A and equal to 0 for membership of subpopulation (B). Find an expression for the ordinary least squares (OLS) estimator of β and explain how one could use this to test the equality of population means.

(d) Provide an interpretation for the ordinary least squares (OLS) estimators $\widehat{\lambda}$ and $\widehat{\beta}$.

(e) Why might an analyst prefer to utilise the method used in part (c) over (b).

(f) "If individuals are randomly allocated to subpopulations A and B, the OLS estimator $\widehat{\beta}$ will be unbiased for β." Evaluate this statement.

END OF PAPER