KEYNES, PIGOU, AND THE *GENERAL THEORY*

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CONTENTS

1. Introduction 2
2. Keynes and Pigou 3
   2.1. Beginnings 3
   2.2. Collegiality 4
   2.3. Controversy 5
3. The *General Theory* in a post-Pigovian perspective 8
   3.1. Keynes’ “Classical Postulates” 8
   3.2. Keynes’ acceptance of “well behaved” production functions 10
   3.3. Keynes’ acceptance of “conventional” utility functions 11
   3.4. The common problem of macroeconomic aggregation 12
   3.5. Keynes’ Pigou-oriented criticism 14
4. The *General Theory* in an anti-Pigovian perspective 15
   4.1. The Z-functions 15
   4.2. Effective demand 20
   4.3. Walras’ Law *versus* Say’s Law in the *General Theory* 26
   4.4. Linking up with the Z-curve 31
   4.5. In search for the “right” representation 35
   4.6. The “frozen land” metaphor 39
5. Concluding remarks 42

Appendix A. The problem of aggregation and Keynes’ wage units 45
   A.1. Some analytical issues 45
   A.2. Generalisations and simplifications 49

Appendix B. Consumption and time preference 52
   B.1. Keynes on time preference and consumption 52
   B.2. A two-period model of time preference and consumption 52
   B.3. Extending the two-period model 55
   B.4. The classical stationary state 59
   B.5. “Personal” and “subjective” factors in time preference 62

Appendix C. The microfoundations of effective demand 64

References 73

**Abstract.** The paper discusses the *General Theory* under the perspective of Keynes’ methodological similarities and controversies with his Cambridge colleague A.C. Pigou.

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**Key words and phrases.** Keynes, Pigou, General Theory, Macroeconomics, Classics.

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1. Introduction

The vast literature about Keynes and the General Theory demonstrates that there are many ways to approach this important book. In the following I will attempt to show that a particularly fertile approach is to see Keynes in relation to his Cambridge colleague A.C. Pigou. Other contributions to this seminar will relate Keynes to Karl Marx (Sardoni 2008) or to Alfred Marshall (De Vroey, 2008). Joan Robinson (1978) once wanted to see Keynes in relation to David Ricardo and to Pierro Sraffa. I myself tried to follow this “Cambridge post-Keynesian” approach while I was a research student at Cambridge many decades ago. But I found that it did not help much for a better understanding of the General Theory. As the “proof of the pudding” is in the eating so, similarly, the “proof” of an interpretation is in the reading, that is: in the way of elucidating lines of thought which originally might have appeared to be rather difficult to make sense of. In this regard I tend to give priority to a Pigou oriented approach to Keynes, but it is not my intention to denigrate other ones. An important corollary to a Cambridge oriented view of Keynes is to see him in relation to the controversies which he had with several representatives of the London School of Economics (LSE). But this is a line of thought which will appear only briefly below.

Apart from the potentially disturbing plethora of analytical approaches to Keynes there is the problem that along with such alternative and complementary interpretations we have a rather strange “folklore” of seemingly “well-known facts” about Keynes. This is really pre-analytic stuff which should have been beyond economic discourse from the beginning. But since such folklore gives false plausibility to otherwise unconvincing interpretations of Keynes, I will briefly deal with this problem, too.

Efforts to “make sense of Keynes” pre-suppose that there is at least a rudimentary interest in his writings. It is well known that the writing and the reception of the General Theory was motivated by the inability of politicians and economic advisors to find an appropriate answer to the Great Depression of the 1930ies. On the time scale of today’s young students this seems to be ages away. But as far as economic problems are concerned, we do have recent bankruptcies of once renowned banks and worries about repercussions which remind many commentators of that Great Depression (Eichengreen, 2008). Politicians and administrators nowadays do try to deal with this situation in ways quite different from those which their predecessors had tried 70 years ago. This shows that economic practitioners have learned since then (Leijonhufvud, 2008). They have learned to a considerable extent from Keynes.

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1Joan Robinson (1978, p.147) “Keynes hardly ever peered over the edge of the short period . . . All the same, he hankered after the concepts of a normal rate of profit and value of capital, though he could not get them clear. Here it was Ricardo who could have helped him out.”

2Joan Robinson (1978, p.148); Sraffa (1960) “was not published until 1960. Sraffa had shown a draft to Keynes in 1928. Keynes evidently did not make much of it and Sraffa, in turn, never made much of the General Theory. It is the task of post-Keynesians to reconcile the two.”
Even the American public, originally rather wary of Keynesian “leftish” ideas now widely believes in the importance of Keynes’ concept of “aggregate demand” as is witnessed by the “Inaugural Preview” edition of *Time Magazine*, heralding the new presidential era of the USA under Barack Obama. The issue starts with the words (Grunwald, 2009):

> John Maynard Keynes, the trendiest dead economist of this apocalyptic moment, was the godfather of government stimulus. Keynes had the radical idea that throwing money at recessions through aggressive deficit spending would resuscitate flatlined economies . . .

> As President-elect Barack Obama prepares to throw money at the current downturn . . . we all really do seem to be Keynesians now. Just about every expert agrees that pumping $1 trillion into a moribund economy will rev up the ethereal goods-and-services engine that Keynes called “aggregate demand” and stimulate at least some short-term activity . . .

To try to “understand Keynes” is therefore not just a book-wormish pastime. It is an endeavour to understand better an analytical approach which is now considered to be particularly relevant for our present economic problems.\(^3\)

### 2. Keynes and Pigou

#### 2.1. Beginnings.

There seems to be a common understanding among economists that Keynes was in fundamental opposition to Pigou. But such a fundamentalistic view is not quite appropriate. The fact is that for most of their lives, both were in rather friendly relations with each other and that they either supported or at least accepted their respective analytical positions.

There were good grounds for *personal* mutual acceptance. They both had a similar background as far as the social status of their families and their public school education was concerned. Pigou had his formative years at the public school of Harrow, Keynes at the public school Eaton. Both were one-time presidents of the students’ Cambridge Union Society (Pigou: 1900; Keynes: 1905). Both were pet students of the founder of the Economics tripos at Cambridge, Alfred Marshall. Keynes was a favourite of Pigou as well, because when Pigou was elected to the chair formerly held by Alfred Marshall, he offered Keynes a lectureship paid by Pigou himself (Skidelsky, 1983, p. 185).

In an early assessment of Keynes, published in *Granta* magazine of June 1904, Pigou wrote (Skidelsky, 1983, p. 123)

> Mr. Keynes . . . contrives to be clear-headed without making muddle-headed people hate him. That is a remarkable thing, which demonstrates that, besides the minor gift of cleverness, he has the major one of sympathy.

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\(^3\)This remark and its potential impact on a hitherto deficient understanding of Keynes’ original analytical intentions will be substantiated below, p.12

\(^4\)See also Thirlwall, 2009 in this context.
With the hind-sight of the later Keynes-Pigou controversy, this appears to be a clear
sighted assessment of the potential hatred against Keynes coming from muddle-
headed people. But it is a less clear assessment of Keynes’ competence to mollify
the anger of such people against Keynes’ cleverness.

2.2. Collegiality. Pigou accepted and respected Keynes as fellow economist and
vice versa. Even when he later criticized the inherited “classical theory”, [Keynes
(1936a, p.260) proclaimed:

Professor Pigou’s Theory of Unemployment seems to me to get out
of the classical theory all that can be got out of it;
thereby conveying that although his criticism is directed against a piece authored
by Pigou, Keynes did not want to negate his analytical abilities but he wanted to
criticise the paradigm to which Pigou’s abilities were applied.

As far as Pigou’s positive assessment of Keynes is concerned, there is a very re-
markable expression of esteem for Keynes as an able and helpful partner in economic
discussion. It is to be found in Pigou’s very first book (Principles and Methods of
Industrial Peace). In the foreword [Pigou (1906, vii) acknowledges “valuable help”
which Keynes gave him “in connection with Appendix A” of that book. This is
remarkable because “Appendix A” deals with the workings of labour markets. It
starts out from a conventional labour market model and then leads to an offer curve
analysis of wage bargains very much in the tradition of Alfred Marshall (1923,
fig.26, p.357). Marshall himself suggested in that context that this type of analyt-
cal apparatus should be employed for analyzing wage bargains. That Keynes and
Pigou worked together on this piece of Marshallian analysis is an interesting testi-
mony to common discipleship. Another such testimony is given by their common

Pigou’s acknowledgement of co-operation with Keynes on labour markets strangely
contrasts with their later controversy on the causes of and remedies for unemploy-
ment. Pigou’s acknowledgement of Keynes’ assistance in labour market analysis in
1906 suggests that Keynes cannot be easily accused of having had little knowledge
in 1936 about the basic tenets which were held by Pigou in this field.

Keynes’ biographer Skidelsky (1983, p.211) states: “As colleagues, Keynes and
Pigou had their differences.” But in the context of this remark, the differences
were about the fulfillment of teaching duties in economics, not about its substance.
Skidelsky (ibid.) relates that for unknown reasons Keynes “had to take over Pigou’s
main lecture course on Economic Principles in 1910-11.” We may read this not as
an indication of substantive conflict but of agreement. Certainly Keynes would not
have been permitted to lecture on Economic Principles instead of Pigou if he had
not been considered as an appropriate, i.e. equivalent, replacement of Pigou. We
may take this episode as endorsement for Keynes’ own assessment of his relation to
orthodox Marshallian and implicitly to orthodox Pigovian economics:

In that orthodoxy ... I was brought up. I learnt it, I taught it, I
wrote it. ... subsequent historians of doctrine will regard this book
as in essentially the same tradition (Keynes 1936a. p.xxi)
Thus, according to Keynes’ own judgement we, acting now as historians of economic thought, might well see him predominantly in analytic vicinity with Pigou and others who subscribed to the same economic orthodoxy – in spite of Joan Robinson’s (Robinson, 1979a, p.28) proclamation “The General Theory called for a radical reconsideration of the whole corpus of orthodox doctrine”.

Outside pressure is often regarded as a strong agent for collegiality but it is not a reliable one. In any case, it is a good test for cohesion in a school of thought. In this regard it is interesting to read the account recently given on the website of the London School of Economics (LSE, 2004):

The 1930s economic debate between LSE and Cambridge is well-known in academic circles. Rivalry between academic opinion in LSE and Cambridge’s case goes back to the School’s roots when LSE’s Edwin Cannan (1861-1935), Professor of Economics, and Cambridge’s Professor of Political Economy, Alfred Marshall (1842-1924), the leading economist of the day, argued about the bedrock matter of economics . . . LSE’s Robbins and von Hayek and Cambridge’s Keynes were chief figures in the intellectual disagreement between the institutions.

The academic rivalry between Cambridge Marshallians and London Cannanites is too complex and involved to be dealt with adequately in the present context. But the short account of the LSE just quoted gives important clues about some interesting issues: 1) It is remarkable that Pigou is absent in this juxtaposition. 2) Keynes is considered as the spearhead of opposition from the Cambridge side 3) The LSE side perceives itself as a close-knit ensemble, an “institution”, and treats the Cambridge side analogously. The problem between Keynes and Pigou was, however, that at a time of outside pressure against Keynes, this collegial cohesion turned out to be deficient on the Cambridge side of the debate.

2.3. Controversy. The reason why Keynes and Pigou entered economic folklore not as colleagues but as foes is of course closely connected with the writing and debating of the General Theory. There is a rupture indeed between the two in this connection and there has been much speculation why it occurred.

Most of those who comment on this controversy put the responsibility almost exclusively on Keynes. His motives are often stated as having been of rather low ethical standard. Some authors propose salesmanship as one of Keynes’ motives: creating a scandal by attacking Pigou in order to have better sales for the General Theory. Others claim plagiarism: Keynes said the same as Pigou but by attacking him he wanted to distract from this fact. In any case there is ingratitude involved: “poor Pigou” who once supported Keynes out of his own pocket finds himself later vilified by Keynes who turned against his benefactor with cheap polemics. In his relation to Pigou, Keynes thus appears as the proverbial “dog that bit the hand that fed him”. The truth is far off such caricatures.

In a recent book I have tried to show, among other things, that it was rather Pigou who disassociated himself from Keynes in several ways. It was not Keynes who was the first mover (Ambrosi, 2003). Victoria Chick (2008), in an unpublished paper,
reproduced my arguments maybe in a more readable and condensed way. I think she uses the appropriate characterization when she writes that this controversy grew out of Pigou’s aggression and Keynes’ frustration. But how could it come about that a collegiality which grew over decades of mutual respect eventually changed to a condition of mutual bad feelings?

The answer lies indeed in the Great Depression of 1929/30 and in the subsequent responses of the academic economic establishment to this challenge. Since 1929 Keynes was member of the Treasury’s “Macmillan Committee” investigating Finance and Industry. He also was member of the governmental Committee of Economic Advisors. Within this latter framework he obtained authority from Prime Minister Ramsey MacDonald in 1930 to set up a “Committee of Economists”. Keynes was authorized to propose its composition and to formulate its brief, given below in table 1.

<table>
<thead>
<tr>
<th>Members</th>
<th>Brief</th>
<th>Proceedings</th>
<th>Report</th>
</tr>
</thead>
<tbody>
<tr>
<td>J.M. Keynes, chair</td>
<td>To review the present economic conditions of Great Britain, to examine the causes which are responsible for it and to indicate the conditions of recovery</td>
<td>appointed on 24 Jul. 1930</td>
<td>24 Oct. 1930</td>
</tr>
<tr>
<td>H.D. Henderson, Prof. A.C. Pigou, Prof. L. Robbins, Sir Josiah Stamp</td>
<td></td>
<td>13 meetings from 10 Sept. to 23 Oct. 1930</td>
<td></td>
</tr>
<tr>
<td>Secretaries: R.F. Kahn and H.F. Hemming</td>
<td></td>
<td></td>
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</tbody>
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Keynes was convinced that the committee thus composed could come up with a well considered answer to the problems expressed by the brief. With Henderson he had previously published a pamphlet foreshadowing some of his future ideas. Pigou was his Cambridge colleague. Josiah Stamp was a director of the Bank of England and an experienced public servant and banker. Robbins was very young and just appointed as professor at the LSE.

Keynes wrote that “his” committee members

... are well accustomed to the most up-to-date academic methods and ways of discussing these problems, are essentially reasonable and good members of a committee, and happen to have given already a good deal of time and thought to the problem which would be set them. (quoted from Howson and Winch[1977] p.47)

In fact, the outcome of the committee work turned out to be a disaster for Keynes as chairman.

Keynes was just about to grasp the importance of the concept of effective demand in 1930. He introduced it in a rudimentary form through the proposal of a “revenue tax” on British imports in the hope of stabilizing domestic production. This provoked vociferous opposition from Lionel Robbins, who refused to sign the

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5See Kent[2007] for a detailed discussion of the historical and intellectual context of this article.
final report and insisted on an own dissenting minority report to be published with the final committee report (Robbins 1930). Many important members of the LSE camp followed suit in a publication edited by William Beveridge, then director of the LSE (Beveridge 1931). None of the rest of the members of the Committee of Economists, Keynes' putative allies, did support his position to a certain extent. They did support the idea of public works, but only on the condition that that would not raise “doubts as to the general soundness of the public finances”. (Howson and Winch 1977, p.201). They did not support Keynes’ position in principle but they limited their opposition to occasional dissenting passages in the final report without going to Robbins’ extreme who disassociated himself entirely from the final report. Pigou was cooperative in some technical respects, especially in spending some time with Keynes editing the final report, but he was totally un-cooperative as far as Keynes' ideas about tariffs were concerned. On this point, Pigou had a one-page dissenting note inserted into the final report (Howson and Winch 1977, p.225f.).

Eventually, Pigou was fed up with having been dragged by Keynes into committee work which he did not like (Pigou was asked to testify to the Macmillan Committee where Keynes was a member and Keynes made Pigou a member of the Committee of Economists where Keynes was chairman). This was not the stuff an academic economist should busy himself with, according to Pigou. When Pigou went so far as to publish this sentiment as a hardly veiled public rebuke against Keynes (Pigou 1933), he overstepped the boundaries of collegiality.

Keynes could have overlooked the personally demeaning passages in Pigou’s (1933) foreword. But along with the personal disappointment which they must have meant for Keynes, there went Pigou’s disappointing obtuseness in substance. This obtuseness was two-fold: 1) Pigou did not have the faintest idea what lay behind the concept of “effective demand” and 2) Pigou was not in the least receptive for the idea that Cambridge had to cultivate a cohesiveness and specificity in an analogous way to the one cultivated by the LSE-scholars. It is this disillusionment with Pigou which eventually lead Keynes to bring in a controversial tone in the debates with all his analytically prominent economic colleagues – not just with his opponents associated with the LSE.

We have an interesting statement by Keynes (1931, p.505), summarizing his exasperating experience in the discussion with his LSE-colleagues in which he declared:

Perhaps controversy with one’s friends and colleagues is an essentially barren thing. But I come to the end of my attempt to deal with the controversy which I have provoked [with the “revenue tariff proposals” of 1930-31, GMA] … with an unusually arid flavour in my mouth.

At this time – after the fiascoes with the Committee of Economists in 1930 and the barrage of almost all of British economics against his first attempts to articulate viewpoints of effective demand – after these depressing experiences Keynes must have asked himself how to proceed further. To give up controversy altogether would have meant admitting defeat. To continue controversy with the LSE as before would
have been futile. It is an interesting topic of research into the proto-history of the General Theory to document why exactly he decided for extending controversy to include his Cambridge colleague A.C. Pigou. But such research really belongs to fields of “meta-economics”. At least as interesting is the question of basic economic tenets which were at issue.

3. The General Theory in a post-Pigovian perspective

3.1. Keynes’ “Classical Postulates”. Among Keynesians it has become customary to denote analytical discipleship with the prefix “post”.

We might do likewise with regard to Keynes himself. Since Keynes openly confessed that he had been a “High Priest” of orthodox economics, we may call the orthodox aspects of his theory as “post-Pigovian” – with apologies to the PKSG. Apologies seem due, since Keynes begins the General Theory with a critique of the “Classical Postulates”. Does this not speak “for a radical reconsideration of the whole corpus of orthodox doctrine” (Robinson, 1979a, p.28)? Is it therefore not a gross misrepresentation to refer to Keynes as having “post-Pigovian” aspects in his theory? Should one not be clearer and should one not refer to him as outright anti-Pigovian?

Before answering such questions, let us recapitulate the type of criticism which Keynes did direct against the “classical postulates” in Ch.2, GT. Both postulates which he lists refer to “the” real wage. The first one postulates that for firms the real wage is equal to the marginal product of labour, the second one postulates that for households the real wage is equal to the marginal rate of substitution between effort, i.e. labour time experienced as causing negative utility on the one hand, and goods supplying positive utility of consumption on the other hand. Keynes expressed these postulates in a somewhat different manner, but it is clear what he meant. The geometrical locus of the respective postulates are “the” labour demand and supply curves. What Keynes criticizes in this context is: Classical economics claims that we are in a position where demand equals supply on the labour market – or rather labour markets, in plural, since Keynes was very much aware of heterogeneity in economics. But if we want to address the problem of unemployment, we must be clear that we do not mean laziness of workers or ignorance about realistic job opportunities, but involuntary unemployment proper. This means that workers must be off their supply curve. But Keynes appears to many readers as being tortuously complicated in this context.

Why didn’t Keynes express the classical postulates more simply so that one understands better what he meant? The answer to this question is that the analytical matters involved are not that simple. The formulation had to cover several analytical renderings of the labour market(s). At least since the time when he worked with Pigou (1906, Appendix A) on discussing supply and demand on the labour market, he knew that there are alternative representations – supply and demand curve analysis versus offer curve analysis – of labour markets. A further possibility

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6I skip here a review of variants of the usage of this prefix among followers of Keynesian ideas. For a brief exposition of them see Hartwig (2007, p.726).
for representing economic ideas about the labour market is to argue in the context of a simplified general equilibrium model of an entire economy in analogy to the pure theory of foreign trade. In that approach the economist confronts directly the respective countries' preference systems, represented by indifference curves, with the productive system, represented by the production possibilities. Analogously one could confront directly households preferences concerning work effort and consumption with the production possibilities of an economy. All these alternatives of representing the workings of the labour market, namely

1. supply and demand curves as loci of optimal firm and household decisions
2. offer curves of firms and households in the Marshallian tradition
3. production possibilities of firms and preferences of households

were known to Keynes as analytical instruments. It seems that with his sometimes tortuous sounding formulations he tried to avoid subsequent criticism that he used expressions which limited the applicability of his characterization to just one type of these approaches.

In addition to the problems posed by the multitude of possible formal representations of the workings of the labour market, there were long-standing debates in the literature about the relation between the level of employment and the degree of competition and about frictions in the allocation of labour as causes for unemployment. Keynes' intention was to cut through the thicket of such old debates and to lay bare the basic question: is there the analytical possibility for “involuntary unemployment in the strict sense”? Keynes clearly gave an affirmative answer to this question and in interpreting the *General Theory* it is one task to get to this basic question of “involuntary unemployment”.

Keynes' criticism cannot have been that the Classics had no idea what involuntary unemployment is. He certainly knew that Pigou (1914, p.16) gave a definition of unemployment very similar to what Keynes meant himself:

The amount of unemployment ... which exists in any industry, is measured by the number of hours' work ... by which the employment of the persons 'attached to' or 'occupied in' that industry fall short of the number of hours work that these persons would have been willing to provide at the current rate of wages under current conditions of employment.

The implication of Pigou’s definition is, of course: if the workers wanted to work more at the going real wage but found not sufficient employment, then, if there is a small decrease in the real wage but more jobs are offered, then it is not plausible that workers will work less than before. It is most likely that they will take up additional jobs. One could argue, however, that there are trade unions limiting labour supply. The seemingly involuntarily unemployed workers could then be treated as not “really” being unemployed. They withhold labour services voluntarily in order to exert monopolistic power as a group. The workers as a class – so to speak – “invest” the
hardship of unemployment for the benefit of higher real wages for the time when they do get employment. But this is an unrealistic claim made by economists who deny the relevance of involuntary unemployment in the sense just quoted.

Keynes' seemingly tortuous treatment of labour market analysis in GT, ch.2 is also motivated by the attempt to meet the "trade union objection" against the existence of "involuntary unemployment in the strict sense" as Keynes defined the situation described in the passage just quoted. Keynes’ insistence that such involuntary unemployment does exist and is highly relevant in situations of protracted mass unemployment is not the denial of orthodox analysis but a denial of apologetic distractions from the phenomenon described in the quote. Concerning the choice theoretic definition of "involuntary" unemployment proper there was no serious rift between Keynes and Pigou, however.

But the main question is not how to define unemployment – it is how to explain the existence of involuntary unemployment once it is properly defined. If analytical reasons for the protracted existence of involuntary unemployment – in the strict, household disequilibrium sense – were not given, then, in spite of its title, Pigou’s Theory of Unemployment had been a misnomer, according to Keynes. Pigou’s book just did not deal with the subject matter which the title of the book announced. But to point out that there was this deficiency in Pigou (1933) does not necessarily imply a denial of “classical” economic analysis as such.

3.2. Keynes’ acceptance of “well behaved” production functions. In the prefaces to the General Theory Keynes was maybe somewhat nebulous about his allegiance to “classical” economics. But in the main body of the text he was – in part at least – very concrete concerning his acceptance of “orthodox” neoclassical analysis (Keynes, 1936a, p.17):

In emphasising our point of departure from the classical system, we must not overlook an important point of agreement. For we shall maintain the first postulate as heretofore, subject only to the same qualifications as in the classical theory . . .

[A]n increase in employment can only occur to the accompaniment of a decline in the rate of real wages. Thus I am not disputing this vital fact which the classical economists have (rightly) asserted as indefeasible.

These passages mean that Keynes accepted the “well behaved neoclassical production function” without any reservation save one: it is not valid on the macroeconomic level but on a sectoral level. But this reservation was also made by Pigou (1933).

It should be very clear that the type of linear technology which later was debated in the Joan Robinson inspired Cambridge Capital Controversies has nothing to do with Keynes’ technological assumptions. That is not to say that Joan Robinson did not make an important point concerning the problems of measuring capital. But it

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8For an illustration of Keynes’ argument in the context of a market diagram see Hayes (2006, p.52, fig.1.1).
was not a point which Keynes wanted to make and it was not based on the technological assumptions which Keynes subscribed to. As far as production assumptions were concerned, Keynes was a stout orthodox economist according to the express statement just quoted from the GT.

3.3. Keynes’ acceptance of “conventional” utility functions. But Keynes not only accepted the contemporary conventional assumptions about “well behaved” technology. He also knew and accepted the contemporary theory of labour supply. He also knew the doctrine of time preference extremely well and made it the basis of his theory of consumption. It is important to stress this because there are several cases in which it is claimed that any such knowledge was remote from Keynes’ argumentation.

Take for example Grossman (1972 p.28) who claimed:

...Keynes’ specification of the consumption function was simply ad hoc, and no passage in the General Theory explicitly avers otherwise.

But that is clearly not the case. Keynes does not derive the consumption function ad hoc. He derives it conceptually and analytically from “time-preference”. This term is explicitly mentioned by Keynes (1936a, p.166) in the General Theory and he explicitly relates his consumption function to this concept, referring to that aspect of time-preference which I have called the propensity to consume, which, operating under the influence of the various motives set forth in Book III [GT], determines for each individual how much of his income he will consume and how much he will reserve in some form of command over future consumption.

Thus Keynes is as clear and as explicit as one can be that the whole structure of the discussion of his consumption function is an application of time preference theory. A comparison between his “Book III – The Propensity to Consume”, especially ch. 9, GT, will show that this discussion closely mirrors the work published shortly before by Irving Fisher (1930), a locus classicus of time preference analysis. Of at least the same authority with regard to time preference analysis is Frank Ramsey who published an investigation of optimal savings in an intertemporal context. It is therefore significant that it was this very Ramsey (1928, p.545) who attributed mastership to Keynes in this field:

The more we save the sooner we shall reach bliss, but the less enjoyment we shall have now, and we have to set the one against the other. Mr. Keynes has shown me that the rule governing the amount to be saved [and derived from an intertemporal calculus of variations, GMA] can be determined at once from these considerations.

On the basis of this and considerable other knowledge it would be downright ridiculous to claim that Keynes did not know and did not apply the classical theory of intertemporal consumption optimization in its most progressive contemporary form and that he did not want to apply it in the context of his theory of consumption. The opposite is true.

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But there is also the claim that the well established orthodox theory of labour supply was unknown to Keynes or that it was conscientiously left out by him when writing the General Theory – or that he should have let it out in that context. His position in this regard comes out quite clearly in his correspondence with R.G. Hawtrey shortly after the publication of the GT. [Hawtrey (1936, p.18)] claimed that he could not understand Keynes’ theory of involuntary unemployment because I cannot see how to apply the theory of the marginal disutility of labour to a community in which there is unemployment. So far as I recollect, the marginal disutility of labour plays a very restricted part in the works of great economists . . . Pigou [1933, GMA] does not mention it. To this [Keynes (1936b, p.25)] replied in exasperation:

When . . . you tell me that ‘the marginal disutility of labour plays a very restricted part in the works of the great economists’ I am again simply staggered. The whole of Pigou’s Theory of [Un]Employment which is a book you have reviewed, is based on it and on nothing else. Pigou’s theory in its entirety is based on the level of employment being the resultant of what he calls the real demand for labour and the supply of labour given by the schedule of the different quantities of labour, the marginal disutility of which is balanced by differing amounts of wage goods. Moreover, the whole of Marshall’s theory is based on this and, above all, the whole of Jevons’s theory. In fact, there is no other theory that I am acquainted with.

It is thus clear that it is essential to have a good understanding of the “theory of the marginal disutility of labour” as inherited from orthodox theory since Jevons, if one wants to understand Pigou and Keynes – at least this is so as it emerges in Keynes’ correspondence.

When restating the General Theory towards the end of the book in ch.18 [Keynes (1936a, p.245)] expressly lists among his “givens” “the tastes and habits of the consumer” – in other words: his preferences and his basis for intertemporal decision making – and “the disutility of different intensities of labour”. This means that he clearly thought in these terms and that he expected his theory to be understood in these categories (see also below, page 50).

3.4. The common problem of macroeconomic aggregation. Mark [Hayes (2007)] recently observed (in an article about Keynes’ Z-function with which we will also deal below):

Much of modern theory assumes an aggregate production function for output as a whole, blithely ignoring Keynes’s methodological warning that the assumption of homogeneous output and capital is fundamentally inadmissible in a macroeconomic theory of the monetary economy. In this respect, Classical theory has not moved very far beyond Ricardo’s corn model.
Let us therefore stress: this was not a bone of contention between Keynes and Pigou. They both disliked the idea of a single “macroeconomic production function”.

As is well known, this dislike poses great problems. It is an enormous analytical challenge – which is unresolved to this very day, vide Hayes’ comment – to combine individual or sectoral economically plausible relationships to an aggregate relationship of a similar nature. If we have, for example, a total economy with one wage-goods sector and one non-wage-goods sector, then there is the problem that the workers in the entire economy will calculate their real wage – and depending on the real wage their labour supply – in terms of just these wage goods. But total employment has also some workers in the non-wage-goods sector. The real wages as given by the marginal product in this other sector are, however, in terms of non-wage goods. For firms it is the real wage in terms of this sector which is relevant for employment. For workers working in this sector it is, however, the real wage in terms of wage goods which is relevant. We thus have a heterogeneity in real wages entering the economic decisions.

Pigou (1933, p. 90) got around this problem on the labour demand side of his disaggregated model economy. He postulated a functional relationship between sectoral and total employment of the type:

\[ N = \phi(N_1) \]

where \( N \) is total employment and \( N_1 \) is sectoral employment in the wage goods sector (symbols, except for \( \phi \) follow Keynes’ convention, not Pigou’s, who uses, e.g. \( x \) instead of our \( N_1 \)). It is quite a challenge to trace the genesis of Pigou’s \( \phi \)-function. It can be shown, however, that under special conditions (production and utility functions of Cobb-Douglas type) this function will have an elasticity of unit value (Ambrosi, 2003, p. 65, equ. (9.20)). In the Cobb-Douglas case, however, sectoral employment \( N_1 \) is a well defined (negative) function of the sectoral real wage \( F'_1(N_1) \), the derivative of the sectoral production function \( F_1(N_1) \). It can then be seen – in spite of arguing in a disaggregated context – that there will be a reliable (negative) relationship between the real wage in terms of wage goods on the one hand and total employment on the other hand. This is Pigou’s “real demand for labour” – the centerpiece of his Theory of Unemployment. Keynes (1936a, p. 273) reconstructs this concept and declares:

Professor Pigou’s “elasticity of the real demand for labour in the aggregate” is a concoction similar to some of my own, depending partly on the physical and technical conditions in industry (as given by his function \( F \)) and partly on the propensity to consume wage-goods (as given by his function \( \phi \))

It is through such passages that we may confirm that Keynes very consciously saw his own constructions in relation to those of Pigou (1933) and accepted many of its elements in principle. Thus it is “legitimate” in this sense to take this piece of “orthodox” analysis as a starting point for reconstructing the analysis on which Keynes’ own argumentation in the GT may be reconstructed.
3.5. **Keynes’ Pigou-oriented criticism.** It could be said that Pigou thus solved the aggregation problem: he showed in a disaggregated context that the workers’ real wage *must* be reduced if there is to be increased employment. Thus his model was indeed more sophisticated than even many modern representations of the economy which do not bother with any sectoral disaggregation.

But we saw above that Keynes does not deny the “vital fact” of orthodox economics that higher employment goes with lower real wages. The problem with Pigou’s model is, however, that we have here an entry door for a number of fallacies and misconceptions:

(1) If we are in a well functioning labour market, we must say something about labour demand and labour supply. But if we regard the comparative statics of labour market equilibria, only very limited observations can be made about changes of the level of employment. The ensuing theory is not about unemployment in the strict sense, but about equilibrium employment – plain and simple.

(2) If the labour market does not work properly – and if there is involuntary unemployment in the strict sense this is the case – then we must have a proper theory why that is so.

(3) If we have a temporary phenomenon like extraordinarily high rates of unemployment, then we must have a theory which can take account of these temporary effects. There must be at least conceptually an element in the theory of employment which admits for some degree of volatility. This element is missing in [Pigou (1933)](https://example.com).

Important parts of the GT are elaborations of and reactions to the peculiarities of the Pigovian classical analysis just listed. Point (1) of this list is the basis for [Keynes (1936a, p.274f.)](https://example.com) calling Pigou’s ‘Theory of Unemployment’ “something of a misnomer” and of stressing that employment in that framework can only change when the parameters describing technology or preferences change.

Point (2) of this list is behind Keynes’ claim that Pigou has three variables – sectoral labour demand \( (N_d^1 \text{ and } N_d^2) \) and total labour supply \( (N_s) \) ([Keynes (1936a, p.274)](https://example.com)) but only two equations, namely his \( \varphi \)-function (above, equa.(1) ) and a labour supply function \( N_s = \chi(N_i) \). In other words: if you want to show that there is involuntary unemployment in the sense of \( N_d^i < N_s \), then you cannot use \( N_d^i = N_s \) as under point (1). But then you need a new relationship – and that is missing. It is Keynes’ implicit claim that he does supply this “missing equation” and so it is one of the main tasks of interpreting Keynes to discuss this “missing equation”.

Point (3) is put by [Keynes (1936a, p.279)](https://example.com) in the following way:

I repeat that Professor Pigou has altogether omitted from his analysis the unstable factor, namely fluctuations in the scale of investment, which is most often at the bottom of the phenomenon of fluctuations in employment.

This again is a programmatic statement because Keynes believes, of course, that he can supply this very point of aggregate economic analysis.
It is now the point in our inspection of the relation between Keynes and Pigou to regard in more detail how Keynes developed his anti-Pigovian positions while relying on post-Pigovian analysis.

4. The General Theory in an Anti-Pigovian Perspective

4.1. The Z-functions. As Mark Hayes (2007, p.742) pointed out recently, Keynes’ construction of a linear Z-function depicting the value of aggregate supply, “has caused no end of trouble”. Particular chagrin is caused by “the Footnote”:

[1] For example, let us take \( Z_w = \phi(N) \) . . . as the aggregate supply function . . . Then, since the proceeds of the marginal product is equal to the marginal factor-cost at every point on the aggregate supply curve, we have

\[
\Delta N = \ldots = \Delta Z_w = \Delta \phi(N),
\]

that is to say \( \phi'(N) = 1; \)

[2] provided that factor cost bears a constant ratio to wage-cost, and that the aggregate supply function for each firm (the number of which is assumed to be constant) is independent of the number of men employed in other industries, so that the terms of the above equation, which hold good for each individual entrepreneur, can be summed for the entrepreneurs as a whole. This means that, if wages are constant and other factor costs are a constant proportion of the wages-bill, the aggregate supply function is linear with a slope given by the reciprocal of the money-wage [share, GMA]. (GT, p.55f, n.2 ; omissions and numbering are mine, GMA)

This “Footnote” is indeed quite a challenge. Its grammatical structure is tortuous. Its message seems to be contradictory: in the first half Keynes unequivocally states that his \( \phi \)-function has always a derivative with unit value “at every point” on the curve. In the second half he states that – under certain conditions – the slope of the curve has never a unit value, because it is given by the inverse of the share of the money-wage bill in the money-value of \( Z \), where “share” is my addition, since this is clearly what is meant by Keynes.

[9] We will see this presently when we translate this passage into graphical terms. The meaning of the notation is in part explained

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\[^{9}\text{Compare Hartwig and Brady (2008 p.818,n.2) “The last sentence of the footnote \ldots remains odd, however. Obviously, the slope of } Z_w \text{ cannot be equal to 1 and to } 1/w \text{ at the same time.” Like many other interpreters of this passage, the authors obviously fail to see that the Footnote has two parts, the first referring to points on the curve, the second to points of the curve. They also fail to see the difference between “money-wage” \textit{rates} and “money-wage” \textit{bills}. In the seemingly “odd” sentence Keynes expressly writes about wages on the one hand and “the wages-bill” on the other hand. Their admission of incomprehension which is due to their deficient interpretation is remarkable as coming from authors who \ldots proclaim “a reinterpretation is not necessary at all as the ‘infamous footnote’ can be easily understood without it.” (Hartwig and Brady 2008 p.816)\]
by Keynes himself and should also emerge from the following discussion. The basic idea is that if we have a supply of output \( Q \) and an associated price \( p \) in terms of money (Keynes himself uses \( P \) in the text) then the money value of output is \( Z = p \cdot Q \) in terms of \( £\)-s. The non-monetary “real” value is, of course, the output supplied, \( Q \). But the real value could also be counted differently, namely in terms of the amounts of “real” labour services one could buy for the money value \( Z \) at a given wage \( w \) (Keynes himself uses \( W \) in the text). Then the “real” value in terms of labour is \( Z_w \equiv \frac{Z}{w} \equiv \frac{p}{w} Q \).

One message of the quoted passage and of Keynes’ associated discussion is: do not count “real” values in terms of \( Q \). Because as soon as you have not only one sector of production but two of them, producing, say, \( Q_a \) of apples and \( Q_b \) of bananas, you have problems counting the combined “real” value. It would literally mean to “add apples and bananas”. Use “real” values in terms of “wage units” instead. Then you get \( Z_{wa} \) in terms of labour services that you could buy from the sale of apples and \( Z_{wb} \) in terms of labour services that you could buy from the sale of bananas. It is conceptually not problematic to add the “real” labour services thus calculated, giving \( Z_w = Z_{wa} + Z_{wb} \) of total labour services and hence the total real value of the aggregate supply. Otherwise you would have to add “apples and bananas” and that is quite problematic.

Matters are not quite that simple in this context, of course. One obvious problem is – and some of Keynes’ disciples made quite some fuss about this problem so that virtually no post-Keynesian does count in Keynes’ wage units – that there are different wages because labour services are heterogenous in the two sectors. The heterogeneity of apples and bananas should be conceptionally more of a problem than the heterogeneity of labour services, however. But this is a discussion which cannot be pursued here. (For further discussion of these problems see Appendix A, page 45 below.)

Another problem in this context is: the \( Z_w \)-values are labour time which you can buy with the money value of output. They are “labour commanded” values. They are calculated “real” values oriented at the output of labour \( ^{10} \). But “really real” is the labour input, the “labour embodied” in each output. There is a Ricardo-Marx-Sraffa tradition in economics to shun labour commanded values and to prefer counting economic “real” values in terms of “labour embodied” values. Keynes himself did not think much of this particular tradition, at least as far as its usefulness for combatting involuntary unemployment was concerned. It is most ironic that “post Keynesian Cambridge” became the Mekka of cultivating this particular tradition. But this is again a debate which cannot be covered here. The topicality of the problems associated with “labour commanded” and “labour embodied”

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10 On the basis of these definitions it follows tautologically that a linear curve from the origin of the \( N \) and \( Z_w \)-plane must have the slope \((pQ)/(wN)\) which is nothing else but the inverse of the share of the money wage bill in the money value of output. Since \( N \) is the “wages-bill” in wage units and \( Z_w \) is the value of output in wage units, the same result follows whether accounting is done in money units or in wage units, of course.

11 For further elaboration of this concept see Ambrosi (1988).
in this context is that Keynes, in the quoted passage, has the somewhat veiled message: given some rather “orthodox” conditions concerning production functions and competition we can combine “embodied” labour and “commanded” labour in a linear function for each sector of production. In this particular case we can not only aggregate the $Z_{wi}$-values ($i = a, b, etc.$) quite well but we can also aggregate the underlying functional relations to one aggregate relation. This is quite a neat characteristic of Keynes’ construction because otherwise functional aggregation in economics is virtually impossible, as Joan Robinson has shown very convincingly. It is this functional relation $Z_w = \phi(N)$ which Keynes addresses in “the Footnote” just quoted and it is, of course, of central importance for understanding the General Theory.
The logic of “the Footnote” is best described graphically. For the moment we abstract from disaggregation (but see Appendix A, page 45 below.) Figure 1 starts from a well behaved orthodox production function $Q = F(N)$ drawn in quadrant I. When labour input has the value of $\bar{N}$, then the profit maximal output is $\bar{Q}$ and the real wage is

$$\frac{w}{p} = \tan \angle OR\bar{Q} = \frac{\bar{Q}\bar{N}}{R\bar{N}} = \bar{Q}/\bar{Z}_w$$

The stretch $R\bar{N}$ is equal to $\bar{Z}_w = (p/w)\bar{Q}$ because otherwise the fraction would not solve to $w/p$ as required by equ.(2). The economic reason why $R\bar{N}$ is equal to the value of $\bar{Z}_w$ is: The total value of output in wage units is given by: (1) “wages in wage units” – that is: employment $\bar{O}\bar{N}$ – plus (2) profits in wage units – that is $RO$. Both stretches together give $R\bar{N} = \bar{Z}_w$, the total value of this supply in terms of the total income it generates, measured in wage units. Now mirror $R\bar{N} = \bar{Z}_w$ from its original position into quadrant IV via a 45° line. Mark the point $\bar{Z}_w$ where this line cuts a perpendicular line erected in $\bar{N}$ on the $N$-axis. This is our first point showing a connection between employment $N$ and the value of supply $Z_w$. Other such points can now be constructed by calling out alternative levels of employment $N$ and finding the corresponding co-ordinates in quadrant IV in the same way as had just been done for point $\bar{Z}_w$. The ensuing locus in quadrant IV will be the line starting from $O$ and going through point $\bar{Z}_w$ – if the conditions are fulfilled which Keynes gave in the Footnote, part [2]. The slope of this line, measured along the $Z_w$-axis in quadrant IV is given by

$$\frac{N}{Z_w} = \frac{wN}{p\bar{Q}} = \lambda$$

where $\lambda$ is the share of money wages in total money value. The slope of $OZ_w$, measured with respect to the $N$-axis is then $1/\lambda$. $OZ_w$ lies on the curve representing Keynes’ $Z_w = \phi(N)$-function. Under the conditions listed by Keynes, it has “a slope given by the reciprocal of the money-wage [share]” as we quoted the Footnote, end of part [2] above. It should be clear from the above that the missing word which clarifies the meaning of this passage is indeed “share”. We mark the corresponding curve therefore as “$O[2]$” in fig.1. Since $0 < \lambda < 1$ always holds, the slope of this curve is $1/\lambda > 1$.

For a contrary view see Davidson (2007, fig. 6.1, p.70) where a linear curve from the origin which stands for a function $Z_{w1}(N)$ has a slope < 1 with a fixed wage $w_1$. It is then clear that for any change of employment we will have $\Delta w_1 N > \Delta pQ$. In other words: any increase in employment will never be covered by an associated increase in “expected proceeds”. Thus in a free market economy an increase in employment can only come about if the loss which results from the difference between expected proceeds and the cost of additional employment is paid for by subsidies. Davidson (2007, p.70) claims that this figure represents The Multiplier. But without heavy direct subsidies to employers the minimal loss of any firm will occur at zero employment and zero production – no matter how large investment demand is. This certainly is not what Keynes meant when he wrote about the employment multiplier.
But if curve “O[2]” does represent Keynes’ $Z_w$-function, how can he claim that part [1] of the Footnote is true? The claim is that “at every point on [not of!] GMA the aggregate supply curve” there is a unit slope. But since under perfect competition every point of the $Z_w$-curve can be constructed with a $45^\circ$ line and since every such line has a unit slope, this statement is also true for every such point. In graphical terms: The “O[2]”-curve is in every point cut by a $R[1]$-curve, each having a unit slope but a different starting point “$R$”. Therefore part [1] as well as part [2] of the Footnote is true.

Thus, the Footnote is not really as contradictory as much of the literature made it appear. It quite correctly says that under conditions which amount to Cobb-Douglas technology and perfect competition, the $Z_w$-function has a constant slope that is given by the inverse of the money wage share in the money value of output. Under the stated conditions the $Z_w$-curve is a locus of intersections with $45^\circ$ lines since for every point the marginal conditions for profit maximization must hold, giving

$$\frac{\partial Q}{\partial N} = \frac{w}{p} \quad \text{hence} \quad p \frac{\partial Q}{\partial N} = \frac{p}{w} \frac{\partial Q}{\partial N} = \frac{\partial Z_w}{\partial N} = 1$$

What is now particularly anti-Pigovian in all this? In a way all this is still rather post-Pigovian because none of these elements contradict orthodox theory. They are variants of orthodox economics: profit maximization plus well-behaved production functions. The analytical elements are just arranged differently. As Keynes (1936a, p.89) put it himself in the GT:

The aggregate supply function . . . depends in the main on the physical conditions of supply. [It] involves few considerations which are not already familiar. The form may be unfamiliar but the underlying factors are not new.

Keynes thus invites us to look at his aggregate supply function with post-Pigovian eyes.

The critical aspect of Keynes’ “unfamiliar form” of otherwise familiar orthodoxy is that by these re-arrangements Keynes could make the critical point that Pigou’s

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13 Point $R$ changes when employment changes because if a new level of employment were to the right of $N$ in quadrant I, the real wage would have to be lower under conditions of profit maximization and the corresponding tangent would cut the $N$-axis in quadrant II to the left of the old point $R$. If employment were lower than $N$, the real wage would be higher, the tangent steeper and the new intersection of this tangent would be to the right of point $R$.

14 For a number of disagreeing opinions see Chick [1983, n.9, p.80] and Hayes (2007). See also Hayes (2007, p.749, equ.14) for a somewhat different discussion of what is here reproduced as equ. (4). Hartwig and Brady (2008) object to Hayes that Keynes does not use the symbol $\partial$ in this context. But Keynes states the unitary value of that specific derivative for a specific point “on the aggregate supply curve” – a very literal interpreter should take note that Keynes does not refer to the slope of the aggregate supply curve. A reader who takes “aggregate” very literal should therefore keep in mind that for such a point the real wage has a specific value and should turn to equ. (13), p.33 below for a result which corresponds to the present one in a setting which explicitly takes account of aggregation.
construction of a theory of employment using not Keynes’ “wage units” but Pigou’s “wage goods units” – in other words: consumption goods units – is just a clumsy way of theorising. Keynes’ own approach offered a number of advantages over Pigou: it combined factor inputs in terms of labour on the one hand with output *values*, also in terms of labour (commanded). Thus it was dimensionally more consistent than any production function which combined inputs like labour services with outputs (products, apples, bananas, etc.).

Maybe it is helpful to give a small example (see also Appendix A). Suppose you have two “orthodox” production functions for apples resp. bananas and two corresponding Keynesian supply functions as given by equ. (5)

\[ Q_a = N_a^\alpha, \quad Q_b = N_b^\alpha; \quad \alpha = \lambda; \quad Z_{wa} = \frac{1}{\lambda}N_a, \quad Z_{wb} = \frac{1}{\lambda}N_b, \]

where \( \alpha \) is the elasticity of output with respect to changes in labour input and where this elasticity is equal to labour’s share in output under conditions of perfect competition. In this example one cannot add apples and bananas to one aggregate \( Q \). One can also not add the non-linear corresponding production functions relating a (non-existent) aggregate \( Q \) to a total employment \( N = N_a + N_b \). But since the \( Z_{wi} \) are both in terms of labour, one does not have the apples vs, bananas problem here and one can very well add the two linear relations giving

\[ Z_w = Z_{wa} + Z_{wb} = \frac{1}{\lambda}N_a + \frac{1}{\lambda}N_b = \frac{1}{\lambda}(N_a + N_b) = \frac{1}{\lambda}N \]

It is a strange twist of discipleship that the Cambridge post Keynesians racked their brains about the disadvantages of Keynes’ choice of units. Otherwise they hardly worked with these units and preferred Pigovian-Ricardian-Sraffian consumption goods units.

4.2. Effective demand.

4.2.1. The difficulty of the concept. The doctrine of “effective demand” certainly was the element which Keynes considered to be of utmost importance in connection with the *General Theory*. This comes out maybe most clearly in a letter addressed to R.F. Harrod soon after the publication of the GT (Keynes 1936c, p.85)

You don’t mention *effective demand* or, more precisely, the demand schedule for output as a whole, except in so far as it is implicit in the multiplier. To me, the most extraordinary thing regarded historically, is the complete disappearance of the theory of the demand and supply for output as a whole, i.e. the theory of employment, *after* it had been for a quarter of a century the most discussed thing in economics. One of the most important transitions for me, after my *Treatise on Money* had been published, was suddenly realising this. It only came after I had enunciated to myself the psychological law that, when income

\[ ^{16} \text{If there are sectoral differences in the } \lambda \text{-s one has to weigh them with the sectoral share in total supply. But this is a complication, not a refutation of the aggregation for which this is an example.} \]
increases, the gap between income and consumption will increase, – a conclusion of vast importance to my own thinking but not apparently, expressed just like this, to anyone else's.

But this fundamental point about the importance and originality of effective demand was extremely difficult to bring across at that time, as this letter shows. It still seems to be so, judging by Robert Clower’s contention that Keynes’ claim to have contributed to “effective demand” analysis was just fraudulent (Clower 1994):

I have shown elsewhere (Clower 1989) that the theory of effective demand presented in Chapter 3 of The General Theory is a straightforward variant of the Marshallenesque partial equilibrium macromodel . . . On this showing Keynes’s excoriation of Ricardian economics for neglecting “aggregate demand” [JMK VII, 32-3] is a sham. . . . [Keynes, GMA] committed a (probably intentional) fraud.

4.2.2. Walras’ Law and the orthodox view of unemployment. Consider fig. 2 which is a variant of the standard elementary “orthodox” model of an economy (Varian 1999). The production function again is given by \( F(N) \). If for some reason – to be discussed later – output is fixed at \( \bar{Q} \), then the profit maximizing real wage is \( w/p \) and the economy’s budget line is \( RL \). Labour demand \( N_d \) is profit maximizing as is goods supply \( Q^s \). Given the budget line \( RL \) households want to be on the tangent indifference curve \( U \) with labour supply \( N^s \) and goods demand \( Q^d \). If the economy is fixed by \( \bar{Q} \), we have involuntary unemployment \( N^s > N^d \) and involuntary under-consumption \( Q^d > Q^s \).

Figure 2. Involuntary unemployment

Preliminary version – January 25, 2009
Let us briefly characterize further this situation of involuntary unemployment by regarding the rectangular triangle to the right of point $Q$. Its adjacent side is given by the difference between labour supply and demand, i.e. $N^s - N^d$, its opposite side by $Q^d - Q^s$. Therefore we have

\[ \frac{w}{p} = \frac{Q^d - Q^s}{N^s - N^d}, \text{ hence } w(N^s - N^d) = p(Q^d - Q^s) \]

In other words: the value of excess supply of labour in this situation of involuntary unemployment is equal to the value of excess demand for goods, i.e. equal to the “negative excess supply of goods”. This is plausible if we realize that the households wanted to buy goods with the money they wanted to earn but could not earn because of involuntary unemployment. But this means that the sum of excess supplies in this situation is zero, a condition known in the literature as “Walras’ Law”

\[ w(N^s - N^d) + p(Q^s - Q^d) = 0 \]

Thus we have here a situation in which “Walras’ Law” holds but involuntary unemployment reigns in the sense of Keynes (and Pigou, see above, p.9).

The orthodox neoclassical theory would now say: in this situation there should be a tendency for unemployed workers to bid down money wages and for unsatisfied consumers to bid up prices. The real wage will go down, the economy will leave point $Q$ on $F(N)$ to the right, line $RL$ will tilt and move until a point is reached where a tangent indifference curve will also be tangent with the production possibility curve and full employment will be reached. Thus, when there is involuntary unemployment, the most sensible advice seems to be: wage adjustment (down) and / or price adjustment (up). If wages are rigid (Pigou), public works and easy money might cautiously be recommended: they will drive prices up and real wages down even if money wages are rigid. If we have also rigid prices, we have a “fix-price” economy in which government spending and monetary expansion might also be advisable – but only to a very limited extend (Clower, Leijonhufvud, Malinvaud, etc.).

It is this logic which Keynes considered to be fundamentally flawed – but not because the underlying model is wrong. It is because “effective demand” had no chance to enter the analysis.

4.2.3. Keynes’ effective demand view of unemployment. One of the problems with Keynes’ GT is that in the exposition of its basic tenets there are at least two 45°-lines which have caused much controversy. One of these two has just been discussed in connection with Keynes’ “Footnote”. We showed that the 45°-line mentioned there by Keynes is a representation of the equilibrium condition of profit maximization under conditions of perfect competition in the sense of equ.(4), p.19. The locus of all the equilibria described by such an equilibrium condition – and by its graphical representation through a 45°-line – is then, of course, not a 45°-line but the $Z_w$-curve which was represented in fig.1. The exact construction of that $Z_w$-curve is maybe not so important for the understanding of the GT – apart from the fact that
it illustrates that Keynes, in algebraically describing that 45°-line in his Footnote, was contented to accept “orthodox” assumptions in this respect.\footnote{It should be noted that if the assumption of perfect competition is not made, then eq. (4) does not hold, and the associated 45°-line does not exist — but that is another matter. The question to be discussed above was: what is Keynes’ 45°-line in the context of his Footnote?}

Quite a different 45°-line is based on the tautology

\[(9) \quad Y \equiv pQ \equiv W + P\]

which says: income $Y$ is identical with the total value of output $pQ$ and this is identical with the total value of wages income $W \equiv wN$ plus non-wages income $P$.\footnote{This refers to the standard neoclassical model where output is always net output so that depreciation is no problem in the proper calculation of net income.} This is the “proposition which is indubitable, namely that the income derived in the aggregate by all the elements in the community concerned in a productive activity necessarily has a value exactly equal to the value of the output” (Keynes 1936a, ch.2). From equ. (9) we derive $Y/p \equiv Y^r \equiv Q$ and represent this in quadrant II of fig. 3 as a 45° line between the output axis marked $Q$ and the real income axis $Y^r$. Quadrant I of this figure is essentially the same as the one of the previous figure. It preserves all that had been said before about neoclassical orthodox economics and profit maximization. It is the post-Pigovian aspect of the GT which in my reading of the book remains valid for interpreting the GT.

\[\text{Figure 3. Keynes’ Effective Demand}\]

The essential novelty of the GT, not to be found in any previous orthodox economic writings, is the following passage from GT, ch.3
(1) In a given situation of technique, resources and costs, income (both money-income and real income) depends on the volume of employment $N$.

(2) The relationship between the community’s income and what it can be expected to spend on consumption, designated by $D_1$, will depend on the psychological characteristic of the community, which we shall call its propensity to consume. That is to say, consumption will depend on the level of aggregate income and, therefore, on the level of employment $N$, except when there is some change in the propensity to consume.

(3) The amount of labour $N$ which the entrepreneurs decide to employ depends on the sum ($D$) of two quantities, namely $D_1$, the amount which the community is expected to spend on consumption, and $D_2$, the amount which it is expected to devote to new investment. $D$ is what we have called above the effective demand.

Item (1) in this quote stresses the real aspect of Keynes’ economic analysis as depicted here by “quadrant I”. It relates Keynesian economics to orthodox economics. It stresses that employment $N$, as envisaged in the Keynesian system, must be seen under explicit reference to technology and costs. After all that has been said before, it should be clear now that the elements of Keynes’ theory mentioned in this context are the ones inherited from “orthodox” economic analysis and which he considered to be “indifeasible” (See above, p. 10).

Item (2) describes consumption demand, depicted here by the $D_1^r$-curve in quadrant II. It is a representation of the consumption function. This function is actually not yet quite anti-Pigouvian because it can be shown that it is a more general form of the time-preference theory of the rate of interest which was propagated by Pigou, especially in order to distance himself analytically from Keynes after the publication of the GT [Pigou 1937]. This point is elaborated below in appendix B p. 52f.

Item (3): The revolutionary aspect of Keynesian economics is the subsequent claim that the level of employment is determined by a magnitude “$D_2$”, represented in real terms as $D_2^r$ in quadrant II. This additional magnitude determines the level of total demand $D$ resp. $D^r$. This in turn determines output $Q$. Entrepreneurs’ expectation of this output finally determines employment $N$. Thus, in Keynesian analysis, there is a clear interconnection between the two analytical worlds of quadrant II and quadrant I, but there is no such interconnection in the orthodox case.

In view of the claim that Keynes’ effective demand analysis was a “fraud” we have to be very clear that there are here two very different – but in the Keynesian case interconnected – frameworks for arguing about economic activity and employment: There is (1) a “quadrant I oriented view” which states: the economic system is determined by relative prices, i.e. by the slope of the L-line. If this slope is “wrong” in the sense of generating an excess supply of labour, then the situation can always be rectified by an appropriate lowering of the real wage. Unless there is undue market interference through labour unions, government action, sluggishness of market...
participants, etc. there is always a natural tendency towards market equilibrium, as
was discussed above in connection with equ.(8), p.22.

On the other hand, there is (2) the “quadrant II oriented view” which says: if we
have a definite value of (expected) real turnover \( Q \), then it does not help to lower
wages in order to lower unemployment. Any wage cost reduction will be passed on
to prices because firms might fear to lose their expected turnover if they do not pass
on cost reductions. In this situation wages and prices will drop by equal percentage
points, the real wage will be constant. Involuntary unemployment will be constant
– unless we have an increase in “effective demand”.

The orthodox paradigm of employment policy (1) can very well advocate policy
measures against unemployment – but only insofar as those measures can lower the
real wage ( when technology, capital input, level of competition, etc. are given).

The orthodox position does not necessarily preclude moderate state intervention.
In case of wage rigidity public works and expansion of the monetary base might
do not too much harm and even a little bit of good. They drive up the price level
and thus they might be good for lowering the real wage. If that happens, firms
will respond to higher relative prices and higher profits and they will employ more
workers. So, do not the “orthodox” economists and Keynes make the self-same
policy recommendations? An affirmative answer to this question seems to be a
wide-spread fallacy among Keynes-sceptical economists.

Contrary to the above orthodox argument which allows only for “meek interven-
tion” by economic policy, Keynes’ paradigm (2) suggests “decisive intervention”.

Only Keynes’ approach opens the perspective of the eventuality of deficient effective
demand. If real demand in his sense is indeed missing, then public works would
raise demand directly and through a fortifying multiplier mechanism and not just
indirectly through their impact on the price level. Similarly, in Keynes’ analysis ex-
pansionary monetary policy is not potentially beneficial for the level of employment
because it might drive up prices as in the orthodox argumentation. In the Keynesian
case monetary policy might enable more investment through lower interest rates.
Again there would be an immediate and a fortified employment effect, justifying
this type of active economic policy. Thus, in ways which cannot be elaborated here
in detail since we are dealing here with a very elementary graphical context, the
effects of fiscal and monetary policy enter \( D_2 \) resp. \( D'_2 \).

Figure 3 not only illustrates that if firms expect higher turnover therefore they
would produce more, hire more workers and leave \( Q \) towards the right. In addition,
quadrant II gives answers with regard to two points of critique that Keynes raised
against Pigou (see above, p.14),

- The \( D \)- resp. \( D' \)-function supplies the “missing equation” – the equation
  which the Pigovian model – and hence the conventional classical one – needs
  in order to change it from a model of employment to one of involuntary
  unemployment. Such an additional equation is necessary for showing that in
  order to generate involuntary unemployment as a conceptual possibility, the
  model economy must be capable of generating an economic constraint which
  can explain economically why the system does not move “automatically”
to the full employment position. In other words: the “orthodox” model must be over-determined in such a way that involuntary unemployment is conceivable at all without resorting to ad-hoc arguments about rigidity of wages and prices.

• Since total $D$ is determined by $D_2$ and this magnitude in turn contains investment demand, we have here an answer to Keynes’ critique that Pigou’s system displays far too little volatility. Investment demand is known to be highly volatile over the business cycle and thus we do have in Keynes’ system an element to which the Pigovian system did not give sufficient attention in Keynes’ eyes.


4.3.1. The irrelevance of non-Walrasian worlds. There has been much debate during the last decades whether Keynes’ approach is based on the negation of “Walrasian economies”. Some authors claim that the General Theory requires the postulation of “non-Walrasian” economic worlds. As Frank Hahn (1978, p.1) put it:

If non-Walrasian models are rejected on whatever grounds, then it would seem must properly formulated Keynesian models . . . because as long as market opportunities are described only in the Walrasian way no distinction can be drawn between demand and effective demand.

In other words: if any rejection of non-Walrasian economic setups would mean the rejection of “properly formulated” Keynesian models, then Keynesian models and Walrasian ones must be mutually exclusive. But what exactly are Walrasian and “non-Walrasian” economies? Hahn (1978, p.1) starts from the following definition:

I shall call an economy non-Walrasian whenever the trading possibilities of agents cannot be described as the set of trades which at given prices make the value of purchases no greater than the value of sales. If we eliminate the double negation contained in this definition, we could say that in a Walrasian economy “the trading possibilities of agents” can “be described as the set of trades which at given prices make the value of purchases no greater than the value of sales”. This definition is somewhat puzzling because if taken literally, it seems to contain a tautology: since every sale of any economic good requires a purchase of the self-same good because there cannot be a seller who does not have a buyer, therefore sales and purchases must always be identical in value, in quantity, and in price. They are just different forms of book-keeping of the self-same economic transaction. To say that purchases are “no greater” in an economy than sales seems to be a modest understatement: both are identical – and this identity is valid under any circumstances, no matter whether the sales – purchases identity is situated in a “Walrasian” or in a “Keynesian” world. The important and non-trivial aspect of the quoted definition seems to be the idea that the “trading possibilities” mentioned in the definition are not identical with the conceivably realized turnovers. But that means: trading possibilities may occasionally be one thing, actual turnovers might
be quite a different thing. The essence of a Walrasian economy therefore seems to be that it combines conceptually these two possibilities.

A combination of conceptually determined turnovers with alternative trading possibilities is indeed what emerged when we discussed “Walras’ Law” in connection with equ.(7), p.22 above. The situation discussed in that context was one of “involuntary unemployment in the strict sense”: at the going real wage there were $N^s - N^d$ workers unemployed who would have liked to work at that real wage, as is immediately visible from regarding figure 2. Households also would have liked to buy $Q^d$ of goods. Thus, given the budget line $RL$, there would have been trading possibilities in point $N^s, Q^d$ on that line. These possibilities are in a way nonsensical because the co-ordinates of the household equilibrium describe a point which is beyond the production possibility curve $F(N)$. But in another way these possibilities are quite sensible because they are “anchored” in point $Q$, that is the budget line is consistent with a feasible point, since $Q$ is a point of the attainable set of the economy. If this point is one of “effective demand” it is a point where sales and purchases are identical in the sense of being conceivably observable turnovers. So, point $Q$ gives the tautological part of Hahn’s just quoted definition of a “Walrasian economy” whereas the budget line going through this point gives the non-trivial possibilities of that definition. In any case: no matter whether we regard the household equilibrium point $N^s, Q^d$ as nonsensical or not, the magnitudes of supply and demand which define this point are consistent with “Walras’ Law” as expressed by and discussed in connection with figure 2, p. 21.

Algebraically, “Walras’ Law” was expressed by equ.(8) which, in still different versions may be written as

(10) \[ wN^s + pQ^s = wN^d + pQ^d \]

or, in general \[ \sum S_i = \sum D_i \]

where $S_i$ is the value of individual supply and $D_i$ is the value of individual demand for the respective economic goods (products, services).

We saw at the beginning of this subsection that Frank Hahn justified the call for “non Walrasian” economies in Keynesian analysis with the claim that Walrasian economies would not permit a discrepancy between demand and effective demand. But the statement of Walras’ Law that the sum of excess demands is zero by no means means that every single excess demand is zero as the above discussion should have shown. Figure 2 illustrated the case that there could very well be a discrepancy between supply and demand in a straightforward “Walrasian” world so that “involuntary unemployment” could very well exist in such a Walrasian world. The subsequent figure 3 then illustrated that Keynes’ effective demand analysis made it plausible and analytically tractable that such a disequilibrium Walrasian world may be observable. Thus, Keynes’ effective demand analysis and Walrasian accounting go perfectly well together. The approach laid out in the General Theory by no means calls for an abandonment of “Walras’ Law”. We would rather say: Keynes’ method requires “Walras’ Law”.

In order to clarify the controversial matter of Keynes’ analytical relation to “non-Walrasian economies”, let us suppose for a short while that we are in such an
economy in which Walras’ Law does not hold. An example of such a situation is given in figure 4. This figure is based on the “Walrasian” model of fig. 2, p. 21. But whereas the trading possibilities were on the budget line which here is marked as $RL^U$, the trading possibilities with households are now given by the parallel line $L_{HH}^U L_{HH}$. Thus, the trading possibilities do not cover point $Q$, the only point where profit maximal sales and purchases can occur when the first classical principle is valid, i.e. when the real wage is equal to the marginal product of labour.

The household plans now find their constrained utility maximum in the “non-Walrasian” point $Q^{NW}$.

What is the assumption when such a discrepancy is postulated between the budget line covering $Q$ and the household budget line $L_{HH}^U$? The two budget lines are given algebraically by

$$wN^d + \Pi^U = pQ^s \text{ resp } Q^s = \frac{\Pi^U}{p} + \frac{w}{p} N^d$$

and by

$$wN^s + \Pi^{HH} = pQ^d \text{ resp } Q^d = \frac{\Pi^{HH}}{p} + \frac{w}{p} N^s$$

In other words: if the relative price system is given by $w/p$, then there is a discrepancy of planning budgets and trading plans if firms intend to generate non-wage income $\Pi^U$ whereas households at the same time plan to receive non-wage income.
Π^HH. This difference is represented graphically by the difference of the intercepts \( O^U / p \) resp. \( O^HH / p \).

Now, simple inspection of fig. 4 shows that such a situation does not make economic sense: if households made their plans without any reference to the attainable set of available goods – this set is delimited by the production possibilities as given by \( F(N) \) –, then household plans could be just anywhere in the goods space. As fig. 4 shows, they could quite well be entirely outside of the attainable set of goods. It is then physically impossible in a closed economy to satisfy their plans. But does it make sense in economic analysis to postulate such a planning situation?

Such a situation might be justified with divergent expectations. Divergent expectations are, of course, conceptually plausible. It is possible to imagine a situation in which, say, there are 10 people in an economy who mistakenly all believe that they win at the sweepstakes 1 mio £ each although there are not 10 mio £ in the jackpot but only 1 mio £. But it is inconceivable that when it comes to settling the accounts for this economy at the end of the time period under consideration, that then these mistaken plans will in fact have any economic relevance. The accounts of the economy can only show that – in case the winning number is drawn – there will be 1 mio £ leaving the lottery and this very amount must reach the recipient – at least as far as accounts are concerned. But this means: if we contemplate an economic situation which can be described by respecting rudimentary bookkeeping rules, then it is inconceivable to have a discrepancy between, say, \( \Pi^U \) and \( \Pi^HH \). Consistent accounting which obeys the fundamental double-entry rules concerning firms paying incomes to households can only be along the \( RL \)-line. But if this is the relevant accounting framework for firms and households, we are in an analytic world where Walras’ Law does hold.

Of what relevance is all this to Keynes and the GT? Keynes obviously thought in terms of a point like \( \bar{Q} \) in fig. 4. He expressly subscribed to the First Classical Postulate. That postulate is indeed fulfilled in point \( Q \). Keynes accepted the bookkeeping principle that net production is identical to net income, since any net production must accrue to some household as income received. In addition, Keynes assumed that short period expectations are met so that there cannot be a discrepancy between \( \Pi^U \) and \( \Pi^HH \). All this means that, as far as Keynes and the GT is concerned, we are in a totally Walrasian economy, characterized by a single, consistent budget line for households and firms, say line \( RL \).

This does not meant that involuntary unemployment is an impossibility by assumption. We have shown in the context of the “Walrasian” model of fig. 2 that involuntary unemployment is quite consistent with “Walras’ Law”.

There are a number of important conclusions following from the considerations of this section:

1. Analytically, the General Theory is not situated in a “non-Walrasian world”
2. The General Theory relates to economic situations where the principles of double-entry bookkeeping apply.
3. The General Theory regards changes in an economy when effective demand changes. Its method is “comparative static”.

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Since bookkeeping means to record data of actual transactions (or of conceivably “actual” transactions), it is an ex-post method. But the argumentation of the GT asks: what would happen in an economy if effective demand changed. In this sense the GT is anticipatory. Combining these two characteristics of the GT we conclude:

The fundamental method of the General Theory is anticipated ex-post analysis.

4.3.2. The generality of the General Theory and the irrelevance of Say’s Law. In the title and in many places of the text of the General Theory Keynes made the claim that his theory of effective demand-determined (un)employment is more general than Pigou’s (1933) Theory of Unemployment and the entire orthodox theory of the working of the economic system. This claim has many aspects: is it just full-mouthed braggadocio and is the General Theory really a special one? Publication titles like the one of his disciple Joan Robinson (1954) claiming The Generalization of the General Theory … suggest that Keynes’ own theory is deficient as far as generality is concerned. If one does accept Keynes’ claim to generality, however, then the central question is: what exactly is so narrowly special about the orthodox theory? One answer to such a question was already mentioned in this text: the time preference theory of interest is a special case of Keynes’ consumption function. A further answer can point to the fact that Keynes’ theory does accept the relative price analysis of the orthodox school, but that the opposite does not hold. Therefore, Keynes’ theory is more general in the sense of encompassing more theoretical cases than the orthodox theory does.

The orthodox – relative price oriented – school cannot accept Keynes’ effective demand arguments because that contradicts their claim that it is only relative price adjustments that count and nothing else. This conviction they can maintain by either ignoring “quadrant II”-type analysis or by more or less implicitly making assumptions about what is going on analytically in that context. If they were explicit about those assumptions they ought to say in some detail why the working of the price system guarantees that there is no influence from the effective demand side. In the context of quadrant II such an immunisation against effective demand arguments must amount to the claim that the $D$-curve always is identical with the 45°-output-income curve. This means that every level of output generates its required demand. Keynes dubbed this statement as “Say’s Law”. There is much debate about the appropriateness of that name. But the substance of this “law” is clear: if it is fulfilled, it is indeed just relative prices that determine the outcome of the working of the economy. If it is not fulfilled, however, it is futile to only discuss relative price movements because the really binding constraints might be somewhere else, namely in the deficiency of effective demand. This does not mean that deficient effective demand is always a problem. Indeed, Keynes went out of his way to stress that his system merges with the classical one under conditions of full employment. In figure

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19See Harcourt (2005) p.20 who points out: Joan Robinson “along with Kahn and Kaldor as contemporaries and Sraffa again an enigmatic onlooker, moved to generalize The General Theory to the long period.”

20See Kates (1998)
we respected this view in limiting the length of the 45°-line so that its maximum 
co-ordinate on the Q-axis corresponds to the likely full-employment level of output. 
It has sometimes been said that Keynes just reversed “Say’s Law” law so that there 
is an even more absurd “Keynes Law” claiming that demand creates its own supply. 
That clearly is not Keynes’ claim and it is not the case. By linking the Keynesian 
analysis of quadrant II visually to the orthodox relative-price analysis of quadrant I 
it should become obvious that Keynes’ analysis does respect the essential principles 
of economic thinking.

The reader should note that there were two complaints which Keynes (1936c, 
p.85) articulated against his disciple Roy Harrod after he tried to give a simplified 
account of the GT (later published as Harrod (1937)). The one was that Harrod, 
in presenting Keynes’ theory, did not even mention effective demand. We quoted 
that passage above (p.20) already.

Keynes’ second complaint against Harrod was (ibid. p.85f.) :

You do not show how in conditions of full employment, which I should 
now like to define as the limiting case in which the supply of output 
schedule causes to be elastic, my theory merges in the orthodox the-
ory.

Keynes did not write such a statement in order to belittle Harrod. He went out of 
his way to compliment Harrod for this article. But it should be obvious from this 
passage that Keynes was considerably concerned that his “disciples” do convey that 
his theory “merges in the orthodox one”. This insistence of Keynes’ is a far cry 
from Joan Robinson’s claim that the GT “called for a radical reconsideration of the 
whole corpus of orthodox doctrine” (see above, p.5). Certainly Keynes was aware 
that his claim for a General Theory meant that economists ought to understand 
that his theory was an encompassing theory and not just a negating one.

The Classical School, in taking relative price movements as the essential deter-
mining factor for employment, must claim that \( D = Y \) is essentially no problem at 
all. In not regarding effective demand under any circumstances, the Classical School 
must subscribe to \( D = Y \) as an additional special assumption – whether it is called 
“Say’s Law” or not, or not even mentioned.

4.4. Linking up with the Z-curve. The presentation of Keynes’ doctrine of “ef-
fective demand” with the help of fig.3 is only a very rudimentary approach to his 
ideas. It is, so to speak, the reaction of a reader who knows orthodox economic 
doctrine when he is first confronted with the arguments that Keynes (1936a, ch.3) 
presents at the very beginning of the GT. But that type of approach will not suf-
fice after Keynes (1936a, ch.4) proceeded to propagate macroeconomic analysis 
not in terms of quantities of goods but in terms of labour services, i.e. in terms 
of wage units. Fig. 5 therefore links up the former exposition of effective demand 
with Keynes’ novel concept of a \( Z_w \)-curve. Since the characteristics of this curve 
have been discussed above already, we can limit ourselves to commenting briefly 
the demand side. Fig. 5 depicts the argument of quadrant II in an analogous way 
in quadrant IV in the \( N - Z_w, D_w \)-plane. If we subscribe to a special variant of
the consumption or savings function, namely to a savings function known in growth theory as the “extreme classical savings function”, then workers consume all their wages and non-workers consume nothing at all.\footnote{From the standpoint of intertemporal allocation theory the model world corresponding to the “extreme classical savings function” justifies the jibe that economics is a “dismal science”: workers in this world cannot save because they are starvation constrained, non-workers cannot consume. They are “worldly ascetics” for whom present consumption has zero elasticity of utility so that $\alpha^0 = 0$ holds for all of them.} Then $C = wN$ or $C_w = N$ and
Keynes’ $D_w^1 = \chi(N)$-function becomes a $45^0$-line. The level of employment $\bar{N}$ is determined by total demand $D_w = D_w^1 + D_w^2$ meeting aggregate supply $Z_w$. The “black box” in this analysis is, of course, the exact determination of the magnitude of $D_w^2$. This “black box” is opened later by Keynes when he discusses the “inducement to invest” (GT, Book IV). But before analyzing the determinants of $D_2$ resp. $D_w^2$ in greater detail, Keynes obviously wanted to stress the analytical significance of this component of demand – and the analytical significance of the fact that aggregate demand must meet aggregate supply. This important point is made in quadrant IV in a way which is more faithful to Keynes’ original writing than the exposition in quadrant II, resp. in connection with fig.2. But apart from the – very relevant – issue of aggregation, the two representations are equivalent as we may gather from the synoptic presentation in fig.5.

Fig.5 might be interesting in assessing the proposal to change Keynes’ Footnote (GT, p.56) so that the last sentence there reads “the aggregate supply curve is linear with a slope given by the reciprocal of the .. [real - proposed change, GMA] wage.” (see Hayes [2007] p.751) A look at fig.5 shows that this cannot be correct. Any move along the $Z_w$-curve in quadrant IV implies a simultaneous move along the production function $F(N)$ in quadrant I – or, if we keep in mind the aggregation problem discussed above – it implies changes along the production functions (plural) which went into the aggregate value $Z_w$. In any case, any move along the production function(s) means a change in real wages in the context of the GT, since it is expressly based on the validity of the “first classical postulate” with its implication of falling real wages when output increases. Therefore it is conceptually impossible that a move along the $Z_w$-curve should express constant real wages.

Hayes (2007, p.751) uses in this connection the condition that “at the point of profit-maximization” we have “marginal factor cost equals average factor cost”. But as this quote already states: such a point is just one single point. The average cost curve cuts the marginal cost curve just in one point, thereby determining a single level of costs (total, marginal, average) and a single value of output. Thus, if we use this condition in the discussion of the $Z_w$-function, we move from a derivative of the $Z_w$-function as a locus of a multitude of equilibrium points to a quite different derivative, namely a derivative in one single point of that function or curve. It is that point for which constancy of the real wage is postulated. Such a special derivative gives

$$\frac{dZ_w}{dN} \bigg|_{\frac{w}{p}=\text{const}} = \sum p_i \frac{\partial Q_i}{w \partial N_i} dN_i = \sum \frac{dN_i}{dN} = 1 \quad \text{since} \quad \frac{\partial Q_i}{\partial N_i} = \frac{w}{p_i}$$

22This follows also from the definition of $Z_w \equiv pQ/w$. A linear curve from the origin of the $N$ and $Z_w$-plane must have the slope $(pQ)/(wN)$. But this slope could be equal to the (inverse) real wage only if $Q = N$. Keynes nowhere states or even insinuates that he made such an assumption.

23If labour is not the single factor that varies but all the other relevant factors vary as well and if nominal prices stay constant or vary in an appropriate way, then the real wage may be constant while the input of labour changes. But this assumption was not the one which Keynes made in the context of discussing his $Z_w$-function. Even if economic conditions were so that the real wage were constant, that would not affect the slope of the $Z_w$-curve as will be shown in appendix C below.
and this is what Keynes did suggest in the first part of his Footnote. It is substantially the same condition as the one given already when we derived the condition for profit maximization (equ.(4), p.19). In the Footnote Keynes – maybe clumsily, but validly – stressed that the algebraic discussion of such profit maximizing equilibrium conditions must be discerned from algebraic discussions of movements along the $Z_w$-function as a locus of different equilibrium points. Thus, we see no reason to change the wording of Keynes’ Footnote – apart from the addition of the word “share”, as discussed above.

But it is an interesting suggestion to express the $Z_w$-curve in relation not to employment but in relation to output. Such a curve $Z_w(Q)$ is given in quadrant III of fig.5 where we make use of the 45°-line of quadrant II so that we can say that $Y^r = Q$. In the discussion of this curve we “fall back” to the position that in spite of all that we know about aggregation problems an aggregate production function may be used for some pedagogical points of illustrating economic interrelations. But while making such exercises it must be well understood that if aggregation is a serious issue, then the only consistent relation (from the standpoint of aggregation) is given by the $Z_w(N)$-curve as a curve relating to aggregates.

With the provisos just stated in mind, a most simple algebraic form for a $Z_w(Q)$-curve in quadrant III can be given by the following equ.(14b). It is a nonlinear exponential function. The linear expression $Z_w = (1/\lambda)N$ is changed to a nonlinear form, because if we have a production function $Q = N^\alpha$, where under perfect competition $\alpha = \lambda$ then we get $N = Q^{1/\alpha}$; replacing $N$ in the aggregate supply function with this expression then gives the new expression used in equ.(14b).

A ray from the origin $O$ to a point on the $Z_w(Q)$-curve, say the one going through point $X$, measures average factor cost in wage units. That ray has a slope given by the tangent value as written in the following expression (14a):

\[
Z_w/Q = \tan \angle Y^rOX = \frac{p}{w} ; \quad (b) \quad Z_w = \frac{1}{\alpha} Q^{\frac{1}{\alpha}}
\]

Since any other such ray produces a different tangent value, we have another illustration that $p/w$ must be different for any different point on the $Z_w$-curve. Discussing this curve algebraically, we get two cases, and thus a similar “contradiction” as Keynes did in the Footnote:

\[
\begin{align*}
\left. \frac{dZ_w}{dQ} \right|_{p=\text{const}} &= \frac{p}{w} \frac{dQ}{dQ} = \frac{p}{w} = \frac{Z_w}{Q} \quad \text{and} \quad \frac{dZ_w}{dQ} = \frac{1}{\alpha^2} Q^{(\frac{1}{\alpha} - 1)}
\end{align*}
\]

Equation (15a) expresses the condition for a profit maximizing single point on the $Z_w$-curve. It expressly states that marginal cost is equal to average cost. In contrast to this, equation (15b) expresses the characteristics of movements from one such point to an other such equilibrium point. Thus we have here an alternative framework in which to re-iterate what Keynes wanted to express in the Footnote. Otherwise the alternative $Z_w$ curve of quadrant III brings no additional insight. From the standpoint of aggregation it is inferior to Keynes’ original $Z_w$-curve since,
as was stressed before, the latter curve is dimensionally consistent while the $Z_w(Q)$-curve is not consistent, because it relates quantities of goods to hours of work in an aggregative context in which “quantities of goods” has no clear meaning.

4.5. In search for the “right” representation. There are protracted debates about the “right” way of presenting Keynes’ principle of effective demand. An example for a recent contribution to such a debate is given by Hartwig (2007, p.726, numbering added):

1. $D$ and $Z$ have nothing to do with the ‘Keynesian Cross’. [2] The two curves as defined by Keynes are in nominal terms, while those of the ‘Keynesian Cross’ are in real terms. [3] Furthermore, Keynes talks about expectations and profit-maximizing. Where are those elements in the ‘Keynesian Cross’? Finally, [4] Keynes’ aggregate supply function $Z$ is not the 45°-line.

In contrast to this quote, we propagated a synoptic view of all the elements mentioned here: the Keynesian 45° cross is in our context just a mirror of the $D$ and $Z$-analysis. Questions about expectations and profit maximization are modeled explicitly and related graphically to the ‘Keynesian Cross’-side and to the $D_w$-$Z_w$-side. A brief discussion of this quote might be a good opportunity to point out the specificity of the present interpretation.

In connection with the quoted statement [1] let us remember: the 45°-line of the Keynesian Cross is a simple accounting tautology while $Z$ is the net value of output. In our view it would be strange to claim that the net value of output has “nothing to do” with accounting. Because of accounting is it that Keynes insisted on the “indubitable” equality of the value of output and the value of income. This quote is the basis of the ‘Keynesian Cross’ as we showed above in connection with equ.(9), p.23. Since the value of $D = D_1 + D_2$ depends in its $D_1$-part on income, it is “indubitable” for us that sentence [1] can not be correct. The two representations are – so to speak – two sides of the same coin. They are closely interrelated as has been shown – graphically and algebraically – in the Journal of Post Keynesian Economics decades ago (Ambrosi, 1981a, fig.1, p.505).

A simplified version of that demonstration is given here as fig.6. This figure “short-cuts” quadrant II and quadrant IV of fig.5, p.32, thereby avoiding the problematic macroeconomic production function which we carried with us until now because in this way it was possible to refer in a simple way to the conditions of profit maximization and consistent expectations on which Keynes’ model is based. But as stated already above, the macroeconomic production function is a problematic concept for reasons of aggregation problems and it is maybe preferable to avoid the concept and the figure of an aggregate production function since it is individual resp. sectoral production functions which Keynes has in mind in the presentation of his theory. As in fig.5 quadrant IV we have here in fig.6 quadrant I an “extreme classical” consumption function so that Keynes’ $\chi(N)$-function again is a 45° line, for simplicity’s sake without an intercept – which means that consumers have no wealth. Aggregate demand $D_w = D_{w1} + D_{w2}$ becomes “effective” where it meets aggregate
supply, thereby determining the value of output \( Z_w \) and the level of employment \( \bar{N} \). Because of the already discussed accounting tautology between net values of output and income \( (Z_w \equiv Y_w) \), any realized value of aggregate supply can be transposed into the identical value of aggregate income. If, following Keynes (1936a; p.90), we formulate the consumption function not only with employment as independent variable but, alternatively, also with income as independent variable, then we can draw the corresponding curve \( D_{w1} = C_w = \chi(Y_w) \) in fig.6 quadrant II. With the self-same value of ‘non-consumption demand’, i.e. investment demand \( D_{w2} \) as before, the resulting income \( \equiv \) net output must be identical as the one derived before. It is in this sense that the ‘Keynesian 45° Cross’ of quadrant II is just the other coin of Keynes’ \( D_w-Z_w \) resp. \( D-Z \) analysis.

This interdependence has important analytical implications, generally overlooked in the non-Post Keynesian literature. Thus, in our special case of the “extreme classical” consumption function we have

\[
C_w = c_N N + c_\pi \Pi \quad \text{with} \quad c_N = 1, \quad c_\pi = 0
\]

where \( c_N \) is workers’ propensity to consume and \( c_\pi \) is non-workers’ propensity to consume out of non-work income \( \Pi \). Hence we have

\[
C_w = N = \lambda Z_w = \lambda Y_w \quad \text{since} \quad Z_w = \frac{1}{\lambda} N \quad \text{and} \quad Z_w = Y_w .
\]

We thus have here an interesting result with regard to the consumption function \( \chi(Y_w) \): under special conditions it might depend entirely on the distribution of
income, i.e. on labour’s share $\lambda$ – the higher labour’s share, the higher is real consumption demand. But this result depends on the assumption made concerning income class specific propensities to consume. The analytically important aspect of this result is: there are direct interrelations between the ‘Keynesian Cross’ and Keynes’ $D-Z$-analysis. They might escape those scholars who claim that the two are entirely unrelated. That is why we cannot accept the quoted sentence [1].

With regard to sentence [2] our position is: If the words “as defined by Keynes” in this sentence are meant to suggest that at some point Keynes himself stressed that $D$ and $Z$ can only be interpreted in money terms and never in real terms that certainly is not true. Keynes (1936a ;p.43) proposed to use two types of units, namely money units and labour units. The latter have two manifestations: ‘labour input’ and ‘labour commanded’, the latter of these two being Keynes’ real values in “wage units”. Keynes (1936a; p. 25) does use money units when he writes “$Z = \phi(N)$”. But Keynes (1936a; p.55 n.2) then writes $Z_w = \phi(N)$ where $Z_w$ is in real terms, namely in labour commanded units. When it comes to debating consumption, Keynes (1936a; p.90) uses its monetary value $C$ side by side with its real value $C_w$. As far as the Keynesian Cross is concerned, there also is no restriction to either monetary or real values. Since both sides of the $45^\circ$-line are identical by definition of the accounting process which they express, it does not make any difference whether that identity is in monetary or in real terms. Thus, both the $D-Z$-analysis as well as the $45^\circ$-analysis may be conducted either in real or in monetary terms. The confrontation between money values and real values which is suggested by sentence [2] just does not exist.

Sentence [3] of the quote insinuates that the $45^\circ$-line oriented analysis is possibly entirely unrelated to profit maximization and to expectations. But that only appears to be so. That appearance happens if one fails to see the interrelations between the $45^\circ$-line oriented analysis with other elements of the Keynesian system, especially with the $Z_w$-curve. Thus sentence [3] is a corollary of sentence [1]. If sentence [1] is denied, sentence [3] can find a positive answer. The $Z$- resp. $Z_w$-curve is constructed explicitly under the assumption of profit maximization and of consistent expectations. Sentence [3] is therefore a further illustration that it is most unhelpful if one tries to argue that the $Z-D$ analysis has nothing to do with the $45^\circ$ Cross.

Finally, sentence [4] deals with an old “Aunt Sally”. Why on earth should the $Z$-line and the $45^\circ$-line be the same? But if it is clear that they are not the same, why is it necessary to emphasize the obvious? As was said here before: the $45^\circ$-line is an accounting relationship, the $Z$-function expresses technological conditions, market conditions, and the assumption of profit maximization. The two can not be the same, but they are related. If, however, sentence [4] is meant to deal with “the Footnote” and the $45^\circ$-line mentioned there, then we must remind the reader that all those who are puzzled by this footnote seem to fail to see the difference between a specific equilibrium condition for profit maximizing firms for a given effective demand ($45^\circ$-line) on the one hand and the locus of such points when effective demand changes ($Z_w$-line) on the other hand.
There is now the question: what generates the said $Z_u(N)$-locus? What motivates the firm to move from one equilibrium point to the next? The answer is, of course: expected demand. If expected demand changes, then the firm moves to a different profit maximizing allocation of its inputs, at different relative prices.

There appears now the next pseudo-problem in Keynesian exegesis. In the quoted article there is considerable concern about the possibility that expected demand by firms might diverge from experienced demand \cite{Hartwig2007}.

Keynes' theory requires the division of time into periods so that plans can be compared with realized results.

But this statement is in the first part a platitude, in the second part it is irrelevant: it is a platitude that one has to divide time into periods. If one wants to make accounts like "today's GNP" or "today's net value of sales" or "last year's GNP" as, of course, Keynes' concept of one period's $Z$ or $Z_w$ or $Y$ or $Y_w$ requires to do, then such measurements require time periods – and it is irrelevant whether original plans are frustrated or not. One has to do this periodization not because of expectations, but because of accounting. But as far as a comparison between plans and realized results for a specified period are concerned, in "Keynes' theory" – which the quoted passage addresses – one should consult Keynes (1936a; p.50, emphasis added) himself for enlightenment:

\textit{...to-day's employment can be correctly described by to-day's expectations...}

Thus, for Keynes the short period expectations determining employment are correct in that they correctly relate dispositions based on short-term expectations and realized factor demand. So, for every "to-day" when the observers make the accounts they are urged by Keynes to assume that "to-day's expectations" are not incorrect. One reason is that economists know very little about the exact reactions of economic agents when their expectations are frustrated. Keynes gives a number of further reasons in that context. The only thing which is sure is that when income and production and employment accounts are made, it is "indubitable" that the value of net production is equal to the value of income. It is on the basis of this indubitable ex-post clarity of accounts that Keynes wanted to construct his macroeconomic theory and not on the intractable haze of possibly frustrated ex-ante expectations. Keynes is aware, of course, that entrepreneurs do operate in a haze of several possibilities of eventual outcomes. He bridges this haze of expectations on the one side with the clarity of definite numbers of accounting on the other side with the methodological device of ‘certainty equivalents’ as Keynes (1936a; p.24 fn.3) elaborates:

An entrepreneur, who has to reach a practical decision as to his scale of production, does not, of course, entertain a single undoubting expectation of what the sale-proceeds of a given output will be, but several hypothetical expectations held with varying degrees of probability and definiteness. By his expectation of proceeds I mean, therefore, that expectation of proceeds which, if it were held with certainty,
would lead to the same behaviour as does the bundle of vague and more various possibilities which actually makes up his state of expectation when he reaches his decision.

Thus, in the context of his short-period theory of production and employment, Keynes assumes that expectations are correct and that they have certainty equivalents so that it is possible to argue with definite figures which correspond to the rules of accounting of income and production in an “indubitable” way. The discrepancy of plans and realized results is a chimera as far as Keynes and his short-period theory of employment and production is concerned.

4.6. The “frozen land” metaphor. Wherein lies now the fundamental difference between Keynes and Pigou? There are many minor points of difference which we covered so far concerning functional relations, problems of aggregation, choice of units, etc.. But whether these differences are essential or not is debatable and has been much debated. There is even the charge that Keynes was fraudulent with his claim for originality in dealing with effective demand (Clower 1994).

There seem to be two ways how to bring Keynes’ specificity into better focus: a consideration of historical precedents and a closer look at the time structure of the basic modeling approach. Actually, the two are interrelated.

Concerning historical precedent, it is interesting to note that Keynes (1936c, p.85) himself was quite fascinated by the existence of such a precedent (see above p.20). The historical debate to which he refers in his letter to Harrod in rather general terms is in particular the one between David Ricardo and Robert Malthus. Keynes (1933a, p.97f.) describes this debate in great detail. We can quote only a comparatively small part of it here. But it should emerge from the following passage that knowing about this debate seems indeed to be vital for a better understanding of Keynes’ analytical intentions (emphases in the original; for better recognition of ‘quotes in quote’ I add two-level quotation marks):

“Ricardo is investigating the theory of the distribution of the product in conditions of equilibrium and Malthus is concerned with what determines the volume of output day by day in the real world. Malthus is dealing with the monetary economy in which we happen to live; Ricardo with the abstraction of a neutral money economy. They largely recognised the real source of their differences. In a letter of 24 January 1817, Ricardo wrote:

‘It appears to me that one great cause of our difference in opinion on the subjects which we have so often discussed is that you have always in your mind the immediate and temporary effects of particular changes, whereas I put these immediate and temporary effects quite aside, and fix my whole attention on the permanent state of things which will result from them. Perhaps you estimate these temporary effects too highly, whilst I am too much disposed to undervalue them. To manage the subject quite right, they should be carefully distinguished and mentioned, and the due effects ascribed to each.’”

Preliminary version – January 25, 2009
To which Malthus replied with considerable effect on 26 January 1817:

‘I agree with you that one cause of our difference in opinion
is that which you mention. I certainly am disposed to refer
frequently to things as they are, as the only way of making one’s
writings practically useful to society, … Besides I really think
that the progress of society consists of irregular movements, and
that to omit the consideration of causes which for eight or ten
years will give a great stimulus to production and population,
or a great check to them, is to omit the causes of the wealth and
poverty of nations – the grand object of all enquiries in Political
Economy. A writer may, to be sure, make any hypothesis he
pleases; but if he supposes what is not at all true practically,
he precludes himself from drawing any practical inferences from
his hypotheses. …

But to come to a still more specific and fundamental cause of
our difference, I think it is this. You seem to think that the
wants and tastes of mankind are always ready for the supply;
while I am most decidedly of opinion that few things are more
difficult than to inspire new tastes and wants, particularly out
of old materials; that one of the great elements of demand is
the value that people set upon commodities, and that the more
completely the supply is suited to the demand the higher will
this value be, and the more days’ labour will it exchange for,
or give the power of commanding... I am quite of opinion that
practically the actual check to produce and population arises
more from want of stimulus than want of power to produce.’

One cannot rise from a perusal of this correspondence without a feel-
ing that the almost total obliteration of Malthus’s line of approach
and the complete domination of Ricardo’s for a period of a hundred
years has been a disaster to the progress of economics.”

It is small wonder that Keynes quoted these passages with fascination while working
on the manuscript of the General Theory: Malthus describes here some fundamen-
tal differences between Ricardo’s classical tradition and his own position very much
in a way to which Keynes subscribed wholeheartedly: explicit analysis of aggre-
gate demand, calculations in terms of labour commanded values, an analysis which
puts the stress on the momentary outcome of economic activity rather than on the
construction of hypothetical worlds where most real life problems are assumed away.

Keynes (1933a, p.100f.) reconfirms his allegiance with Malthus and claims him
as first ancestor of Cambridge economics

If only Malthus, instead of Ricardo, had been the parent stem from
which nineteenth-century economics proceeded, what a much wiser
and richer place the world would be to-day! We have laboriously to re-
discover and force through the obscuring envelopes of our misguided
education what should never have ceased to be obvious. I have long
claimed Robert Malthus as the first of the Cambridge economists

Preliminary version – January 25, 2009
[In 1793 Malthus was made a fellow of Jesus College, Cambridge, GMA]...

Keynes (1935 p.107) repeats his claim for Malthus as “first of the Cambridge economists” at the Centenary celebration in Cambridge, commemorating Malthus’ death (23 December 1834):

Malthus’s name has been immortalised by his Principle of Population, and the brilliant intuitions of his more far-reaching Principle of Effective Demand have been forgotten.

Let us, however, think of Malthus to-day as the first of the Cambridge economists – as, above all, a great pioneer of the application of a frame of formal thinking to the complex confusion of the world of daily events.

Alas, Keynes’ repeated pleas were in vain. In the same year when these lines were published, Pigou (1935) published The Economics of Stationary States, a book set entirely in the static equilibrium tradition of the classical school. In the same year when the GT was published Joan Robinson (1936) – his supposed disciple – published “The long-period theory of employment” – an intended extension of the GT into long-period equilibrium. When Pigou (1937) wrote an anti-Keynesian article more or less in the same analytical setting as the one of Joan Robinson, Keynes (1937b) wrote that this model “relates to a frozen land remote in its characteristics from all experience”. Strangely, Richard Kahn (1937) got Keynes to drop the passage again – against his own conviction (see below, p.61). But when we look for the fundamental difference between Keynes on the one hand and Pigou and the rest of orthodox economics, then this metaphor should come to the fore. As I wrote somewhere else and in a more detailed context: (Ambrosi, 2003, p.332f.)

The theme of “frozen land” economics meanders in our view from Robinson (1936) to Pigou (1937) and eventually to Barro (1990) and the beliefs of “New Classical Economics” . . .

It mars Joan Robinson’s look back at her brainchild, the ‘reswitching debate’ which she eventually disowns as being “unimportant” – because it is set in the frozen land of timelessness. This expression stands in a way behind an empty promise given by Richard Kahn [1937] to Keynes in 1937 . [“Frozen land”] is a token for illusions which misled revolutionary fervour into barren land. This expression is contained in one single paragraph which was never published by Keynes. Subsequently this expression was never again discussed by the main actors of the former debate – neither by Keynes nor by Pigou, his authentic Classic. But this does not diminish its significance.

Thus, when asked about the fundamental difference between Keynes and Pigou, I would say: it lies in the “frozen land” metaphor. It is the criticism implicit in this term which Keynes sees applicable from Ricardo up to Pigou and which we could show to be applicable even nowadays.
Keynes’ method is different from the orthodox school in that it tries to give a snapshot of an economy in its proper working but with potentially unemployed resources. This does not mean that marginal analysis must be shunned. Indeed in the index to the GT we have at least 11 entries with the term “marginal”. “Proper working” means that it is assumed that the accounting is done for every one of his “snapshots”. Therefore terms like “ex ante” savings and investments are not relevant. It is not the dynamics between ex ante and ex post in which Keynes is interested but a picture of an economy in ex post accounting. The interesting thing then is to ask: how can economic policy effect a change of economic variables, say, one year hence so that the accounts for that economy will show higher production and higher employment. Setting this problem might seem to be an unambitious aim. In fact, it is the most important outcome that a successful employment policy can aim for if faced by the danger of large-scale and protracted unemployment.

5. Concluding remarks

Keynes’ General Theory of Employment . . . is “post-Pigovian” in several important regards: it is written with much recourse to Pigou’s Theory of Unemployment (Pigou 1933). Keynes expressly wanted that his theory should be received by his readers as containing the orthodox – Pigovian – theory as a special case. Keynes’ theory is a General Theory in an “encompassing” sense. By its very intention it could not reject but had to absorb orthodoxy. Even Keynes’ seemingly revolutionary aggregate supply function explaining the newly defined magnitude “Zw” which occupied considerable space in this essay and which still causes controversy today, Keynes (1936a, p.89) himself considered to be a piece of rather orthodox economics. But if so much of Keynes’ economics patently “encompasses” orthodox economics – why are contrary convictions so widely spread in the exegetical literature to Keynes?

One answer to this question lies maybe in the research agenda of Keynes’ contemporary “post-Keynesian” followers whose most outspoken representative was Joan Robinson (Ambrosi, 2003). But it is often complained by critical economists that the later “Cambridge Keynesians” had little impact on mainstream economics. It is not plausible that their interpretations of Keynes were the only reason for the strange disappearance of the analytical context which Keynes was eager to stress for his own approach.

An interesting additional answer was recently given by Paul Davidson (2007). He attributed great responsibility to economics Nobel-price winner Paul Samuelson for the specific image which the economic community came to form in the post-World War II era concerning Keynesian economics. Davidson pointed out: Samuelson’s reception of Keynes’ GT had to overcome two obstacles: (1) one was his own research programme, (2) the other obstacle came from the McCarthyism of the 1950ies which agitated against “un-American” lefties – and against Keynes who was considered to be one of them.

Concerning point (1) it must be noted that Samuelson’s preoccupation at the time when Keynesian ideas were first debated was to work at his Foundations of Economic Analysis – an orthodox generalized relative price theory which came to be
considered to be his most impressive achievement in economics (Samuelson, 1947). But in a hugely successful textbook Samuelson (1948) published and propagated not only orthodox economics but also Keynesian ideas about income determination. Subsequently he became known as prominent propagator of Keynesianism. But, as Samuelson (1996) later “confessed” he attributed the relevance of Keynes’ ideas to price rigidities, administered prices, imperfect competition and other elements which were not further reflected upon but which suggested that the Walrasian general equilibrium system – which was Samuelson’s main economic theoretical concern – had no challenging analytical connection with the Keynesian system. This saved Samuelson’s central research programme from interference through a reception of Keynesian ideas.

Reflecting on this constellation, Paul Davidson (2007, p.181) commented:

> Obviously, Samuelson’s mind was already so filled with contrary notions of Walrasian equilibrium theory that he never made any attempt to catch the clues to Keynes’s general theory analytical foundations... The complaint here is that in the Samuelson-type approach to Keynes the analytical potential in Keynes’ theory disappeared from the economic community’s consciousness. Davidson’s observation gives an additional answer to the above question why the “encompassing” intention of Keynes’ General Theory was not better received by later generations of economists: Samuelson was able to “sell” the “General ” Theory as a special case of all sorts of rigidities.

There is some irony to this complaint against Samuelson-type Keynesianism because it might well have been that Samuelson’s particular procedure in popularizing Keynes was a blessing for the survival of Keynesian ideas in academic discourse after World War II. With this remark we enter now the consideration of the second type of difficulties which the reception of Keynesianism faced after World War II. Colander and Landreth (1996b) argue and substantiate that Keynesian ideas were highly suspect to influential businessmen in the USA and to the supporters of the fight against “un-American” activities during the post-World War II era of McCarthyism. Samuelson’s careful cocooning of Keynesianism into a world of its own created a modus vivendi not only with orthodox general equilibrium theory but also with McCarthyism. Thus, Paul Davidson’s (2007; p.187) final assessment is:

> Paul Samuelson saved the term “Keynesian” from being excoriated from post-World War II textbooks by the McCarthy anti-communist movement at the time. But the cost of such a saving was to sever the meaning of Keynes’s theory in mainstream economic theory from its General Theory analytical roots. 

> In winning the battle against the forces trying to prevent the teaching of suspected communist-inspired “Keynesian” economics in our universities, Samuelson ultimately lost the war that Keynes had launched to eliminate the classical theoretical analysis...
This diagnosis is interesting because it shows how intertwined academic discourse might be with ideological preoccupations of a particular time. But if Davidson’s diagnosis is right, then currently we might well have reason to hope that a new era of Keynesianism could dawn now that anti-communism is obsolete since communism itself has shown its shortcomings and now that virtually all economic politicians seem to accept that Keynesian demand management is not the downfall but the rescue of capitalist economies. The recent economic-political developments following the financial market crisis of 2008/9 might well liberate some economic minds and motivate a number of academic economists to look again at the “General Theory analytical roots”.

We argued in the above that a number of interesting and important analytic roots of Keynes’ General Theory are to be found in Pigou (1933). But in spite of its very Pigovian roots without an understanding of which important parts of Keynes’ argument would be difficult to comprehend, the General Theory goes far beyond those roots. Let me briefly give a short and selective final list of what I consider to be important points of the specificity of the General Theory.

1. The basic methodology of the General Theory is “anticipated ex-post analysis”.
2. The analytical framework in which the General Theory should be received is post-Pigovianism. It is solidly based on the standard body of neoclassical economics. It incorporates “Walras’ Law” in the disequilibrium version of its formulation. It is best interpreted in a disaggregated Cobb-Douglas context.
3. Keynes’ wage units approach is unfortunately far too little received in post Keynesian literature.
4. Because Keynes’ theory incorporates the validity of “Walras’ Law” in an ex-post setting, it is a theory based on “rationalised expectations” but not one based on “rational expectations”.
5. Keynes dearly wanted to create a “Malthusian Tradition” for Cambridge economics. His breach with Pigou was most probably motivated to a large extent by Pigou’s ineptitude to support Keynes in this attempt. As far as I can see Keynes’ desire is still unfulfilled.

This list is, of course, not exhaustive as far as Keynes’ analytical intentions are concerned. We left uncovered most of the monetary aspects of the GT. But we must note that Keynes himself was eager to point out that monetary aspects can be postponed for a considerable time in the exposition of his thought. Keynes (1936a; p.173 emphasis added, GMA), almost half way through his book, expressly stressed

We have now introduced money into our causal nexus for the first time . . .

It can well appear to be rather strange that Keynes put such stress on the late appearance of monetary considerations in the unfolding of the argument of the GT. After all, he was an economist who was widely considered to be an outstanding expert in monetary economics – if not even THE contemporary British expert on

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24See our relevant quote on p.3.
monetary theory. This passage is also strange for an author who published a precursor of the GT as “The Monetary Theory of Production” (Keynes 1933c). But when Keynes published and lectured under such a title, his Cambridge colleague Pigou (1933, p.V) published a hardly veiled rebuke:

> It is possible to study the problem of unemployment either from the money end or from what I shall call, in contrast, the real end. . . . In recent years . . . economists [read: J.M. Keynes, GMA] have been inclined to concentrate attention on the money end. The result, in my opinion, has been to overstress somewhat the role that money plays in more normal times, and to put in the background very important factors of a non-monetary character. For this reason, among others, I have chosen to write my book from the real end, and to bring in the monetary factor only at a fairly late stage.

Soon after Keynes read these passages, he switched to the same procedure like the one propagated by Pigou. Henceforth Keynes worked at starting his General Theory likewise “from the real end”, eager to stress, if we may paraphrase Pigou, “the very important factors of a non-monetary character” which Keynes saw at work. Thus, the seemingly strange postponement of monetary argumentation in Keynes’ General Theory is not so strange after all – it is a further manifestation of the post-Pigovian character of much of the General Theory – even the sequences in the unfolding of its argumentation.

**Appendix A. The problem of aggregation and Keynes’ wage units**

A.1. **Some analytical issues.** The main issue in choosing the right economic units is to find an approach to consistent economic model building. There is little discord in the literature in this regard. Since Keynes explicitly linked his own model to that of Pigou by relating one of his own magnitudes in wage units, i.e. his $C_w$ to a corresponding magnitude in Pigou (1933), namely to $F(x)/F'(x)$, it is rather astonishing that in the (few) discussions of Keynes’ choice of units there seems to have been not a single Cambridge Keynesian who commented on this correspondence.

As is well known, the object of the General Theory is to have a theory of output as a whole in relation to aggregate employment. It seems also to go without saying that “output as a whole” in the sense of real NNP, net “national product”, is a single scalar value whereas factual output of a nation during one year is a vector of millions of goods and services. There is no logical or mathematical possibility to unambiguously represent a vector as a scalar value. It is, of course, possible to transform two vectors into one single scalar value, namely by multiplying the two. But then one does not have the vector of outputs any more but, obviously, a mish-mash of two vectors. As economists, we must live with this intrinsically insoluble problem. But how do we live best – or rather: least badly – with it?

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The first draft for what later became the GT survived under a very similar title (Keynes 1932).
The customary way of macroeconomic model building is to take one product as “standard of value”, be it now David Ricardo’s “corn” in his corn economy or be it the consumption good in neoclassical growth models or be it “wage goods” in Pigou (1933). One may then take the associated vector of relative values in order to make the unavoidable mish-mash for macroeconomic analysis. In a two-sector economy like Pigou’s (1933) we then get

\[ Y_{p1} = \frac{Q_1}{p_1} \quad \text{resp.} \quad Y_{p1} = \frac{F_1(N_1)}{F_2(N_2)} \]

where \( Y_{p1} \) stands for NNP in terms of product \( Q_1 \) and where equ.(18 a) states the mere accounting relations whereas equ.(18 b) represents the neoclassical theory of production and the ensuing theory of (relative) value under conditions of perfect competition – which is, by and large, the standard theory for the relative price vector of an economy, at least for Pigou and countless modern macroeconomic textbooks based on ‘neoclassical’ economics.

As we have repeatedly argued above, Keynes had absolutely no problem to accept the “orthodox” assumptions of well behaved sectoral production functions and of remuneration by marginal product. But there is a more or less self-evident problem – at least Keynes treated it as such in the General Theory: If we wanted to relate this type of scalar-product measure of “real” output to aggregate employment, then we wanted, of course, a reliable relationship between our measure of real value on the one hand and employment on the other hand.

In order to make that problem more explicit, we consider the following equations:

\[ N_1 = \nu_1 N \quad ; \quad dN_1 = \nu_1 dN \]
\[ N_2 = \nu_2 N \quad ; \quad dN_2 = \nu_2 dN \]

where \( \nu_i \) with \( i = 1, 2 \) is the share of sectoral employment with \( \nu_1 + \nu_2 = 1 \). The respective shares are here treated as constant but in an total-economy model they must be considered as varying. Their sum must invariably be unity, of course.

From equ.(18) we have

\[ Y_{p1} = F_1(N_1) + \frac{F_1'(N_1)}{F_2'(N_2)} F_2(N_2) \]

and hence (for simplicity we write just the symbols of the \( F_i \)-functions omitting the brackets with the respective arguments ) we have the derivative:

\[ \frac{dY_{p1}}{dN} = F_1' + \frac{\nu_1 F_1'' F_2' - \nu_2 F_1' F_2''}{(F_2')^2} F_2' \leq 0 \]

which has an indeterminate sign value because the fraction in equ.(22) can be either positive or negative since the term containing \( F_1'' < 0 \) has a positive sign whereas the term containing \( F_2'' < 0 \) has a negative sign. Thus, for valuation reasons we cannot clearly say whether there is a reliable positive relationship between real (!) NNP on the one hand and employment on the other hand as soon as we argue on
the macroeconomic level. This is, of course, annoying because on a sectoral level there clearly does exist a positive input-output relationship, since by assumption sectoral inputs $N_1, N_2$ increase and hence both sectoral outputs must also increase. Nevertheless, this is the type of economic unit adopted by Pigou (1933) and by much of the post-Keynesian school.

But now consider Keynes’ own ‘wage unit’ analysis in this self-same context. An analogue to equ.(18) reads:

\[
\begin{align*}
(23) \quad \text{a) } Y_w &= \begin{bmatrix} Q_1 \\ Q_2 \end{bmatrix}' \begin{bmatrix} p_{1w} \\ p_{2w} \end{bmatrix} \quad \text{resp.} \\
\text{b) } Y_w &= \begin{bmatrix} F_1(N_1) \\ F_2(N_2) \end{bmatrix}' \begin{bmatrix} \frac{1}{F_1'(N_1)} \\ \frac{1}{F_2'(N_2)} \end{bmatrix}
\end{align*}
\]

where $\frac{p_i}{w_i}$ ($i = 1, 2$) are the inverses of the sectoral real wages. The result of this multiplication is then real NNP not in consumption goods units but in wage units. Hence the analogue to equ.(21) reads now:

\[
(24) \quad Y_w = \frac{F_1(N_1)}{F_1'(N_1)} + \frac{F_2(N_2)}{F_2'(N_2)}
\]

A remarkable difference between the two equations is: equ.(21) has a rather messy term where the $N_1$ and $N_2$ terms are all lumped together – with the ensuing comparative-static problem mentioned in connection with the undetermined sign value of the derivative expressed in equ. (22). In other words: when labour input rises we cannot be sure whether the real value of total output rises as well. The reason could be that with increasing labour input the output of the non-wage goods sector becomes relatively cheap (in comparison to wage goods). The if you add the rise in wage-goods and the real value of non-wage goods which have become relatively cheap, then you might get a total value of output which has not risen, although output in every sector clearly did rise. This is certainly an unsatisfactory result.

In contrast to that, equ.(24) has no comparable term where sectoral employments are mixed up. Consequently, there are not comparable ensuing comparative-static problems. The values for sector 1 or sector 2 are now only determined through $N_1$-employment or $N_2$-employment, respectively, whereas in equ.(21) sector 2 values were generated by a medley of $N_1$-employment and $N_2$-employment.

Thus the analogue to equ.(22) now reads

\[
(25) \quad \frac{dY_w}{dN} = \nu_1 \left( 1 - \frac{F_1 F_1''}{(F_1')^2} \right) + \nu_2 \left( 1 - \frac{F_2 F_2''}{(F_2')^2} \right) > 0
\]

giving a determinate positive sign value.

Since the analogous sign value of the derivative given by equ.(22) was not determinate although output clearly must increase since with increased total and sectoral output every element of the output vector must have increased, according to neoclassical assumptions, the accounting in terms of wage units demonstrably has a clear advantage over accounting in output units. It is therefore quite astonishing that post-Keynesian Cambridge economists have seen little motivation to enlarge on
If she addressed wage units at all in her publications, Joan Robinson was rather nonchalant about whether to count in these units or in price units (Robinson, 1954b, p. 72, reprint p. 6). It is astonishing that she had so little concern for the valuation problems and solutions proposed by Keynes since she herself, his disciple, spent a good part of her endeavours as an economist to demonstrate valuation problems, in particular with regard to the valuation of capital. She seems not to have been aware that Keynes’ approach to wage units offered – to a certain extent, because a satisfactory solution of translating a single vector into the value of a scalar expression is impossible – a solution of a similar problem, namely to the problem of “real” output valuation.

Keynes proposition of wage units is not only particularly well suited when it comes to analyzing changes in employment. This can be shown if we permit ourselves the simplification of using constant elasticities of production functions. In this case we have for any product $Q_i$ the real value of its output in terms of wage units given by:

$$
\frac{p_i Q_i}{w_i} = F_i(N_i; K_i) = \frac{N_i^{\alpha_i} K_i^{\beta_i}}{\alpha_i N_i^{\alpha_i - 1} K_i^{\beta_i}} = \frac{1}{\alpha_i} N_i \quad \text{with} \quad \frac{d}{dK_i} \left( \frac{p_i Q_i}{w_i} \right) = 0
$$

i.e. in this case the quantitative input of capital just cancels and thus we do not need to bother about “quantities of capital” and how to measure them properly – a problem which later was to occupy Joan Robinson very much.

The economic reasons seem to be quite plausible which explain why variations in capital services are maybe quite irrelevant for values in terms of wage units: take, e.g., a rise in capital input in the sector under consideration. Then output will be higher but prices will be lower because the increase capital input will increase also the marginal productivity of workers. There will be thus more output at lower prices in terms of wage-units. Both effects put together might well result in unchanged values in terms of wage units.

In this context one is reminded of the much quoted – but it seems little understood – passage in the *General Theory* (p. 40) that

... Queen Victoria was a better queen but not a happier woman than Queen Elizabeth – a proposition not without meaning and not without interest, but unsuitable as material for the differential calculus.”

Why does Keynes write that in the context of tracing price and quantity changes in his discussion of the choice of units? Our answer is that it might well be that effects on prices and quantities just cancel out and thus drop out of the differential calculus as in the case of equ. (26). Whether this interpretation is just a rationalisation or the original explanation, i.e. whether Keynes was indeed aware of this particular implication of his choice of units, is not known to this author. But after all the years of dealing with index numbers and economic units, it is most likely that he passed his judgement in this matter not without being aware of its implications.
In any case, it is a pity that none of the Cambridge post-Keynesians went into a deeper discussion and application of these aspects of Keynes’ wage unit analysis. On the rare occasions when they do deal with these units they seem to see only problems and inconsistencies in Keynes’ choice of units (Bradford and Harcourt 1997). But upon some reflections it might well appear that Keynes’ units permit quite interesting adaptations.

A.2. Generalisations and simplifications. Equ. (27) states a general formulation of an aggregate measure of output in terms of wage units where the two vectors on the right-hand side represent individual firm products and marginal products in analogy to the previous two-sectoral presentation:

\[
Z_w = \begin{bmatrix}
F_1(N_1) \\
\vdots \\
F_i(N_i) \\
\vdots \\
F_n(N_n)
\end{bmatrix}
\begin{bmatrix}
\mu_1 & \cdots & 0 \\
\vdots & \ddots & \vdots \\
0 & \cdots & \mu_n
\end{bmatrix}
\begin{bmatrix}
\omega_1 & \cdots & 0 \\
\vdots & \ddots & \vdots \\
0 & \cdots & \omega_n
\end{bmatrix}
\begin{bmatrix}
F_1'(N_1)^{-1} \\
\vdots \\
F_i'(N_i)^{-1} \\
\vdots \\
F_n'(N_n)^{-1}
\end{bmatrix}
\]

The matrices in equ. (27) have non-zero elements only on the diagonal. The symbols have the following meanings:

\[
\omega_i \equiv w_i/\bar{w} \quad \text{with} \quad i = 1, \ldots, n
\]

measures “relative money wages”. By introducing this extension of the original formulation we could take account of different productivities and efficiencies of labour which in real life certainly do exist. The symbol \( \mu_i \) with

\[
\mu_i \equiv \frac{1 + \nu_i}{1 + \pi_i} \quad \text{with} \quad i = 1, \ldots, n
\]

measures specific market conditions which can relate either to product markets, characterized by \( \pi_i \) or to labour markets, characterized by \( \nu_i \). The details concerning these latter parameters will be discussed below, however (see equ. (71)f., p.66).

This sketch may help to point out that Keynes’ measure permits as detailed a disaggregation and differentiation as is analytically required in order to reach a meaningful partial analytic resolution level on the one hand while on the other hand this measure leads to a single scalar value of net output and this is, of course, what is required for macroeconomic analysis. The single scalar value thus derived in terms of wage units has a particular advantage: it has the dimension “labour services”. Thus it is a measure which is dimensionally consistent when the problem at hand has to do with labour and employment as inputs on the one hand and output value on the other hand, as is, of course, the case with Keynes’ General Theory of Employment.

The advantages of easy decomposition of aggregate values into component partial analytic values carry over to comparative static analysis. The scalar value given by \( Z_w \) on the left-hand side of expression (27) is the sum of individual sectoral \( Z_{wi} \)-values, each of which being determined by the terms given on the right-hand side of...
Thus we may write

\[ Z_w = \sum_{i=1}^{n} Z_{wi} \quad \text{whence} \quad \hat{Z}_w = \sum_{i=1}^{n} \frac{Z_{wi}}{Z_w} \hat{Z}_{wi} \]

where the symbol \( \hat{ } \) signifies a relative change, e.g. \( \hat{x} \equiv dx/x \).

From expression (27) we know that each individual \( Z_{wi} \) is determined by a number of terms, so that in a comparative-static context we get the further expressions and decompositions as given by:

\[
\begin{align*}
(31) & \quad a) \quad Z_{wi} = \frac{\mu_i \cdot \omega_i}{\alpha_i} \cdot N_i \\
& \quad b) \quad \hat{Z}_{wi} = \hat{\mu}_i + \hat{\omega}_i + (1 - E_{\alpha_i}^N) \hat{N}_i - E_{\alpha_i}^K \hat{K}_i
\end{align*}
\]

where \( \alpha_i \) in (31a) is the elasticity of production. In the Cobb-Douglas case this magnitude is given as a parameter. If there are, however, functions

\[
\alpha_i = \alpha_i(N_i, K_i)
\]

then one can formulate elasticities for the \( \alpha_i \) as expressed in equ.(31b). This means that then the share of labour cost in the value of output is not technologically given but may change. Such changes cannot be excluded in principle.

Eqs.(31a) show that the relationship between the value of sectoral supply on the one hand and respective employment is a rather complicated one – in principle. But simplifying assumptions can make a theory manageable which is based on such relationships. Keynes was quite aware of this. He stated in detail a number of simplifications when restating the General Theory in his ch.18 (Keynes 1936a, p.245):

“\begin{quote}
We take as given
\begin{enumerate}
\item the existing skill and quantity of available labour,
\item the existing quality and quantity of available equipment,
\item the existing technique,
\item the degree of competition
\item the tastes and habits of the consumer,
\item the disutility of different intensities of labour . . .
\end{enumerate}
\end{quote}

The first three of these ‘givens’ determine the shape of the production functions and the marginal productivity functions. The last three determine the extent of the disturbance from the marginal productivity real wage and the preferences.

Keynes eliminates [1] changes in the quality of labour (as here expressed by \( \hat{\omega}_i \)), [2] changes in equipment (\( \hat{K}_i \)), [3] changes in technique (\( \hat{\alpha}_i \)), [4] changes in the degree of competition (\( \hat{\mu}_i \)). With all these rates of change set zero, equ.(31b) reduces to \( \hat{Z}_{wi} = \hat{N}_i \) and thus it may easily be said that sectoral employment is proportional to the value of output in terms of wage units.

It follows from this that Keynes must have been aware of the analytical requirements for his wage unit analysis to consistently generate the type of theory of employment he wanted to offer. He was particularly aware that it is only in the short run that one may reasonably assume the constancy of the just listed magnitudes.

\[ ^{26} \text{Emphasis and numbering added, GMA.} \]

Preliminary version – January 25, 2009
This emerges in an exchange between Keynes and Joan Robinson published by Geoffrey Harcourt (1994, p.21, n.3) who points out that Keynes wrote (27 August 1942):

“For my units to work two conditions must be fulfilled, namely that labour in some sense is the only factor of production and that we are functioning in short-period conditions.”

It is also interesting that this was a response to Joan Robinson’s comment (21 August 1942) ‘that your units work because capital equipment is given’ (ibid.). If the published statement is Keynes’ entire answer, he did not clearly reply to Joan Robinson. When writing that labour “in some sense” is the only factor of production he could not possibly have meant that there is no capital around. Did he think of capital as ‘dated’ labour so that all equipment contains just dated labour? But then clearly the analysis must extend to the first beginnings of capital formation. This does not seem to go well with his stress on ‘short-period conditions’. One could, of course, discern between short-period conditions as the concrete backdrop of the General Theory on the one hand. Here, the concrete assumptions, just stated, come into play. On the other hand, one may think of a long-period dated-labour analysis where changes are contemplated for the magnitudes just assumed to be constant. But Keynes does not indicate that he did indeed want to make such a distinction. In addition, reducing capital to its historical labour inputs would imply a “labour embodied” theory of value as far as capital is concerned. But Keynes, in the General Theory, clearly proposed a labour commanded theory of value. In any case, we know from a further publication (Harcourt, 1996, p.322) that in this context Keynes wrote one further letter to Joan Robinson, namely on 29 August 1942, in which he asked her, apparently in exasperation:

“How could I have protected myself more completely and more wordily from your accusation?”

The accusation to which Keynes here refers seems not to have survived in writing, but, as Harcourt remarks in this context (ibid.), “the issue being discussed is clear”: It seemed to Joan Robinson that Keynes postulated the “wage unit” to be an invariable measure of value. Keynes, in the quoted letter, took exception against this charge with the argument that he took great care to specify the conditions under which this measure of value may indeed be applied. To infer from this that he would have also advocated to apply the self-same measure without any adjustments when those conditions are not met, is, as he insisted, a non sequitur. With this argumentation Keynes insisted once more on the short-period context of his own analysis.

27 GMA: This is tantalisingly evasive – how many senses are there of labour being the only (relevant ?) factor of production. Could it be that Keynes had a case in mind in which capital just could not affect wage units – as discussed above in connection with equ. on page 48.

28 See above. In this letter Keynes draws the attention in particular to the top of p.214 of the General Theory.
This discussion leads us to believe that the present interpretation – the characteristic of which is that capital is present in production as equipment but does not affect economic values in terms of wage units – is the most plausible, or at least the most neutral, one as far as Keynes’ analytical intentions were concerned.

Nevertheless, the question of long-period analysis vs. short-period conditions does bear some further discussion and clarification in the context of the quoted Keynes–Robinson exchange. But we must stop here.

APPENDIX B. CONSUMPTION AND TIME PREFERENCE

B.1. Keynes on time preference and consumption. As stated on p.11, Keynes saw his consumption function as an expression of time preference. This is quite plausible since this function represents the decision how much of the current (and in some cases of discounted future) income to spend in the current time period and how much to save for later consumption. The corresponding choice theoretic foundations of time preference analysis were well established by the 1930ies, Keynes was mentioned as inspirer in the most sophisticated contemporary publication dealing with time preference and optimal savings (Ramsey 1928). Later publications on the consumption function claimed that Keynes was ignorant about time preference, but such claims are without any foundation. I have reconstructed Keynes’ most likely analytical procedure in developing his theory of the consumption function. It appears from that reconstruction that many of Keynes’ utterances concerning the characteristics of the consumption function can be derived as comparative-static results of that model (Ambrosi 2003, p.185f). The argumentation made there does not need identical reproduction here. But it might be helpful to render here some simplifications and ramifications of that argumentation because it is important for an understanding of Keynes’ “frozen land”-criticism against Pigou.

B.2. A two-period model of time preference and consumption. Suppose a household has income $Y^0$ now and contemplates consumption in the current time period of $Q^0_c$ goods and in the next time period of $Q^1_c$ of goods. The latter has to be provided for by savings $S^0$. The relevant budget equations are then:

\[ Y^0 = p^0_c Q^0_c + S^0 ; \quad (1 + r) S^0 = p^1_c Q^1_c \]

where $r$ is the rate of interest and the $p$-s are the respective prices. The money value of consumption is then $C^0 = Y^0 - S^0$. Elimination of $S^0$ in equ.\(33\) then gives the intertemporal budget

\[ Y^0 = p^0_c Q^0_c + \frac{p^1_c Q^1_c}{1 + r} . \]

The corresponding utility function is

\[ U = U(Q^0_c, Q^1_c) . \]

\[29\]Here and in the following the upper indices 0, 1, 2, ... signify time periods, not exponents. In the case of the discount factors $(1 + r)$ and $(1 + \varrho)$ which appear further down in the text, the respective upper numbers are exponents, however. The different meanings should be clear from the context.
For an intertemporal equilibrium of the household it is required that the “equimarginal rule” holds in the form

\[
\frac{\partial U}{\partial Q_c^0} \bigg/ \frac{p_c^0}{1 + r} = \frac{\partial U}{\partial Q_c^1} \bigg/ \frac{p_c^1}{1 + r}.
\]

Expand now the left-hand term in equ. (36) with \( Q_c^0 / U \) and the right-hand term with \( Q_c^1 / U \) and define the “elasticities of utility”

\[
\theta^0 \equiv \frac{\partial U}{\partial Q_c^0} \frac{Q_c^0}{U} \quad \text{and} \quad \theta^1 \equiv \frac{\partial U}{\partial Q_c^1} \frac{Q_c^1}{U}.
\]

We may then re-write the equilibrium condition of equ.(36) as

\[
\theta^0 \frac{U}{p_c^0 Q_c^0} = \theta^1 \frac{U}{p_c^1 Q_c^1 / (1 + r)}.
\]

In this expression \( U \) obviously cancels. The equilibrium condition \( \text{(36)} \) resp. \( \text{(38)} \) may then be solved to give

\[
\frac{p_c^1 Q_c^1}{1 + r} = \frac{\theta^1}{\theta^0} \frac{Q_c^0}{p_c^0 Q_c^0}.
\]

But the left-hand side of equ.(39) is the same expression as the last term in the budget equation \( \text{(34)} \). We may therefore replace that term using equ.(39), thus obtaining the “equilibrium budget”

\[
Y^0 = p_c^0 Q_c^0 + \frac{\theta^1}{\theta^0} p_c^0 Q_c^0 = \left(1 + \frac{\theta^1}{\theta^0}\right) p_c^0 Q_c^0.
\]

But since \( p_c^0 Q_c^0 \equiv C^0 \), we may re-write the “equilibrium budget” equation \( \text{(40)} \) as

\[
C^0 = cY^0 \quad \text{where} \quad c = \frac{\theta^0}{\theta^0 + \theta^1} \quad \text{and} \quad 0 < c < 1.
\]

We thus have generated a “typical” Keynesian consumption function from a rather simple intertemporal model.

One may ask, however: where does time preference come in in this context? After all, Keynes does refer to this term when explaining his consumption function. The answer is that one could say that the elasticity of utility \( \theta^1 \) might contain a specific intertemporal factor of discounting future utility experience so that

\[
\theta^1 = \frac{\theta}{1 + \varrho}
\]

holds where \( \theta \) is the “true” elasticity of utility experienced in the future time period “1”, and where \( \varrho \) is a utility discount factor similar to the interest rate used in the budget equation. Thus, via a “properly re-defined” \( \theta^1 \) we might see that time preference does come into the construction of the consumption function.

But this introduction of time preference is not necessary in order to deduce the consumption function from intertemporal household allocation of consumption. Its essential property, namely that \( 0 < c < 1 \) holds, is valid without the decomposition of equ.(42), namely of \( \theta^1 \), the presently “perceived” elasticity of utility, into a “true”
component \( \theta \) and an intertemporal utility discount component \( \varrho \). The Keynesian
consumption function is plausible without any elaborate intertemporal model. It
follows from the extremely simple consideration: if a household does want to make
provisions for future consumption out of its present income, the household cannot
spend all its present income. Therefore the propensity to consume must be less than
one.

But now that we have introduced the concept of “time preference”, the next
question is: Why does the “orthodox” theory come to a totally different idea about
the significance of this concept, namely that it determines the rate of interest?

A tentative answer – a fuller one will follow after the next section – has to return
to the equilibrium condition of equ.(36), say, in the form of (38). It will not do,
however, just to de-compose \( \theta^1 \) as was done via equ.(42). A number of additional
assumptions are necessary. Thus, if we postulate (!) the assumptions

\[
p^0_c Q^0_c = p^1_c Q^1_c \quad \text{and} \quad \theta^0 \triangleq \theta = \theta^1 (1 + \varrho)
\]

for equ.(39), which is one variant of the household’s equilibrium condition, then it
can easily be seen that the household equilibrium expression reduces to the

\[
r = \varrho
\]

classical case \( r = \varrho \)

In addition, the “equilibrium budget” reduces in this case not to the consumption
function of equ.(41) but to the

\[
C^0 = \frac{1}{2 + \varrho} Y^0 \quad \text{resp.} \quad S^0 = Y^0 - C^0 = \frac{1 + \varrho}{2 + \varrho} Y^0
\]

and hence to the belief that there is a direct negative influence of interest rates
on consumption resp. a positive influence of interest rates on savings, since we
just derived that \( r = \varrho \) holds. Thus interest rate changes are identical with time
preference changes. But one must be well aware that a necessary step in deriving
this result was to make the assumptions given in equ.(43). There is little of intrinsic
economic plausibility for these assumptions. Why should it be assumed, e.g., that
elasticities of utility should be identical now and in the future and that they may
differ from the standpoint of present utility maximization only in so far as the rate
of time preference \( \varrho \) has a positive value? The formal reason is that otherwise we
would not get the desired result \( r = \varrho \). But this is not a convincing theory of the
rate of interest. The exercise just shows on what specific assumptions the time
preference theory of interest can depend. It does not show that this theory is a
robust theory which covers a wide range of economic circumstances. In addition, it
should be noted that the time preference parameter \( \varrho \) is a parameter describing the
utility function, i.e. the preferences of the household being modeled. But preferences
are normally assumed to be given and stable during a comparative-static discussion
of household equilibria. If that is so, there can be hardly much variation in the rate
of interest, if this is determined by time preference. The classical belief that the
rate of interest is determined through time preference obviously collides with the
fact that interest rates do change considerably in real life.

Preliminary version – January 25, 2009
If the orthodox time preference theory of interest rates were indeed derived in the way just described, then it would be easily understood by the students of this theory on what outrageously specific and limited assumptions it is based. It is quite clear that the “Keynesian” consumption function is a far more general version of the self-same model.

But the adherents of the orthodox theory of time preference could argue that the model presented in the above section is not at all theirs. After the next section we will move nearer to the more complex orthodox model. But although the full orthodox model of time preference determined interest rates is more complex, it nevertheless is highly specific, as we will see from the following.

B.3. Extending the two-period model. The model of equs. (34) and (35) may be extended by one period and by additional income streams in the two future time periods, giving

\[
Y^0 + \frac{Y^1}{(1 + r)} + \frac{Y^2}{(1 + r)^2} = p^0_cQ^0_c + \frac{p^1_cQ^1_c}{(1 + r)} + \frac{p^2_cQ^2_c}{(1 + r)^2}.
\]

The corresponding utility function is

\[(47)\]

\[U = U(Q^0_c, Q^1_c, Q^2_c).\]

The analogue to the equilibrium condition of equ.(38) is given by

\[(48)\]

\[\frac{\theta^0 U}{p^0_cQ^0_c} = \frac{\theta^1 U}{p^1_cQ^1_c/(1 + r)} = \frac{\theta^2 U}{p^2_cQ^2_c/(1 + r)^2}.
\]

The equilibrium present values of expenditures are now, in extension of equ.(39)

\[(49)\]

\[\frac{p^1_cQ^1_c}{(1 + r)} = \frac{\theta^1}{\theta_0 p^0_cQ^0_c}; \quad \frac{p^2_cQ^2_c}{(1 + r)^2} = \frac{\theta^2}{\theta_0 p^0_cQ^0_c}.
\]

The equilibrium budget values can now be written as

\[(50)\]

\[Y^0 + \frac{Y^1}{(1 + r)} + \frac{Y^2}{(1 + r)^2} = p^0_cQ^0_c(\frac{\theta^0 + \theta^1 + \theta^2}{\theta^0})
\]

and the consumption function is now

\[(51)\]

\[C^0 = c'W + c'Y^0
\]

where

\[(52)\]

\[c' = \frac{\theta^0}{\theta^0 + \theta^1 + \theta^2}; \quad W = \frac{Y^1}{(1 + r)} + \frac{Y^2}{(1 + r)^2}.
\]

If we now discern between “short-period” and “long-period” expectations and treat the latter – pertaining to the future time periods – as given, then we could say: as long as the interest rate does not change, \(W\) “wealth” is constant. Then changes in present consumption plans with regard to \(C^0\) will be mainly influenced by changes in short-period expectations concerning \(Y^0\). There is now a difference between the
“marginal propensity to consume” $c'$ as given by $c'$ in eq.(52) and the average propensity to consume as given by:

$$c = \frac{C^0}{Y^0} = c' \frac{W}{Y^0} + c' = c' (1 + \frac{W}{Y^0}) \quad \text{whence} \quad \frac{c}{c'} = 1 + \frac{W}{Y^0}.$$  

A strictly proportional savings function thus requires $W = 0$. Otherwise $c > c'$ always holds.

The distinction between $c$ and $c'$ is insofar of interest, as it enters another of Keynes’ puzzling footnotes which has recently been discussed by (Hayes 2006, p.129). Keynes (GT, p.126, n.2) defines an ‘investment-elasticity of income’

$$\varepsilon_{Y,I} = \frac{dY}{Y} \left/ \frac{dI}{I} \right. = 1 - \frac{c}{1 - c'} = 1 - \frac{c'}{1 - c'} \frac{W}{Y^0}$$

where the last term is a re-formulation of the term to its left, based on our definitions of $c'$ and $c$ as used in eq.(53). The characteristics of this elasticity, the “proportional multiplier” are: If $c = c'$, i.e. if the consumption function has no intercept, the elasticity has a unit value (– this requires $W = 0$ as was just seen above). Otherwise, the value of the elasticity will be less than unity. But there are two opposing influences on this magnitude as follows from the last term of eq.(54):

1. A “poor” community with zero or very small wealth $W$ will have a comparatively large value of $\varepsilon_{Y,I}$
2. But for any given value of $W$, a “wealthy” community in the sense of having a large $Y^0$ will also have a comparative large $\varepsilon_{Y,I}$.

The terms “poor” and “wealthy” are rather unprecise but they seem to correspond to what Keynes (GT, p.126) wrote in this context:

Thus whilst the multiplier is larger in a poor community, the effect on employment of fluctuations in investment will be much greater in a wealthy community, assuming that in the latter current investment represents a much larger proportion of current output.

Keynes (ibid.) considers his “proportional multiplier” of our eq.(54) as a generalisation of this quote. He claims, however: “As wealth increases $c'$ diminishes, but $c$ also diminishes.” (symbols adapted, GMA). This remark is puzzling because it follows from the above discussion that $c'$ is independent of wealth in the sense of $W$ (see eq. 52) and that $c$ increases with $W$ (see eq. 53). Especially this latter result is quite plausible: if a community has more wealth, it could consume more in any future time period, and there is “now” no need to provide more for future consumption, so that higher wealth “now” in the sense of $W$ can very well go with

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The numerical examples which Keynes gives in this context have a mixture of absolute and percentage changes. The argument concerning the latter community is therefore that there $S^0 = I^0$ must be large due to the large level of income. Hence an “x% increase” of investment in such a community signifies a far larger absolute amount of investment than in the former one and hence also a correspondingly large employment effect – and vice versa in the case of an investment downturn. “Rich” communities (i.e. communities with a comparatively high $Y^0$) therefore have more pronounced booms and slumps than “poorer” ones – according to this consideration.

Preliminary version – January 25, 2009
increased average consumption. The USA during the years just before the financial crisis of 2008 gave a good example for this observation: during a time of ever increasing stock market values the average propensity to consume was just about unity. This positive dependence of consumption on the perception of the present value of wealth was indeed what Keynes wrote a few pages before this discussion of the proportional multiplier (see GT, p.92f., discussion below).

At the end of the footnote here under discussion, Keynes (GT, p.126, n.2) then draws attention to the term \( c = C^0/Y^0 \) (my notation, GMA) in equ.(54) and claims that \( \varepsilon_{Y,J} \) “increases or diminishes according as consumption increases or diminishes in a smaller or greater proportion than income”. (Hayes 2006, p.129) comments: “This final statement is correct if the marginal propensity to consume is constant . . .”. But that is indeed the case in our model, since it depends entirely on elasticities of utility (see again equ.(52)) . Keynes does use numerical examples in which the marginal propensity to consume does change. In order to accommodate for such an assumption one could, of course, make those elasticities variable in the appropriate way. One could also introduce distributional effects. One could also scrutinize the reasons for changes in the perceived wealth and analyze specific influences on the marginal propensity to consume.

Consider e.g. equ.(52) once more. Its formulation was based on the perception of two future time periods \( T = 2 \). In the case of \( T = 1 \) the analogous expressions would read:

\[
(55) \quad c' = \frac{\theta^0}{\theta^0 + \theta^1} ; \quad W = \frac{Y^1}{(1 + r)} .
\]

In this case \( c' \) is larger than before, since the denominator is smaller and \( W \) is smaller than before, since there is one income term less. We thus have

\[
(56) \quad \frac{\Delta c'}{\Delta T} < 0 ; \quad \frac{\Delta W}{\Delta T} > 0 \quad \text{hence} \quad \frac{\Delta c'}{\Delta W} = \frac{\Delta c'}{\Delta T} / \frac{\Delta W}{\Delta T} < 0 .
\]

Thus, if the planning horizon of a household is relatively long, it has to contemplate more future time periods and thus it will consume less out of current income in order to “stretch” provisions a bit into the future. But with a longer time horizon, the household might well perceive more future income streams and thus its perceived present wealth will increase. The perception of increased wealth will therefore mean – in this special case – a decline in the marginal propensity to consume.

These considerations are rather speculative. Keynes does not mention variable planning horizons. But he does contemplate changing levels of insecurity, variations in liquidity preference being one indicator for these changes. Indeed high liquidity preference may be interpreted as a capitulation of the investor’s capability to calculate likely returns in the face of overwhelming complexities. Therefore he will stay with the no-yield asset “money”.

One further aspect of the intertemporal allocation model should be mentioned. Until now, interest rate changes were not contemplated. If, however, the rate of interest does change, then the present money value of future income as given by \( W \)}
in equ. (52) changes and there will be an “indirect” influence on consumption which Keynes (GT, p.92f.) described in the following way:

\[(3) \ldots \text{The consumption of the wealth-owning class may be extremely susceptible to unforeseen changes in the money-value of its wealth. This should be classified amongst the major factors capable of causing short-period changes in the propensity to consume.}\]

The wording of the quoted passage might appear to be somewhat nebulous. But it becomes clear what is meant when we read it in connection with the following passage in which Keynes (GT, p.94) elaborates and refers back to this passage in the following way:

Perhaps the most important influence, operating through changes in the rate of interest, on the readiness to spend out of a given income, depends on the effect of these changes on the appreciation or depreciation in the price of securities and other assets. For if a man is enjoying a windfall increment in the value of his capital, it is natural that his motives towards current spending should be strengthened, even though in terms of income his capital is worth no more than before; and weakened if he is suffering capital losses. But this indirect influence [of changes in the level of interest rates, GMA] we have allowed for already under (3) above.

Keynes obviously regards in this context “wealth” or “capital” as a flow “in terms of [future, GMA] income” as expressed above in equ. (52) and again below in equ. (64). In the last quote he expressly refers to changes in the present value of this flow due to interest rate changes, as is, of course, directly visible in our just mentioned equations. With given long term expectations these changes can come only through changes in the present level of the rate of interest and this is what Keynes discusses in the second quote. Otherwise, if the “wealth-owning class” have reason to change their long-term expectations concerning the numerator(s) of the wealth expressions, there also is a change in the present value of “wealth” unless there are compensation changes of the level of interest rates.

It is clear that for the consumption of the “wealth-owning class” this influence would be a negative one: an increased interest rate “now” will mean a lower present value of a given future stream of incomes. The lower wealth will mean lower consumption “now” and this is what Keynes describes at some length in these quotes.

This argument might appear to be similar to the classical argument concerning the negative impact of higher interest rates on consumption but it is not in the least dependent on time preference. Our household model of intertemporal allocation shows clearly that when \(W\) changes there is no change in the parameters describing intertemporal preferences. Keynes’ “indirect” effect on consumption is based on the calculation of present values of future incomes of pre-determined money value. If, however, the expected flow of future payments is not positive, but negative, the discounting effect on consumption would be not negative but positive because given...
future obligations to pay will then have a lower present value and the present net value available for consumption might have increased.

Thus, in this case, we would have the opposite of the orthodox prediction: higher rates of interest could mean not lower but higher current consumption out of a given level of current income. Keynes does not mention this and therefor does not seem to base his critique of the classical model on this logical possibility. But it is an obvious implication of his discussion in the quotes just related.

B.4. The classical stationary state. In a recent article, the well-known macroeconomics textbook author Gregory N. Mankiw (2000, p.120) referred to “the Barro-Ramsey model of infinitely-lived families” as one of the two “canonical” models known in the economic literature of intertemporal allocation. Although in that context Mankiw was critical of the deficiencies of those models, it is important to note that the “infinitely-lived families” model does have this authoritative status in economic literature and that it did play an important role in the “classical” position with which Keynes was confronted when publishing the GT.

Let us now extend the intertemporal model so far discussed to the case of “infinitely-lived families”. In this case the time horizon \( T \) has to be extended appropriately so that the case \( T \to \infty \) can be covered. We re-formulate therefore the budget equation of equ.(46) so that it becomes

\[
(57) \quad Y^0 + \sum_{n=1}^{T} \frac{Y^n}{(1+r)^n} = p_c^0 Q_c^0 + \frac{p_c^1 Q_c^1}{(1+r)^1} + \cdots + \frac{p_c^n Q_c^n}{(1+r)^n} + \cdots + \frac{p_c^T Q_c^T}{(1+r)^T}.
\]

The corresponding utility function is now

\[
(58) \quad U = U(Q_c^0, Q_c^1, \ldots, Q_c^n, \ldots Q_c^T).
\]

The analogue to the equilibrium condition of equ.(48) is given by

\[
(59) \quad \theta^0 \frac{1}{p_c^0 Q_c^0} = \frac{1}{p_c^1 Q_c^1} = \cdots = \theta^n \frac{1}{p_c^n Q_c^n} = \cdots = \theta^T \frac{1}{p_c^T Q_c^T}.
\]

if we drop \( U \) which cancels as we have seen above. The equilibrium present values of expenditures are now, in extension of equ.(39) and equ.(49)

\[
(60) \quad \frac{p_c^1 Q_c^1}{(1+r)} = \frac{\theta^1}{\theta^0} p_c^0 Q_c^0; \ldots; \quad \frac{p_c^n Q_c^n}{(1+r)^n} = \frac{\theta^n}{\theta^0} p_c^0 Q_c^0; \ldots; \quad \frac{p_c^T Q_c^T}{(1+r)^T} = \frac{\theta^T}{\theta^0} p_c^0 Q_c^0.
\]

Let us now assume that all future incomes are identical \((Y^f)\) and that \( T \to \infty \). It is well known that then the sum on the left-hand side of equ.(57) converges to \( Y^f/r \) (Allen 1967, p.59) so that we obtain the following left-hand side

\[
(61) \quad Y^0 + \frac{Y^f}{r} = \frac{1}{c^} C.
\]

---

31Barro (1974), GMA
32Ramsey (1928), GMA
33The other model is the “overlapping generations model”.
The right-hand side of eqn.(61) follows from the right-hand side of eqn.(57) since we can replace all the discounted values of future consumption depicted there by using the equilibrium conditions of eqn.(60), thereby expressing them all in relation to $C = p^0Q^0$ as in the previous two variants of our intertemporal model. In analogy to eqn.(52) the value of $c'$ in eqn.(61) is given by

\begin{equation}
(62) \quad c' = \frac{\theta^0}{\theta^0 + \sum_{n=1}^{T} \theta^n}.
\end{equation}

This may be re-written as $c' = \frac{\theta}{\theta + \theta \Sigma_\theta} = \frac{1}{1 + \Sigma_\theta}$. 

As before, the denominator of $c'$ is here the sum of all the relevant elasticities of utility. The alternative formulation in eqn.(62) of $c'$ as $c'_c$ – index “c” for “classical” – uses the special assumption discussed already above in connection with eqn.(41), p.53. This specific assumption is that there is only one unique elasticity of utility “$\theta^0$” in all time periods but they are perceived differently “now” due to “the” rate of time preference “$\varrho$” giving the following row of discount factors

\begin{equation}
(63) \quad \Sigma_\theta = \frac{1}{(1 + \varrho)} + \ldots + \frac{1}{(1 + \varrho)^n} + \ldots + \frac{1}{(1 + \varrho)^T} = \frac{1}{\varrho} \quad \text{for} \quad T \to \infty.
\end{equation}

This series converges to $1/\varrho$ for $T \to \infty$ as shown.

Thus, if households expect an infinite (given) stream of yearly future income $Y^f$, they have a present value of “wealth”

\begin{equation}
(64) \quad W = Y^f/r
\end{equation}

where $r$ is the rate of interest. If their current income is $Y^0$, then we may write the consumption function for $C^0$ as

\begin{equation}
(65) \quad (a) \quad C^0 = c'W + c'Y^0 \quad \text{resp.} \quad (b) \quad C^0 = c'_cW + c'_cY^0 \quad \text{with} \quad c'_c = \frac{1}{1 + 1/\varrho}.
\end{equation}

Thus, after a number of somewhat cumbersome steps – which were not our intention but which are a tribute to the “canonical model” here under consideration – we arrived again at fairly simple expressions for consumption functions. As in the previous models of the consumption function we have marginal propensities to consume as expressed (a) by $c'$ or (b) by $c'_c$. It is obvious from these expressions that they are positive and smaller than unity, as the Keynesian theory requires. These marginal propensities to consume are determined by either (a) rather general conditions, namely by the belief in the existence of elasticities of utility $\theta^n$ which may be identical over time or not (see $c'$ in eqn.(62)). Or they are determined by (b) rather special conditions, namely the belief that there exists just one unique elasticity of utility $\theta$ in all time periods and the additional belief that there is one unique and time-period-invariant intertemporal discount factor for utility $\varrho$ expressing “time preference” (see $c'_c$ in eqn.(62)).

In the case of $c'_c$ one could say that the marginal propensity to consume is an expression of time preference and there are passages in the GT which show that Keynes was prepared to accept that type of parlance in rationalizing his proposal of a consumption function. We saw above (see p.52) that he was well acquainted with Frank Ramsey’s (1928) paradigmatic model of optimal intertemporal allocation. But
in which relation do such expressions stand with regard to the orthodox theory of
interest rates being determined by time preference? The crucial step for finding such
a relation in the present analytical context is: postulate the existence of a stationary
economy with following characteristics

1. a time horizon approaching infinity
2. a unique elasticity of utility of consumption “θ” in all time periods
3. a unique rate of intertemporal discounting of utilities ϱ
4. a unique money rate of interest r
5. a unique level of income Y over all time periods
6. an eternally constant capital stock so that I = S = 0 always holds
7. this implies C = Y “eternally”

if all these rather abstruse assumptions are accepted, then equ.(65b) now reads

\[ C = Y = c_t \left( \frac{Y}{r} + Y \right) \]

hence \( 1 = c_t \left( \frac{1}{r} + 1 \right) \) or \( \frac{1}{c_t} = 1 + \frac{1}{r} = 1 + \frac{1}{\rho} \); \( r = \rho \)

and we get the “orthodox” result: the rate of interest \( r \) is equal to the rate of time
preference \( \rho \).

This result is highly remarkable for a number of reasons:

1. The time preference theory of interest is a very special case of Keynes’ con-
sumption function.
2. This time preference theory of interest follows with algebraic necessity from
Keynes’ consumption function given the assumptions just stated.
3. The assumptions just stated which transform Keynes’ consumption function
into the time preference theory of interest were the basis of Pigou’s “simpli-
fied model” (Pigou 1937) where he did state that the rate of interest was
determined through time preference.
4. R.F. Kahn (1937) was adamantly convinced that Pigou (1937) and others
failed “to see the fundamental fallacy – which is the determination of the rate
of interest by the rate of discount of the future (Piero [Sraffa] agrees about this)”. He complained about the “grossness of this error”.
5. Keynes (1937c) rejected Kahn’s verdict and pointed out what was stated in
this list under item (2). [Keynes (1937b, p.238) points out, however: Pigou’s
time-preference model – although logically correct – “relates to a frozen land
remote in its characteristics from all experience”. Thus he criticized that
type of “long period equilibrium” as an entirely outlandish concept devoid
of any practical meaning for policy-making.
6. Kahn (1937) successfully impleaded Keynes to publish his comment on
Pigou (1937) without the criticism that this type of models imply a “frozen land”-economy. “But I still believe that it represents something at the bot-
tom of his head” (Keynes 1937c, p.261, nr.2)
7. Although Kahn offered that “all of us [Cambridge Keynesians, GMA]” could
write a criticism of Pigou (1937) and Keynes invited them to do so after his
own note was published, not a single line of the promised kind was published
by Keynes’ Cambridge followers.

Preliminary version – January 25, 2009
As Victoria Chick (2008) observed recently, this analytical complex and the related debates – or lack of them – involving Keynes, Pigou, Kahn, and Kaldor have been neglected far too much by post-Keynesians – maybe to the detriment of a proper understanding of the generality of Keynes’ General Theory.

B.5. “Personal” and “subjective” factors in time preference. Concerning the relation between “time preference” and consumption out of a given income, Keynes (1936a, p.166) wrote (emphasis in the original):

> The psychological time-preferences of an individual require . . . decisions . . . concerned with that aspect of time-preference which I have called the *propensity to consume*, which, operating under the influence of the various motives set forth in Book III, determines for each individual how much of his income he will consume and how much he will reserve in some form of command over future consumption.

We argued above that such passages are a clear indication that it was Keynes’ express intention that his readers considered the “propensity to consume” in a context of intertemporal choice – and in particular in a context which was similar to Keynes’ contemporary analytical approaches to time preference analysis. But in this passage Keynes also refers to his “Book III” of the GT where “The Propensity to Consume” is discussed. In that context there is a chapter on “The Subjective Factor” in determining the propensity to consume. There we find allusions to all sorts of concepts which seem far removed from any contemporary choice theory. Thus Keynes (1936a, p.108) refers to “eight motives” which supposedly enter the determination of the propensity to consume. He calls these motives: “Precaution, Foresight, Calculation, Improvement, Independence, Enterprise, Pride and Avarice”. But then he goes on to mention a number of other motives like: “Enjoyment, Shortsightedness, Generosity, Miscalculation, Ostentation and Extravagance” (ibid.). Whatever the names and the number of these motives, they all do not seem to come from a choice theoreticians’ vocabulary. Therefore it might seem that Keynes was not really serious when he alluded to time preference analysis in the above context.

<table>
<thead>
<tr>
<th>“The Personal Factor”</th>
<th>“The Subjective Factors”</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1) Foresight</td>
<td>(i) Precaution</td>
</tr>
<tr>
<td>(2) Self-control</td>
<td>(ii) Foresight</td>
</tr>
<tr>
<td>(3) Habit</td>
<td>(iii) Calculation</td>
</tr>
<tr>
<td>(4) Chance of death</td>
<td>(iv) Improvement</td>
</tr>
<tr>
<td>(5) Welfare of one’s heirs</td>
<td>(v) Independence</td>
</tr>
<tr>
<td>(6) Fashion</td>
<td>(vi) Enterprise</td>
</tr>
<tr>
<td></td>
<td>(vii) Pride / to bequeath a fortune</td>
</tr>
<tr>
<td></td>
<td>(viii) Avarice / unreasonable miserliness</td>
</tr>
</tbody>
</table>

Preliminary version – January 25, 2009
There are two replies to that sort of scepticism about Keynes’ ‘real’ interest in
time preference analysis: (1) With the discussions of “motives” to which we just
referred, Keynes did not negate but he confirmed contemporary time preference
analysis. A list very similar to that of Keynes – in one case ( (1) resp. (ii) )
and in much of the substance of the arguments even identical with it – may be
found in Irving Fisher (1930, pp.80-89). But this was one of the most detailed and
best known contemporary accounts of time preference analysis. We give a brief
overview of the “core” lists of the two authors in table 2 where Fisher’s list relates
to the determinants of time preference and Keynes’ list relates to the propensity
to consume. It seems that in some parts Keynes accepted Irving Fisher’s point
without any reservation (“Foresight”), in some parts Keynes gave Fisher’s arguments
– maybe teasingly – quite a different twist, for example with regard to inheritance.
Fisher, point (5) dignifies this motive by referring to the welfare of the heirs. Keynes,
point (vii), treats this motive more as an expression of personal pride. It would be
interesting to have a detailed comparison of these two famous authors’ views of such
motives but we must stop here at this short juxtaposition. We note that both of
them had more or less the same type of discourse in this field and that Keynes’
entering this type of debate by no means sets him aside from Irving Fisher. But in
noting the similarity, we should be aware of the difference: whereas Irving Fisher
wanted to explain the rate of interest in this context, Keynes wanted to show that
under realistic conditions time preference analysis can “only” derive the propensity
to consume.

(2) The second reason why Keynes’ discussion of various motives entering the
propensity to consume does not warrant to believe that he thereby wanted to negate
the relevance of time preference analysis is: his ultimate reason for discussing them
is to show that in the context of the GT these motives are quite unlikely to change.
They are just one other aspect of the “givens” of the GT. Keynes (1936a, p.109f.)

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34 In case there is doubt about Irving Fisher’s relevance for Keynes, one may note that Keynes (1937a, p.101, n.1) wrote: “I have thought it suitable to offer a short note … in honour of Irving Fisher, since his earliest and latest contributions have been concerned with it [the rate of interest, GMA], and since during the whole of the thirty years that I have been studying economics he has been the outstanding authority on this problem.”

35 Irving Fisher’s fourth motive is quite interesting in connection with our brief discussion con-
cerning changes in the time horizon “T”. Fisher (1930, pp.84f.) elaborates: “(4) The fourth
personal circumstance which may influence impatience for immediate real income has to do with
the uncertainty of life of the recipient. … [T]he time preference of an individual will be affected by
the prospect of a long or short life, both because the termination of life brings the termination of
the income from labor, and because it also terminates the person’s enjoyment of all income.” This
quote obviously alludes to intertemporal differences of planning horizons and it brings into focus
the two types of changes discussed in relation with eqn. (56) above: changes in wealth (our W)
and changes in time preference (our c’). It is interesting that in table 2 there is no counterpart in
the “Keynes-column”. This suggests that – at least in the context of the discussion of “subjective
factors” Keynes himself was not so much interested in variable lengths of intertemporal planning
horizons.

Preliminary version – January 25, 2009
We shall, that is to say, take as given the main background of subjective motives to saving and to consumption respectively. Since, therefore, the main background of subjective and social incentives changes slowly, we are left with the conclusion that short-period changes in consumption largely depend on changes in the rate at which income (measured in wage-units) is being earned and not on changes in the propensity to consume out of a given income.

In other words: the gist of Keynes’ argument about “motives” is not that we should forget about all our $\theta$-s of intertemporal analysis but quite the contrary: we should be confident that for the purpose of the GT we might very well take the characteristics of intertemporal preferences as constant and as given. Because that is the case, it is not changes in the propensity to consume that bring fluctuations into the economy, but quite different factors. Thus this whole debate assures the reader that – if he so wishes – he may well count with given intertemporal preferences. If, however, the reader does not wish to consider time preference analysis at all, he should refrain from making comments whether the GT is “really” more general than the classical theory because such a reader misses an important part of Keynes’ argument.

APPENDIX C. THE MICROFOUNDATIONS OF EFFECTIVE DEMAND

We have seen above that effective demand is a central concept for Keynes. It might appear to be almost a banality to repeat this statement once more. Yet, there seems to be astonishingly little in the way of modeling entrepreneurial behaviour which translates such a statement into the canon of conventional microeconomics of constrained extreme value problems. But GT, p. 55 does suggest to go along such lines:

\[ \ldots \text{the effective demand is simply the aggregate income (or proceeds) which the entrepreneurs expect to receive} \ldots \text{[it] becomes effective because, taken in conjunction with the conditions of supply, it corresponds to the level of employment which maximizes the entrepreneur’s expectation of profit.} \]

According to this description the paradigmatic entrepreneur formulates an expectation about the likely “proceeds”, i.e. the sales turnover, and subsequent to that expectation being formulated and taken as given, he decides how much to produce, how much of which factor to employ, with which prices to calculate etc. All these calculations are done in the light of the aim of profit maximization under given conditions of competition, of demand and supply and of technology.

I am not aware that the literature has produced a specific formal maximizing model which takes account of Keynes’ ideas about effective demand-constrained entrepreneurial decisions.\textsuperscript{36} I myself had made such an attempt quite some time

\textsuperscript{36}Hartwig (2007) and Hartwig and Brady (2008) draw attention to the “excellent reconstruction of the microfoundations of Keynes’ theory” (Hartwig, 2007 p.733) by Koenig (1980). The difference between Koenig’s (1980) very interesting approach and our’s is that Koenig does not explicitly formulate effective demand as constraint as we do here with equ.(67). Hence the rich

Preliminary version – January 25, 2009
ago (Ambrosi, 1981b). The basic idea of the present microfoundation of Keynes’ ideas about turnover oriented entrepreneurial decisions is: for each individual firm there is a Lagrangian $L_i$ with expected turnover $\bar{D}_i$ as constraint. Individual profit maximization takes account of this constraint. The outcome of the model is also subject to the relevant supply functions for factor services and demand functions for goods. Market conditions must also be taken account of. The corresponding Lagrangian is given in the following equ.(67) where it is assumed that market supply of product $Q$ is made up of a number of individual supplies $Q_i$ where $n$, the number of individual suppliers, can be either small or large or in between ($1 \leq n < \infty$).

The first constraint identifies the expected turnover, the second the product demand conditions, the third the labour supply conditions ($\omega$= here not relative wage but a shift parameter), the fourth the capital services conditions ($\rho$ = shift parameter), the last constraint is the production function. (It should be noted that in equ.(67) the symbols $\lambda_1$ to $\lambda_5$ are Lagrange multipliers and not – as in other places of this essay – measures of the share of wages.)

\[ L_i = pQ_i - wN_i - rK_i + \lambda_1(D_i - p \cdot Q_i) \]
\[ + \lambda_2(p - p(Q(Q_i))) \]
\[ + \lambda_3(w - w(N_i; \omega)) \]
\[ + \lambda_4(r - r(K_i; \rho)) \]
\[ + \lambda_5(Q_i - Q_i(N_i, K_i)) \]

The first-order maximum conditions generate a fairly conventional looking set of equilibrium conditions for such a firm given by equ.(68) to (70) under the assumption of a Cobb-Douglas function as given by equ.(68b) where the elasticities of production of labour ($\alpha$) and of capital ($\beta$) are assumed to be constant and where the relative prices are written in terms of wages as $p/w$ and $r/w$ in keeping with our accounting in wage units.

\[ \frac{p}{w} = \frac{1 + \nu_i}{1 + \pi_i} \cdot \frac{1}{\alpha} \frac{N_i}{Q_i} \quad (a) \]
\[ \frac{r}{w} = \frac{1 + \nu_i}{1 + \gamma_i} \cdot \frac{\beta}{\alpha} \frac{N_i}{K_i} \quad (a) \]
\[ Q_i = N_i^\alpha K_i^\beta \quad (b) \]
\[ \frac{r}{w} = \frac{r(K_i; \rho)}{w(N_i; \omega)} \quad (b) \]
\[ D_{wi} = \frac{p}{w} Q_i \quad (70) \]

The other parameters in this system of equations describe market conditions which are expressly assumed by Keynes to be given and to be unchanging. They are comparative-static results which can be generated once effective demand is explicitly identified as a given constraint and hence as exogenous variable and which are reproduced below in table 3. They do not appear in Koenig’s microfoundations. Also, Koenig (1980) does not go into a debate of the proportionality of labour with regard to effective demand which is, of course, the corollary of “the Footnote” which we discussed extensively above.
defined as:

\[ \pi_i \equiv E_Q^i \cdot \epsilon_{Q_i} \quad -1 < \pi_i < 0 \]

\[ \epsilon_{Q_i} \equiv \frac{Q_i}{Q} \frac{dQ}{dQ_i} \]

\[ \nu_i \equiv E_N^i \cdot \epsilon_{N_i} \quad \nu_i > 0 \]

\[ \epsilon_{N_i} \equiv \frac{N_i}{N} \frac{dN}{dN_i} \]

\[ \gamma_i \equiv E_K^i \cdot \epsilon_{K_i} \quad \gamma_i > 0 \]

\[ \epsilon_{K_i} \equiv \frac{K_i}{K} \frac{dK}{dK_i} \]

Thus the market parameters are composed of two elasticities: the “\(E\)”-elasticities represent the goods demand and the factor supply conditions as they appear “objectively” on the respective markets. The “\(\epsilon\)”-elasticities, however, are “subjective” in that they pertain to the individual firm, its market position as expressed by the respective market share \(Q_i/Q\) etc. and by its subjective estimate of the reaction of the competitors as expressed by the derivatives \(dQ/dQ_i\) etc. Normally these derivatives should have a unit value since the derivative of \(Q = Q_1 + \ldots + Q_i + \ldots + Q_n\) with regard to \(Q_i\) normally should be one. But if competitors withdraw from the market because of the action of the firm under consideration, the derivative would be smaller. If competitors follow suit with the firm and also change their respective quantities, then the derivative would be larger than unity.

The decomposition of the “degree of competition”-parameter into two components has a number of taxonomic advantages. One advantage is that we can relate the components to some debates about the character of competition (Hayes, 2008). Thus, the \(E\)-elasticity, relating to the product of a particular industry represents aspects of “competition between industries” (ibid.p.280) (but it also represents consumer’s preferences). The \(\epsilon\)-elasticities represent “competition within industries” (ibid.p.280). A similar – but not quite the same – distinction was made by Torr (1984, p.936) who, in discussing the microfoundations of Keynes’ Z-D-analysis, distinguished between a “PGA (price guesswork assumption)” – this corresponds to our \(E\)-elasticities – on the one hand and a “IGA (interfirm guesswork assumption)” – this corresponds to our \(\epsilon\)-elasticities – on the other hand. The latter takes account of individual firms’ estimate of reactions of other firms to the individual firms own initial action. Thus our algebraic decomposition covers a number of past points of discussion in Keynesian literature and it preserves these points as specific assumptions about respective magnitudes of parameters so that these points can be introduced in comparative-static and possibly in empirical analysis.

A further advantage of our decomposition is that the identification of the “subjective” \(\epsilon\)-elasticity covers the long debate about the possibility that a firm perceives a “kinked” demand curve, depending on the reactions of its competitors. This is, however, only relevant for oligopolistic conditions. If the market share of the firm is minimal, then \(Q_i/Q \to 0\) and the respective elasticity as defined in equs. (71) to (73) also will be zero. Thus the assumption of perfect competition and atomistic small market shares means a great simplification for the formulation of such models.

But the simplification of assuming perfect competition in the just mentioned sense is not tolerable when specific market conditions might have a significant influence on
the important structural equations describing an economy. This will be seen when we take equ. (68a) and formulate the respective firm’s $Z_{wk}$-curve, obtaining:

$$Z_{wi} = \frac{p}{w}Q_i = \frac{1 + \nu_i}{\alpha + \pi_i}N_i = \frac{1}{\lambda_i}N_i$$

(74)

The slope of the $Z_{wi}$-curve will still be given by the inverse of the share of wages in the value of output, as the last term shows and as Keynes’ Footnote stated. Alternatively it can be said that for any given expectation of turnover in wage units, employment $N_i$ will be the larger the smaller the numeric value of the fraction to the left of $N_i$, i.e. the smaller the value of $\mu_i$ as defined above in equ. (29), p. 49.

Thus, for high employment we should have a large elasticity of production $\alpha$, a large value of $1 + \pi$ and/or a low value of $1 + \nu$. Since normally $\pi$ is negative as stated above, the largest value of $1 + \pi$ will be when $\pi$ is zero. That is the case of atomistic competition on the product market. If the paradigmatic firm operated also under atomistic competition as demander on the labour market, then its $\nu$ would also be zero and we would have the desired low value for $1 + \nu$ as well. But what can be said for the case that the firm does not operate on the labour market as atomistic unit and if it has a sizeable weight as demander on the labour market? In this case the $\epsilon$-part of $\nu$ has a considerable numerical value. The desired low value of $\nu$ could be obtained, however, if the $E$-part were low. In other words, the less wages react to changes in employment demand, the better it is for employment under non-atomistic labour market conditions. Put differently, we may conclude: under non-atomistic labour market conditions wage rigidity – and not flexibility – is excellent for a high level of employment!

Money wage changes are, of course, a rather complex matter to which Keynes devoted an entire chapter (Ch 19). Although our arguments here do not quite mirror the ones which Kenyes (1936, p.265) used, it might be interesting that he argued:

...it would be much better that wages should be rigidly fixed and deemed incapable of material changes, than that depressions should be accompanied by a gradual downward tendency of money-wages, a further moderate wage reduction being expected to signalise each increase of; say, 1 per cent in the amount of unemployment. ...It follows that with the actual practices and institutions of the contemporary world it is more expedient to aim at a rigid money-wage policy than at a flexible policy responding by easy stages to changes in the amount of unemployment;

There is an analytical difference, however, between the context of that passage from the GT and the present one: here we argue on a microeconomic basis whereas Keynes in that context had in mind the macroeconomic effects on the real value of money. In spite of this difference, we note that the present model is in agreement with some of the major conclusions of the GT in this field.

But it is not only labour demand which is endogenous in the model of equs. (68) to (70). Those five equations can be solved for the five endogenous variables given...
Table 3. A Keynesian microfoundation of effective demand analysis

<table>
<thead>
<tr>
<th>endog.</th>
<th>$\hat{D}_{wk}$</th>
<th>$\hat{\alpha}$</th>
<th>$\hat{\beta}$</th>
<th>$1 + \pi$</th>
<th>$1 + \nu$</th>
<th>$1 + \gamma$</th>
<th>$\hat{\omega}$</th>
<th>$\hat{\rho}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\hat{Q}_k$</td>
<td>$\alpha + \beta \frac{1 + \nu}{1 + \gamma} + E_{Qi}^\alpha$</td>
<td>$\alpha + \beta \frac{1 + \nu}{1 + \gamma} + E_{Qi}^\beta$</td>
<td>$\alpha + \beta \frac{1 + \nu}{1 + \gamma} - \alpha - \frac{\beta \nu}{1 + \gamma}$</td>
<td>$-\frac{\beta \nu}{1 + \gamma}$</td>
<td>$\frac{\beta}{1 + \gamma} E^\nu$</td>
<td>$\frac{\beta}{1 + \gamma} E^\nu$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\hat{N}_k$</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>-1</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$\hat{K}_k$</td>
<td>$\frac{1 + \nu}{1 + \gamma}$</td>
<td>$\frac{1 + \nu}{1 + \gamma}$</td>
<td>$\frac{1 + \nu}{1 + \gamma}$</td>
<td>$\frac{1 + \nu}{1 + \gamma}$</td>
<td>$\frac{1 + \nu}{1 + \gamma}$</td>
<td>$\frac{1 + \nu}{1 + \gamma}$</td>
<td>$\frac{1 + \nu}{1 + \gamma}$</td>
<td></td>
</tr>
<tr>
<td>$\hat{P}^w$</td>
<td>$1 - \alpha - \beta \frac{1 + \nu}{1 + \gamma} + E_{Qi}^\alpha$</td>
<td>$-\frac{\alpha - \beta \nu}{1 + \gamma}$</td>
<td>$\frac{\beta}{1 + \gamma}$</td>
<td>$\frac{\beta}{1 + \gamma}$</td>
<td>$-\frac{\beta}{1 + \gamma}$</td>
<td>$\frac{\beta}{1 + \gamma}$</td>
<td>$\frac{\beta}{1 + \gamma}$</td>
<td></td>
</tr>
<tr>
<td>$\hat{F}^w$</td>
<td>$\frac{1}{1 + \gamma}$</td>
<td>$\frac{1}{1 + \gamma}$</td>
<td>$\frac{1}{1 + \gamma}$</td>
<td>$\frac{1}{1 + \gamma}$</td>
<td>$\frac{1}{1 + \gamma}$</td>
<td>$\frac{1}{1 + \gamma}$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

in the first column of table 3. Their “hat” symbol ($\hat{}$) stands for relative change of the respective variable so that $\hat{Q} = dQ/Q$ etc. The analogous convention holds for the exogenous variables listed in the horizontal row of the table. The elements of the other fields of table 3 then represent comparative static elasticities which show how the endogenous variables react to changes in the exogenous variables. The most important exogenous variable is, of course, in the present context the expected turnover as given by $D_{wk}$. We see now that employment will react proportionally to changes in $D_{wk}$ which here is “effective demand” because it is met by the firm’s supply. Thus we confirm Keynes’ postulate of a linear $Z_w$-curve because a linear curve from the origin generates a unit elasticity of response to changes in the independent variable.

It should be noted that our model does not rely on the assumption that labour is the only variable factor. Quite to the contrary: this model shows that along with an additional employment of labour, employment of capital services will also increase when expected turnover increases. Whether the input of capital services does vary or not depends on the cost of such additional services. This is, of course, quite

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37 This aspect of our model is in partial agreement with [Hayes 2006, p.87, emphasis added]: “...individual entrepreneurs may vary their individual capital equipment as well as their employment of labour and other factors in deciding to vary their output ...”. The question is, however, what are “other factors” in this quote? Is it capital services? Then we have the interesting – and realistic – case that along with instances of involuntary unemployment of labour (stock of workers) we could well have parallel unemployment of the existing stock of capital equipment due to a deficient demand for capital services by firm managers. But the question of unemployed capital equipment is rarely addressed in Keynesian literature. Neither is there a clear distinction between the stock of capital equipment and the usage of the flow of capital services. But this distinction is of great significance for the theory of investment which, however, is not covered here. Nevertheless, let us be clear that the allocation of capital equipment is a question of portfolio composition of wealth owners while the question of the demand for capital services (paradigm: leasing contracts)
plausible: if parameter $\gamma \to \infty$, i.e. if this parameter describing the elasticity of the price of the services of this factor is exorbitantly high, then the fraction in the $K_k-D_wk$-cell will tend towards zero. This means that the elasticity of the reaction of $K_k$ with regard to changes in effective demand would be zero. If, however, the elasticity in the $K_k-D_wk$-cell, i.e. the elasticity of capital services input is positive, it will be more so, the larger is $\nu$, i.e. the more flexible the money wage appears to be. In economic interpretation this means: the less rigid the wages, the more the firm will fear wage rises and the more it will substitute additional labour by additional capital services. Labour would still increase proportionally, since none of the parameters of the $K_k$-row enter the $N_k$-row. But the factor of proportionality will be lower the less rigid the wages are – a fact which we discussed already in connection with equ. (74).

We see now from the $Q$-row that output will increase either due to increases in the elasticity of output with respect to labour or also due to increases in the elasticity of output of capital services.

The model generates an interesting result with regard to changes in real wages. Since Keynes claimed that he accepted the classical principle that any increase in employment would mean a decrease in real wages, the model should generate the statement that the reciprocal of the real wage $p/w$ as shown in the second row from below will increase when turnover and employment increase. This expectation is confirmed if conditions are so that no or little additional capital services are employed. If, however, the employment of additional capital services is quite elastic – whether that is the case depends on the constellation of parameters as shown in the $\hat{K}/\hat{D}_w$-field –, then it could well be that the relative price of goods falls so that the real wage does not fall but rises with increased employment. This was indeed quite an issue after the publication of the GT, as is now documented in the never editions of the GT by an “appendix 3”. The empirical results discussed there suggest that capital services are far more variable in the short run than is commonly assumed so that real wages could follow a non-orthodox pattern in the business cycle. Our model shows that this result can be easily incorporated in a microfoundation of Keynes’ effective demand doctrine along the present lines.

The next column of our table shows the reactions in the model when the prices of capital services become – or appear to be – more flexible so that $\hat{\gamma}$ increases. As far as reciprocal real wages – or: relative prices in general – are concerned, they might rise or fall, depending on further aspects of market conditions. If employment of capital services does not change because $\gamma \to \infty$ holds, then the relative (and absolute) price of capital will be constant.

If we want to turn from real wages to money wages we can go now to the two last columns of table (3). They let us discuss the effects of abrupt shifts of factor prices, namely of wages (shift parameter $\omega$) or of capital services (shift parameter $\rho$). Looking at employment $N$, we notice that none of these changes have any

\[ \text{is a question of factor demand by firm managers for the purpose of supplying the flow of current production.} \]
effect on employment. The reason is that effective demand in terms of wage units is here exogenous and given. As Keynes stressed in his discussion of changes in money wages (GT, ch.19), one must be clear whether one contemplates nominal wage changes when effective demand is variable or when it is given. It is in the latter context that we have the present discussion. One interesting effect in this context is that an upward shift in wages will lower the relative price of capital services as shown in the last element of the \( \omega \)-column. This will make capital input more attractive and therefore we will have more employment of capital, a drop in the relative price of goods (\( p/w \)) and higher output as will appear in the first element of the \( \omega \)-column. Thus, if we could count on constant effective demand, an upward shift of money wages should have a stimulating effect on output and a dampening effect on relative (!) prices – if additional capital services are readily available. Absolute prices might move up, however, in response to an upward shift of money wages and Keynes showed that this could mean a decrease in the real value of money supply, hence an increase in interest rates and thus effective demand might be affected negatively. But this is a matter quite different from the microeconomic one which is to be discussed here.

Notice that the last column shows that analogous shifts in the remuneration of capital services have effects opposite to the ones just discussed for changes in nominal wages – except with regard to employment which will not change at all. Therefore a discussion of this aspect of the model might appear to be not particularly interesting. But there is one aspect of the cost of capital services which warrants some further comment: We have not discussed Keynes “user cost” in the above. But user cost (rates) are in a way quite similar to the “normal” cost (rates) of hiring capital services. In order to incorporate them into the present context we change the previously formulated target function in equ. (67) so that this part now reads

\[
L_i = pQ_i - wN_i - r(\rho)K_i - vK_i \text{ etc.}
\]

where we have replaced \( r \) by \( r(\rho) \) since we just mentioned \( \rho \) as shift parameter which might affect the level of the remuneration rate of capital services. We also added \( v \) as a rate of depreciation due to the usage of capital – surely an important part of Keynes’ “user cost”. The rest of the Lagrangian is as before. What effect does now a change in user cost, thus defined as \( v \), have on employment and on other endogenous variables of the model? We will not go through the tedious comparative statics once more. Instead, we point to the fact that an increase in \( v \) is analogous to an increase in \( \rho \). We can therefore say by using this analogy: changes in user cost \( v \) will have no direct effect on employment (when effective demand is given). It will make capital services costlier, however, and lead to their reduced use and to a reduced output and higher relative price of produced goods. All of this is rather plausible, since increased user cost mean that the usage of capital will become more costly – but there will be no (direct) effect on employment, since \( N_k \) will be unaffected by any change in a shift parameter for wage rates or for remuneration rates for capital services. We therefore consider changes of user cost of being not of paramount interest in the
reconstruction of this aspect of Keynes’ theory, although Keynes did mention these costs explicitly.

As the further rows of table 3 show, the present model is rich in potential comparative static results. Concentrating on labour demand $\hat{N}$, we see that increases in the productivity of labour as expressed by ($\alpha$) have a positive effect on employment, since the value in the $\hat{N}, \hat{\alpha}$-cell of table 3 has the value “1”. Moving one cell to the right, we see that analogous changes with regard to capital services have no effect on employment. Moving one cell further to the right shows that an increase in competition on the goods market will make the negative $\pi$ smaller and thus will make $1 + \pi$ larger. This will have a positive effect on employment, since this cell has the value “1”. An increase in wage rigidity will make $\nu$ larger. Table 3 shows in the next cell that then employment will increase, since this element has value “1”. The last cell in the $\hat{N}$-row of this table shows that changes of the market conditions for capital services will have no effect at all on employment.

More comparative static results could be discussed, since table 3 has many more cells to comment upon. But the exercises so far should have shown to a sufficient extent that a model of a firm along “Keynes’ post-Pigovian” lines can generate a number of interesting results. It might be interesting to return in the context of this model to Keynes’ “Footnote” and to Keynes’ claim in that place that the $Z_w$-curve has a slope which is given by the inverse of the wage [share]. But this curve is a locus of equilibrium points which are each characterized by a slope with unit value. The latter claim was quoted as “part [1]” above, the former was quoted as “part [2]” above (see above page 15). We have established above already, that in the present context the Keynesian $Z_w$-curve is given by equ.(74). Hence the inverse of the share of wages is

$$\frac{pQ_i}{w\hat{N}_i} = \frac{Z_{wi}}{\hat{N}_i} = \frac{1 + \nu_i}{\alpha (1 + \pi_i)} \quad (76)$$

This is also the slope of the $Z_w$-curve and it is constant, thus giving again a linear curve, It starts at the origin of the $\hat{N}, Z_w$ co-ordinates. Thus “part [2]” of Keynes’ Footnote is valid in this model as it was before.

But if we look at “part [1]” of the Footnote the result is different. We saw via equ.(4) (p.19) that the claim of a unitary slope follows from the fact that in equilibrium the real wage is equal to the marginal product of labour. But that is the case only under perfect competition. In the present model we have parameters expressing the degree of competition and thus the marginal product of labour is not necessarily equal to the real wage. On the basis of equ.(67) we get now as profit maximizing condition:

$$\frac{\partial Q_i}{\partial \hat{N}_i} = \frac{w (1 + \nu_i)}{p (1 + \pi_i)} \quad (77)$$

Hence, after multiplying both sides with $p/w$ we get:

$$\frac{p}{w} \frac{\partial Q_i}{\partial \hat{N}_i} = \frac{p}{w} \frac{\hat{Z}_{wi}}{\hat{N}_i} = \frac{\partial Z_{wi}}{\partial \hat{N}_i} = \frac{1 + \nu_i}{1 + \pi_i} \geq 1 \quad (78)$$

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This result is interesting for two reasons: (1) it generalizes Keynes characterization of the supply function in wage units of part [1] of the Footnote for cases in which the assumption of perfect competition is not warranted, (2) The fact that Keynes was content with a unitary value of this derivative shows that Keynes had no reservation against the assumption of perfect competition in his models.

The interpretive use of this model of effective demand must not be limited to Keynes, however. It might also be interesting in connection with Keynes’ praise for Malthus which we quoted above in the text. Walter Eltis (2005, p.674), after noting this praise, writes

Malthus’s rhetoric unquestionably appears “Keynesian”, but those of us who have attempted to rediscover the underlying theory that lies behind his enormously persuasive statements of the crucial significance of effective demand have suggested a variety of theoretical approaches, but we have failed to agree on a single convincing re-statement.

To ask for a “single convincing restatement” of Malthus’s original “rhetoric” is an unrealistic and unfair requirement, however. Neither Adam Smith nor David Ricardo nor Jean Baptista Say nor John Maynard Keynes have found such a restatement. Let us rather ask: can the present Keynesian model of effective demand shed some light on passages from Robert Malthus which without this – possibly – “underlying theory” would pose “logical difficulty”? This question is suggested by the following passage from Eltis (2005, p.675):

...Malthus was in apparent logical difficulty wherever he insisted, as he frequently did, that a lack of effective demand could produce inadequate wages and inadequate profits at the same time. Profits might be insufficient to justify further investment, ...

Logical difficulties could be of two types: either “too simple” or “too complex”. Of the first type is the following observation: The quoted passage alludes to the value of output as given by the incomes account. Let there be wages income \( W = wN \) and non-wages factor income \( R = rK \) and \( \Pi \) “extra profits” so that we have

\[
(79) \quad Y = W + R + \Pi \quad \text{with} \quad \dot{Y} = \lambda \dot{W} + \kappa \dot{R} + \vartheta \dot{\Pi},
\]

where \( \lambda \) is labour’s share as before, \( \kappa \) is other factor’s share and \( \vartheta \) is extra profit’s share so that \( \lambda + \kappa + \vartheta = 1 \) and where the hat symbol signifies relative change, as before. It is now an extremely simple idea that if there is a positive change in \( Y \) due to an increase in effective demand, then on the right hand side there must also be positive changes. If there is deficient effective demand, the reverse is true. The decrease in income might then well result in a decreased wages bill as well as decreased “profits” from other factor income and decreased extra profits. This is almost a truism – a logical banality – but this observation begs the question how exactly this change spreads to the different categories of income and how all those changes would affect investment decisions. It could be that Eltis considered this latter problem as Malthus’ “logical difficulty”. In order to overcome this latter
type of “logical difficulty” one would need a detailed model which can represent the multitude of possible assumptions by a number of appropriate parameters and elasticities. The effective demand model which produced table 3 might be a good candidate for such an enquiry, but it would go beyond our present topic to enter a detailed discussion of Malthusian tenets in the present context.

References


URL http://findarticles.com/p/articles/mi_qa3620/is_199410/ai_n8732776&tag=rel.res1

Preliminary version – January 25, 2009


Harrod, Roy F. (1937): “Mr. Keynes and Traditional Theory”, in: *Econometrica*.


Howson, Susan and Winch, Donald (1977): The Economic Advisory Council 1930-1939 - A study in economic advice during depression and recovery (Cambridge, UK and others: Cambridge University Press).


Keynes, John Maynard and Henderson, Hubert (1929): Can Lloyd George Do It?, quoted from JMK IX, pp.86-125.


URL http://www.jstor.org/stable/3501261


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